Open Economy Macro

Risk Premium

Jouko Vimunen

AaltoSB/Dept 0f Economics

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1. Intro

- We have referred to risk premia associated with international speculation on various occasions during this course, without making any attempt to analyze the factors that determine their size
- Now we will try to fill in the gap
- Different tools: microeconomics, ie. constrained optimization tools
- Start by listing a number of basic assumptions that allow us to focus on the issue at hand

Assumptions

- Representative agent ('speculator')
- Perfect capital market, no transaction costs, in particular no margin requirements on forward trades
- Only two periods are relevant for decisions: 'present' and 'future' (period 0 and 1)
- Speculator is an expected utility maximizer; utility depends, for simplicity, on future consumption C_1 (ie. present consumption is zero), utility function has standard properties
- No inflation
- Wealth W_0 (fixed 'endowment'); the only way to shift resources to support future consumption is through speculation on forward contracts (ie. no other assets available)

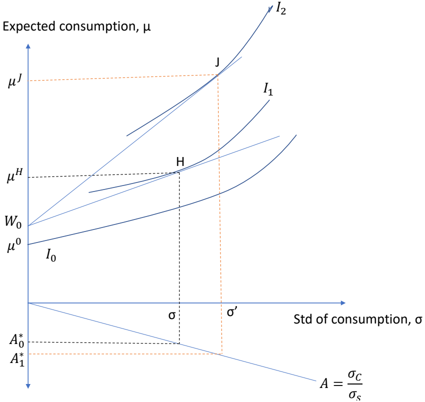
Assumptions cntd

- Every euro spent on buying dollars forward costs (ie. reduces consumable rerources by) €F_0, which is the current price of a dollar delivered in period 1
- On the other hand, consumption in period 1 increases by €S_1, the spot price of dollars in period 1, when the currency is delivered and is available for exchange back into euros on the spot market
- For each euro of forward dollars, future consumption benefits/suffers by €$S_1 - F_0$; if the investor spends €A on forward purchases consumption will increase/fall by €$(S_0 - F_1) A$
- Total consumption: $C_1 = W_0 + (S_1 - F_0) A \longrightarrow$Consumption is uncertain (unless $A = 0$)!

2. A simpe model of risk premium: mean-variance analysis

- Use indfifference maps in the mean-std space:

Figure : Indifference curves in mean-std space



Indifference curves

- Standard micro: two desirable goods $x, y \rightarrow$ indifference curves slope downwards in the (x, y) space
- Now: only one desirable characteristic - expected consumption μ_C - and one undesirable characteristic - consumption risk as measured by the std of consumption σ_C
- Start from the riskless choice μ_0 and move to the right; this involves an increase in consumption risk, which needs to be compensated (risk aversion!) for the agent to choose to bear the risk, ie. expected consumption has to increase $\rightarrow$ indifference curves have to slope upwards, more steeply so, once the amount of consumption risk increases
- Also: higher risk aversion means that the slope of the I-curve will be larger; the intuition is that as risk aversion increases, higher expected consumption is required for a unit increase in consumption risk

Indifference curves cntd 1

- As with standard indifference curves, here the indifference curves are convex: compensation per unit of risk increases with the quantity of risk already taken
- Utility increases once we move up to higher indifference curves: hence $I_0 < I_1 < I_2$
- Investor seeks the highest possible indifference curve
- *Constraint: speculative opportunity line* The speculator faces a constraint on his/her choice and this constraint is derived from total consumption

$$\mathbb{E}_0 C_1 = \mu_C = W_0 + (\mathbb{E}_0 S_1 - F_0) A \tag{1}$$

- Std of consumption σ_C: compute the variance of consumption

$$\begin{aligned} \sigma_C^2 = \mathbb{E}_0 \left[C_1 - \mu_C \right]^2 = \mathbb{E}_0 \left[A \left(S_1 - \mathbb{E}_0 S_1 \right) \right]^2 \\ = A^2 \mathbb{E}_0 \left[S_1 - \mathbb{E}_0 S_1 \right]^2 = A^2 \sigma_S^2 \end{aligned} \tag{2}$$

Indifference curves cntd 2

- Since the std is the positive square root of the variance, we have

$$\sigma_C = |A|\,\sigma_S \longleftrightarrow A = \pm\frac{\sigma_C}{\sigma_S} \tag{3}$$

- Sign of A is the same as that of the risk premium, otherwise (cf. eq. (1)) we would have a situation where e.g. the risk premium is negative $(\mathbb{E}_0 S_1 - F_0) < 0$ and with a positive A expected value of consumption of a risk averse consumer would be lower than his/her initial wealth for a position involving risk - totally unacceptable; assume, for simplicity, that A is positive (ie. $(\mathbb{E}_0 S_1 - F_0) > 0$)
- Substitute (3) into (1)

$$\begin{aligned} \mu_C &= \tfrac{\sigma_C}{\sigma_S}\left(\mathbb{E}_0 S_1 - F_0\right) + W_0 \\ &= \left(\tfrac{\mathbb{E}_0 S_1 - F_0}{\sigma_S}\right)\sigma_C + W_0 = \rho_{SR}\sigma_C + W_0 \end{aligned} \tag{4}$$

- ρ_{SR}= Sharpe ratio

Equilibrium

- If the Sharpe ratio is ρ^0_{SR} our speculator finds an equilibrium at $H = (\mu^H, \sigma)$ with the optimal forward position A^*_0
- Higher Sharpe ratio, e.g. $\rho^1_{SR} > \rho^0_{SR}$ gives a speculator equilibrium at $J = (\mu^J, \sigma')$; optimal forward position is now A^*_1
- Note that we have drawn the new equilibrium J to the north-east of the initial equilibrium H; it is possible that our speculator's preferences take him/her to the north-west of the original equilibrium so that he/she could enjoy a major increase in expected consumption with a reduction in consumption risk; however, these preferences would make the forward market completely unstable
- In the following we will analyze a more general model of the risk premium

3. A more general model of the risk premium

- The previous model is very restrictive, although the simplifications underlying it makes it possible to use a diagrammatic framework to determine the speculator's equilibrium
- We will now generalize the analysis: more specifically the speculator's program is

$$\max_{A} \mathbb{E}_0 \left[U \left(C_1 \right) \right] \tag{5}$$

such that

$$C_1 = \left(S_1 - F_0 \right) A + W_0 \tag{6}$$

- Note that previously we assumed that the speculator is concerned about maximizing expected consumption; in the current context this would mean a risk neutral investor, but if the *typical* investor is risk neutral, the risk premium would be arbitraged down to zero!
- Optimum requires: $\partial \mathbb{E}_0 \left[U \left(C_1 \right) / \partial A \right] = 0$ or

More general model cntd 1

- $$\mathbb{E}_0\left[\frac{\partial U}{\partial C_1}\frac{\partial C_1}{\partial A}\right] = \mathbb{E}_0\left[U'(C_1)(S_1 - F_0)\right] = 0 \tag{7}$$

  that is

  $$F_0\mathbb{E}_0\left[U'(C_1)\right] = \mathbb{E}_0\left[U'(C_1)S_1\right] \tag{8}$$

- Interpretation: l.h.s. marginal opportunity cost (in terms of expected utility) per euro speculated in the forward market: the price of a forward dollar times the expected marginal utility; r.h.s. expected marginal benefit in utility terms
- Hence the familiar result in constrained choice: marginal cost is equal to marginal benefit
- Rewrite (8) as

  $$F_0 = \frac{\mathbb{E}_0\left[U'(C_1)S_1\right]}{\mathbb{E}_0\left[U'(C_1)\right]} \tag{9}$$

  and use the covariance rule
  $cov(X, Y) = \mathbb{E}(XY) - \mathbb{E}(X)\mathbb{E}(Y)$

More general model cntd 2

$$F_0 = \mathbb{E}_0(S_1) + \frac{cov(U'(C_1), S_1)}{\mathbb{E}_0[U'(C_1)]} \tag{10}$$

so that the (negative of the) ratio on the r.h.s. is the risk premium

- We have already encountered formulas similar to (10) and interpreted them from the point of view of insurance or hedging: what matters is the covariation between marginal utility of consumption and the spot rate, not the variance of the spot rate as such
- If the covariance term is negative, so that period 1 euro is strong/weak (S_1 low/high) when marginal utility is high/low (C_1 low/high), the forward price of dollars is lower than expected future spot price (positive risk premium) and investors need to be compensated for this additional risk
- Note: if the marginal utility is constant - risk neutrality - then the covariance term is zero; same conclusion holds, if the spot rate is non-stochastic

The simple model revisited: a mean-variance formulation

- Previous analysis is very general as it imposes no form for the utility function
- However, for the purpose of e.g. empirical analysis this is awkward as is often the case with general formulations
- So, assume the following quadratice utility function

$$U(C_1) = \alpha\mu_C - \frac{\gamma}{2}\sigma_C^2 \qquad \alpha, \gamma > 0 \tag{11}$$

- Note: $C_1 < \frac{\alpha}{\gamma}$ for positive marginal utility
- Solve for the forward rate

$$F_0 = \mathbb{E}_0(S_1) - \frac{\gamma A \sigma_S^2}{\alpha} \tag{12}$$

so that the risk premium depends (on top of α) on the risk aversion parameter γ, volatility of the spot rate σ_S^2 and the forward position

Simple model cntd 1

- Instead of assuming a particular functional form, we could impose restrictions on the distribution of the future spot rate
- If this distribution is approximately normal, it can be shown that in this case

$$F_0 = \mathbb{E}_0(S_1) - \Theta \frac{A}{C_1} \sigma_S^2 \tag{13}$$

  where

$$\Theta = -C_1 \frac{\mathbb{E}_0\left[U''(C_1)\right]}{\mathbb{E}_0\left[U'(C_1)\right]} \tag{14}$$

  is the coefficient of relative risk aversion
- To derive (13), we need to utilize Rubinstein's (1976, "The valuation of uncertain income streams and pricing of options", The Bell Journal of Economics, vol 7(2), 407-425) result: for two normal random variables X, Y and at least once differentiable function g, then, subject to mild regularity conditions

Simple model cntd 2

- $$cov\left(X, g(Y)\right) = \mathbb{E}\left[g'\left(Y\right)\right] cov\left(X, Y\right) \tag{15}$$
- Now, choose $g\left(\bullet\right) = U'\left(\bullet\right)$ and $Y = C_1$, $X = S_1$, so that
  $$cov\left(U'\left(C_1\right), S_1\right) = \mathbb{E}_0\left[U''\left(C_1\right)\right] cov\left(C_1, S_1\right) \tag{16}$$
  and since period 1 consumption is a linear function of the spot rate, $C_1 = \left(S_1 - F_0\right)A + W_0$,$cov\left(C_1, S_1\right)$=$A^2\sigma_S^2$; (14) follows immediately
- Note: both quadratic utility and normality lead to formulation of demand in terms of mean and variance $\longrightarrow$ mean-variance approach

Risk premium and portfolio balance models

- Much of the literature on risk premium carries a slightly different orientation to the one above
- There is a relationship, however
- Instead of focusing on future consumption C_1, we could have concentrated on future wealth W_1 for consistency, so that the investor wants to maximize expected utility of future wealth given his/her initial wealth W_0
- In (13) we could then interpret A/C_1 as the proportion of wealth allocated to foreign assets
- In fact most of the literature on the determination of risk premia see it as a portfolio allocation problem
- It is now easy to see the connection between risk premium and the asset demands in the portfolio balance (and currency substition) model

see it

Portfolio balance cntd

- Solve (13) for the desired share allocated to foreign assets

$$\frac{A^*}{C_1} = \frac{A^*}{W_1} = \frac{1}{\Theta\sigma_S^2}\left[\mathbb{E}_0\left(S_1\right) - F_0\right] \tag{17}$$

- The term in the square brackets on the r.h.s. of (17) is the risk premium, the expected return of the asset; hence, given the restricted specification here, we have arrived at a formulation of the asset demand function used in the portfolio balance model
- In order to fully cover the portfolio balance model, we would need to deal with a multi-asset model, which, in turn, would force us to replace the variance of the single asset with a variance-covariance matrix of the asset (returns); we will not follow this route
- If the period 1 domestic price level is stochastic, it can be shown that the risk premium is

Empirical evidence

- $$(-)\frac{cov\left[\frac{U'(C_1)}{P_1}, S_1\right]}{\mathbb{E}_0\left[\frac{U'(C_1)}{P_1}\right]} \tag{18}$$
- Hence risk neutrality is no longer sufficient to drive the risk premium down to zero, if there is non-trivial interaction between the price level and spot rate
- Three questions about the empirics of the risk premium
  - is there a (non-zero) risk premium?
  - is it variable?
  - if variable, is it sufficiently large and volatile to explain a substantial part of the fluctuations observed in floating rates in recent years?
- Overall, the evidence seems to suggest risk premia are too small and not variable enough to explain exchange rate fluctuations