# ADVANCED INVESTMENT THEORY

Lecture 1: INTRODUCTION

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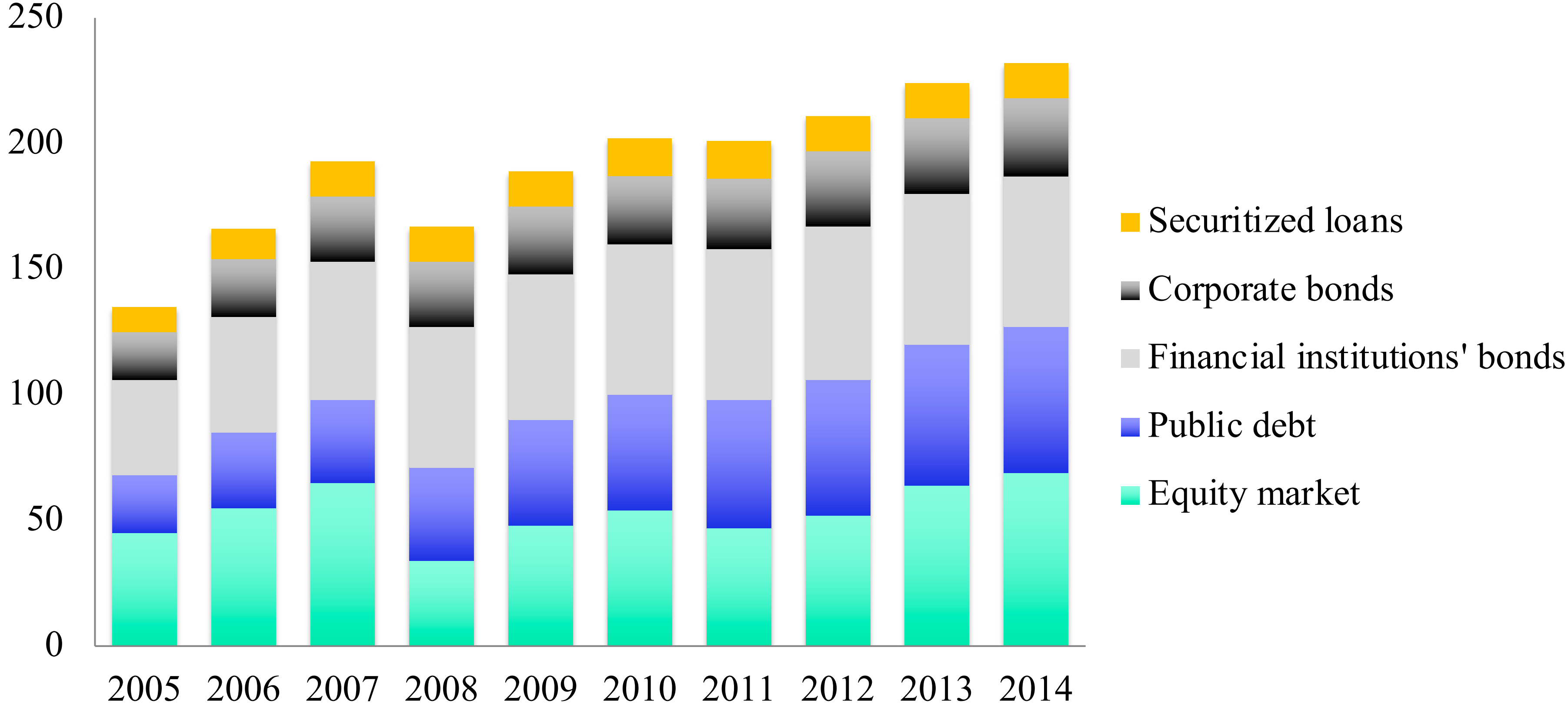
# SOME STATISTICS:
# World Financial Markets (trillion USD)




Source: Deutche Bank

## Global Derivative Markets

**Source:BIS**

## Notional amounts outstanding at year-end, in trillions of USD

| | 1998 | 2006 | 2014 |
|---|---|---|---|
| **OTC Instruments** | **80** | **418** | **691** |
| **Foreign exchange contracts** | **18** | **40** | **74** |
| Forwards and forex swaps | 12 | 20 | 35 |
| Currency swaps | 2,3 | 11 | 26 |
| Options | 3,7 | 10 | 13 |
| **Interest rate contracts** | **50** | **292** | **563** |
| Forward rate agreements | 5,8 | 19 | 92 |
| Interest rate swaps | 36 | 230 | 421 |
| Options | 8,0 | 43 | 49 |
| **Equity-linked contracts** | **1,5** | **7,5** | **6,9** |
| Forwards and swaps | 0,1 | 1,8 | 2,4 |
| Options | 1,3 | 5,7 | 4,5 |
| **Commodity contracts** | **0,4** | **7,1** | **2,2** |
| Gold | 0,2 | 0,6 | 0,3 |
| Other commodities | 0,2 | 6,5 | 1,9 |
| Forwards and swaps | 0,1 | 2,8 | 1,3 |
| Options | 0,1 | 3,7 | 0,6 |
| **Credit default swaps** | **-** | **29** | **19** |
| Single-name instruments | - | 18 | 11 |
| Multi-name instruments | - | 11 | 8,6 |
| of which index products | - | | 7,9 |
| **Unallocated/OTC** | **10** | **43** | **25** |

## Global Derivative Markets (Source BIS)
## Notional amounts outstanding at year-end, in trillions of USD

| | 1998 | 2006 | 2014 |
|---|---|---|---|
| **Exchange-traded instruments** | **14** | **69** | **75** |
| | | | |
| **Futures** | **8,4** | **26** | **27** |
| Interest rate | 8,0 | 24 | 25 |
| Currency | 0,0 | 0,2 | 0,2 |
| Equity index | 0,3 | 1 | 1,6 |
| **Options** | **5,6** | **44** | **38** |
| Interest rate | 4,6 | 38 | 31 |
| Currency | 0,0 | 0,1 | 0,1 |
| Equity index | 0,9 | 6 | 5,6 |

## Turnover of financial Derivatives traded on organized exchanges
## Notional amounts in trillions of US dollars

| | | | |
|---|---|---|---|
| **Total turnover** | 388 | 1806 | 1936 |
| Interest rate futures | 296 | 1169 | 1267 |
| Interest rate options | 56 | 446 | 334 |
| Currency futures | 2,6 | 17 | 29 |
| Currency options | 0,5 | 1,1 | 3 |
| Equity index futures | 18,9 | 74 | 155 |
| Equity index options | 14,3 | 99 | 148 |

# Part I

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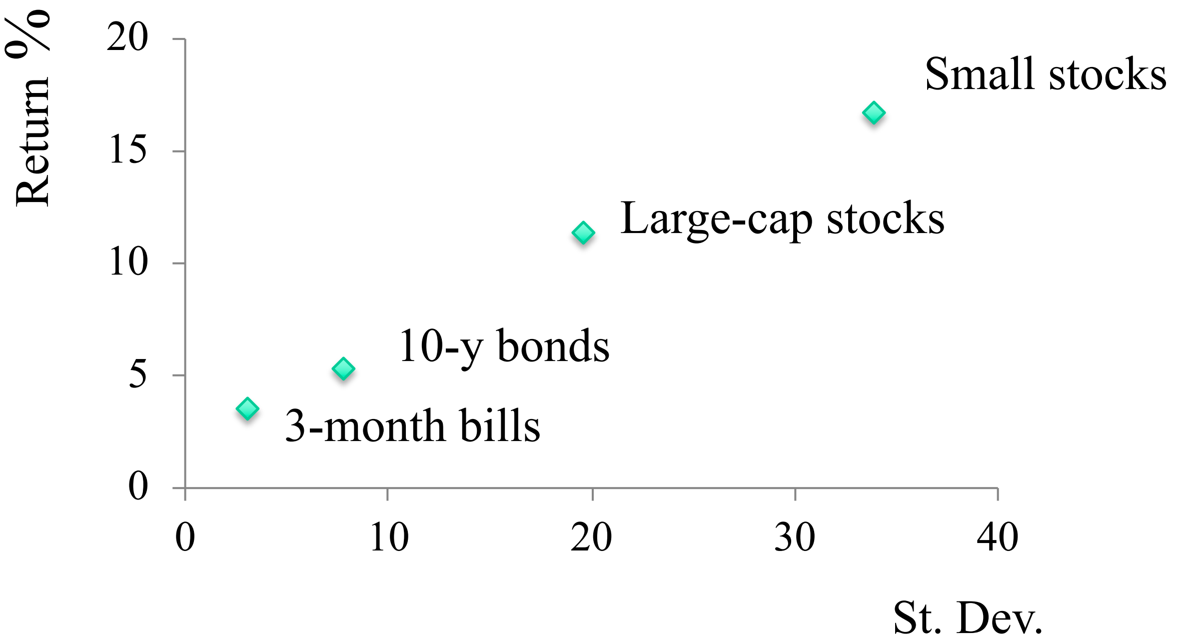
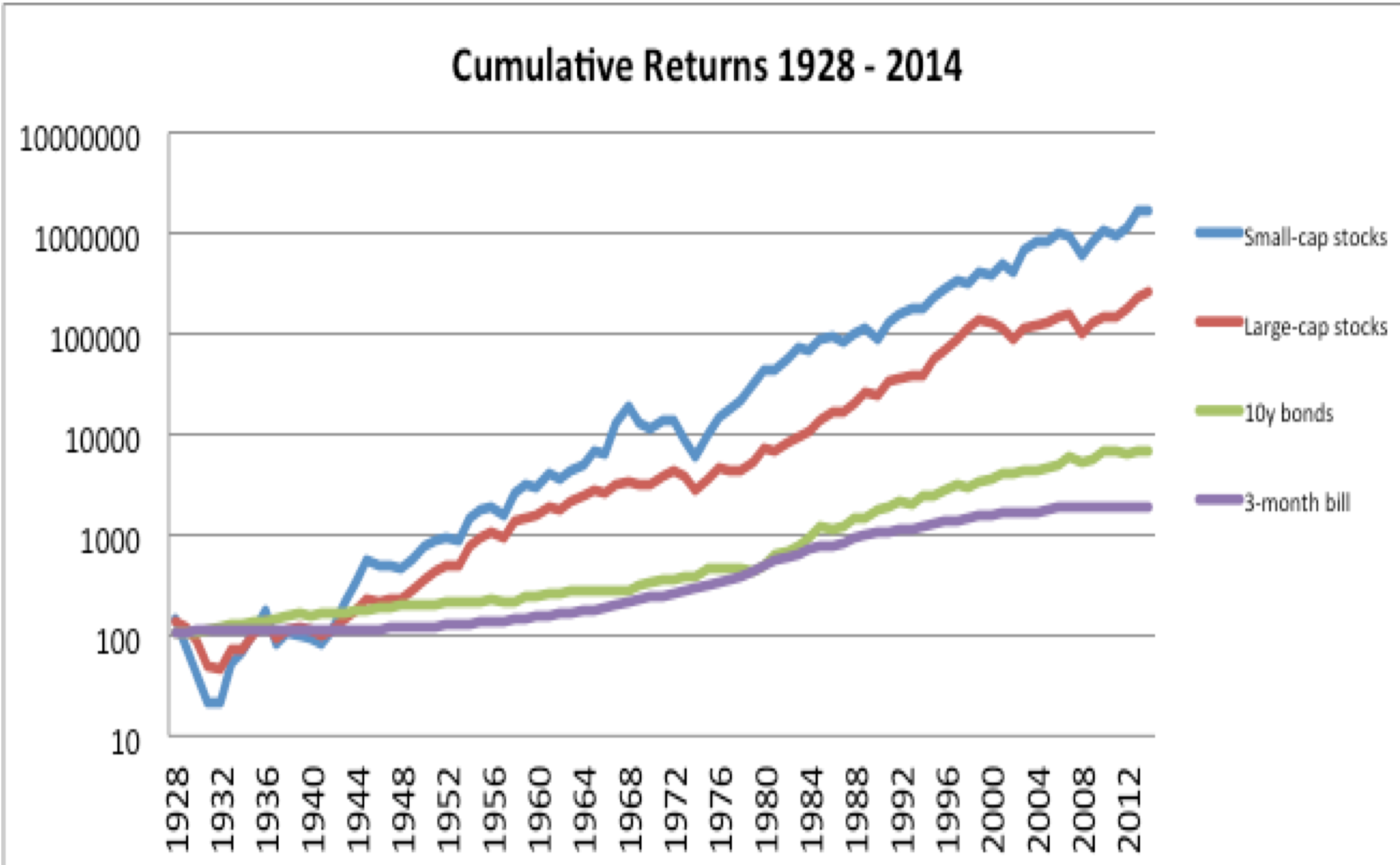
- Security returns history
- Portfolio theory and the price of risk
- Risk aversion and portfolio choice
- Computing portfolio characteristics
- Portfolios in the risk – expected return - diagram
- Benefits of globalization
- Summary

# Mean Returns of Different Asset Groups



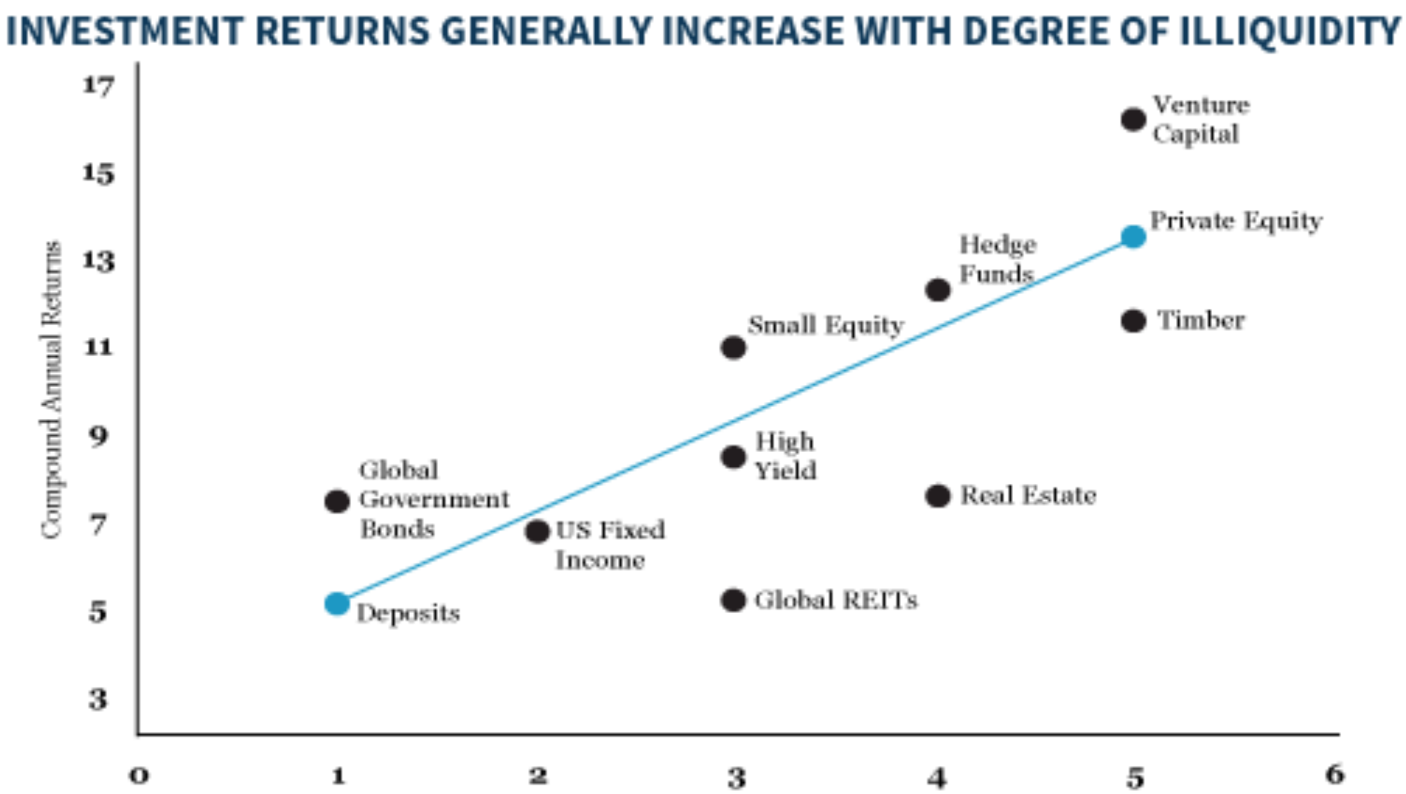

1928-2014

| | MEAN RETURN | STANDARD DEVIATION |
|---|---|---|
| SMALL STOCKS | 16.75 | 33.89 |
| LARGE STOCKS | 11.36 | 19.57 |
| BONDS 10Y | 5.28 | 7.8 |
| T-BILLS 3M | 3.53 | 3.1 |



Cumulative Returns 1928 - 2014
10000000
1000000
100000
10000
1000
100
10
1928
1932
1936
1940
1944
1948
1952
1956
1960
1964
1968
1972
1976
1980
1984
1988
1992
1996
2000
2004
2008
2012
Small-cap stocks
Large-cap stocks
10y bonds
3-month bill

# HIGH PAST RETURNS IN ALTERNATIVES LOOK ATTRACTIVE AND PROVIDE DIVERSIFICATION




Ilmanen, A. (2011). *Expected returns: An investor's guide to harvesting market rewards*. John Wiley & Sons.

# Computing Portfolio Characteristics

- Random variable $\widetilde{R}$: A list of possible realizations $R_{s=1}$, $R_{s=2}$, $R_{s=3}$ , …for different (mutually exclusive) states of nature $s$ = 1,2,3... with respective probabilities $p_{s=1}$, $p_{s=2}$, $p_{s=3}$ ...

- Expectation E(·) of a random variable:

$$E(\widetilde{R}) = p_{s=1}R_{s=1} + p_{s=2}R_{s=2} + p_{s=3}R_{s=3} + .... = \sum_s p_s R_s$$

- Variance and standard deviation of a random variable

$$Var(\widetilde{R}) = E\left\{ \left[\widetilde{R} - E(\widetilde{R})\right]^2 \right\} = \sum_s p_s \left[R_s - E(\widetilde{R})\right]^2 = \sigma^2$$

$$Std(\widetilde{R}) = \sqrt{Var(\widetilde{R})} = \sigma$$

# Computing Portfolio Characteristics

- Portfolio weights: Percentage of value invested in each asset;

$$x_1 + x_2 = 1$$

- Portfolio return: Weighted arithmetic average of individual asset returns;

$$\tilde{R}_p = x_1\tilde{R}_1 + x_2\tilde{R}_2$$

- Expected average portfolio return: Weighted average of expected asset returns

$$E(\tilde{R}_p) = x_1 E(\tilde{R}_1) + x_2 E(\tilde{R}_2)$$

# Computing Portfolio Characteristics

- Portfolio risk: Typically associated with the standard deviation of the portfolio return;

$$\sigma_p = \sqrt{E\left[\tilde{R}_p - E(\tilde{R}_p)\right]^2}$$

- Portfolio risk for two assets: It depends on the covariance between the two returns

$$\sigma_P = \sqrt{x_1^2\sigma_1^2 + (1-x_1)^2\sigma_2^2 + 2x_1(1-x_1)\,\mathrm{cov}(\tilde{R}_1, \tilde{R}_2)}$$

- Recall:

$$\mathrm{cov}(\tilde{R}_1, \tilde{R}_2) = E\left\{\left[\tilde{R}_1 - E(\tilde{R}_1)\right]\left[\tilde{R}_2 - E(\tilde{R}_2)\right]\right\}$$

$$\text{correlation} = \rho_{12} = \frac{\mathrm{cov}(\tilde{R}_1, \tilde{R}_2)}{\sigma_1\sigma_2}$$

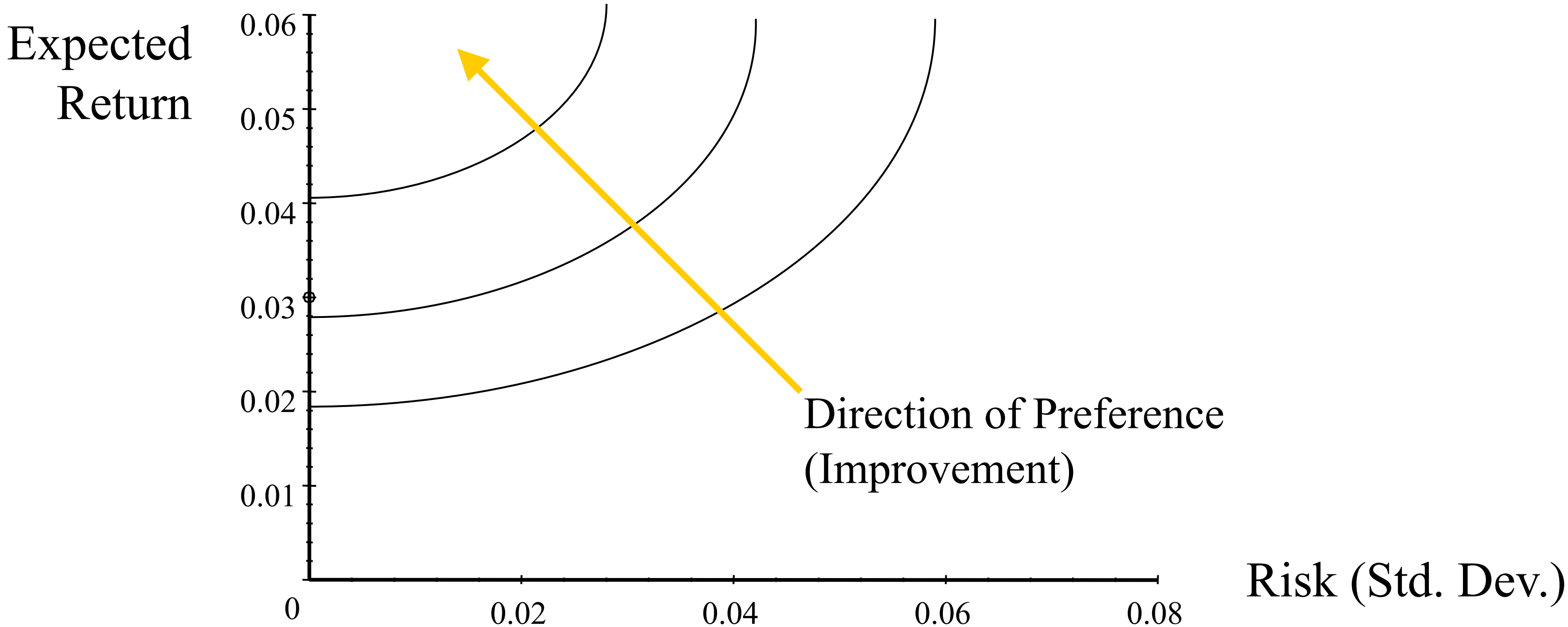
# Example

| **Economy** | **Prob.** | **Return stock 1, *$R_1$*** | **Return stock 2, *$R_2$*** | **Portfolio of both stocks $x_1=x_2=.5$** |
|---|---|---|---|---|
| **Good (s=1)** | 1/3 | 0.20 | 0.16 | 0.18 |
| **Fair (s=2)** | 1/3 | 0.14 | 0.10 | 0.12 |
| **Bad (s=3)** | 1/3 | 0.10 | 0.10 | 0.10 |
| | | | | |
| **Expected return, *E[R]*** | | 0.14667 | 0.12 | 0.13333 |
| **Variance, *VAR[R]*** | | 0.00253 | 0.0012 | 0.00173 |
| **Standard Deviation** | | 0.0503 | 0.0346 | 0.0416 |

# Risk - Expected Return - Diagram

- Two securities:
  - Case I: Combine one riskless and one risky asset
  - Case II: Combine two risky assets with same std. deviation, but different expected returns
  - Case III: Combine two risky assets with different std. deviations and different expected returns
- Many securities (Part II)

# Investor Preferences and Optimal Choice

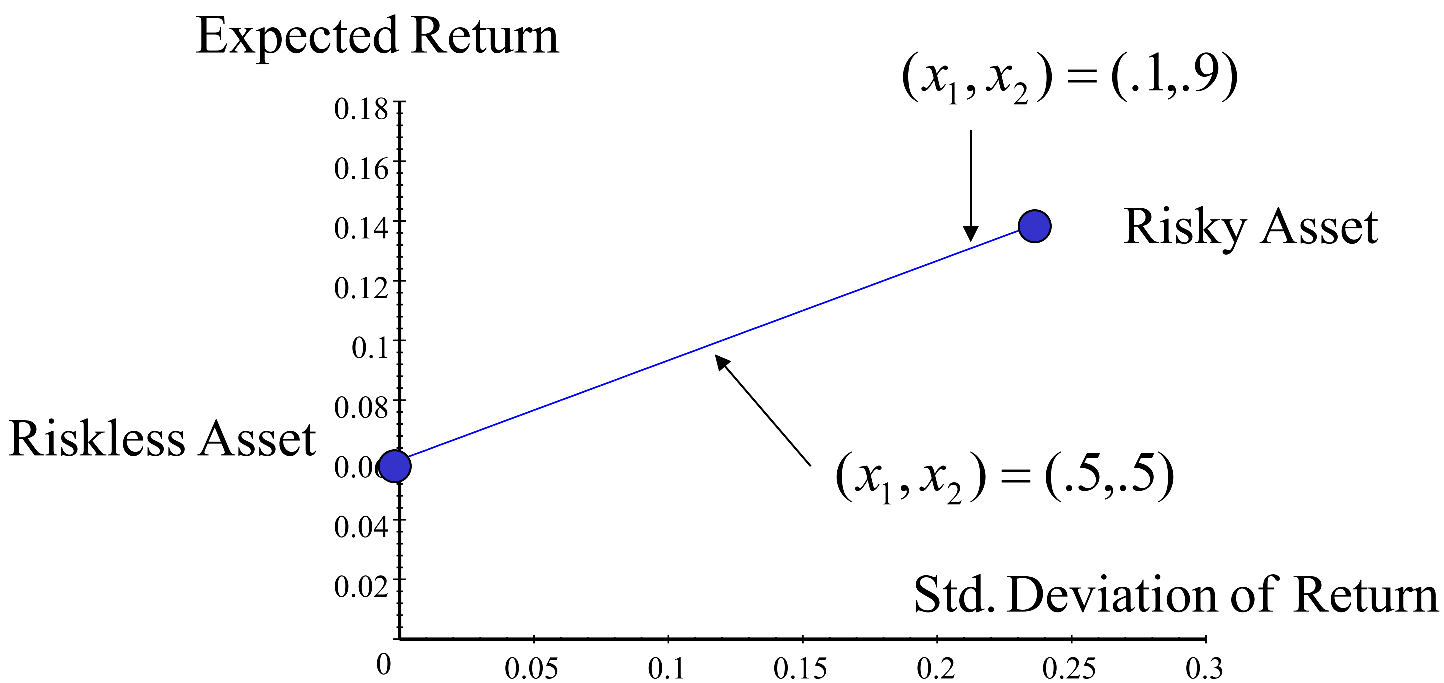



Investors prefer

- •Higher expected returns
- •Lower standard deviation of returns

Lines represent indifference curves (combinations of equal utility)

# Case I: Riskless and Risky Asset




- Variations in the portfolio weights $(x_1,x_2)$ move us along the line
- Covariance of riskless and risky asset is always zero
- Investors can choose optimal point (risk return trade-off)

# Combining Risky Assets

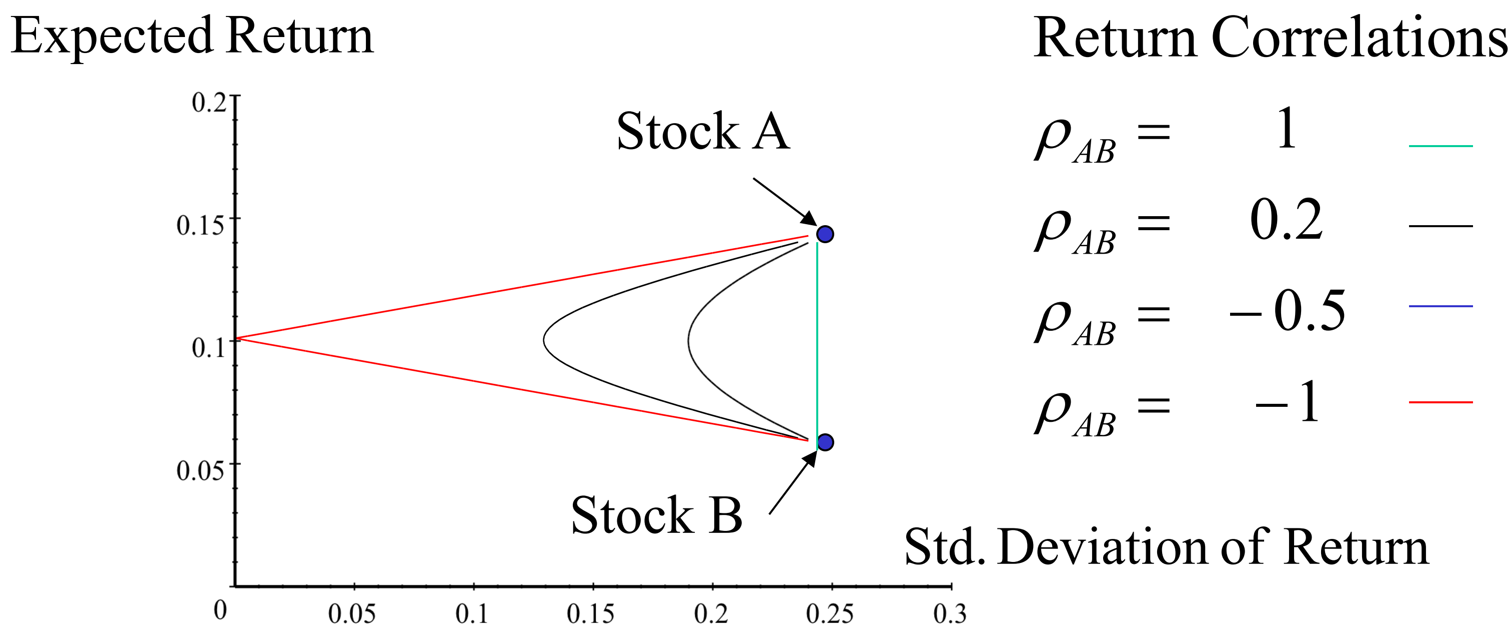
- Example: Two stocks A and B have the same individual risk of 24% return std. deviation; the expected return of stock A is 14% and that of stock B only 6%.
- Question: Would anyone ever hold stock B?
- Answer: Yes, positive amounts of stock B contribute to an overall lower portfolio return variance (as long as their return correlation is lower than one)
- This is the portfolio diversification "miracle"

# Combining Risky Assets

| Stock A<br>$E(R_A) = 14\%$<br>$\sigma_A = 24\%$<br>Portfolio Weight<br>$x_A$ | Stock B<br>$E(R_B) = 6\%$<br>$\sigma_B = 24\%$<br>Portfolio Weight<br>$x_B$ | Expected Return of Portfolio,<br>$E(R_p)$ | Std. Dev. of Portfolio (if correlation $\rho_{AB} = 0$),<br>$\sigma_p$ | Std. Dev. of Portfolio (if correlation $\rho_{AB} = 1$),<br>$\sigma_p$ |
|---:|---:|---:|---:|---:|
| 1.00 | 0.00 | 14% | 24% | 24% |
| 0.75 | 0.25 | 12% | 19% | 24% |
| 0.50 | 0.50 | 10% | 17% | 24% |
| 0.25 | 0.75 | 8% | 19% | 24% |
| 0.00 | 1.00 | 6% | 24% | 24% |

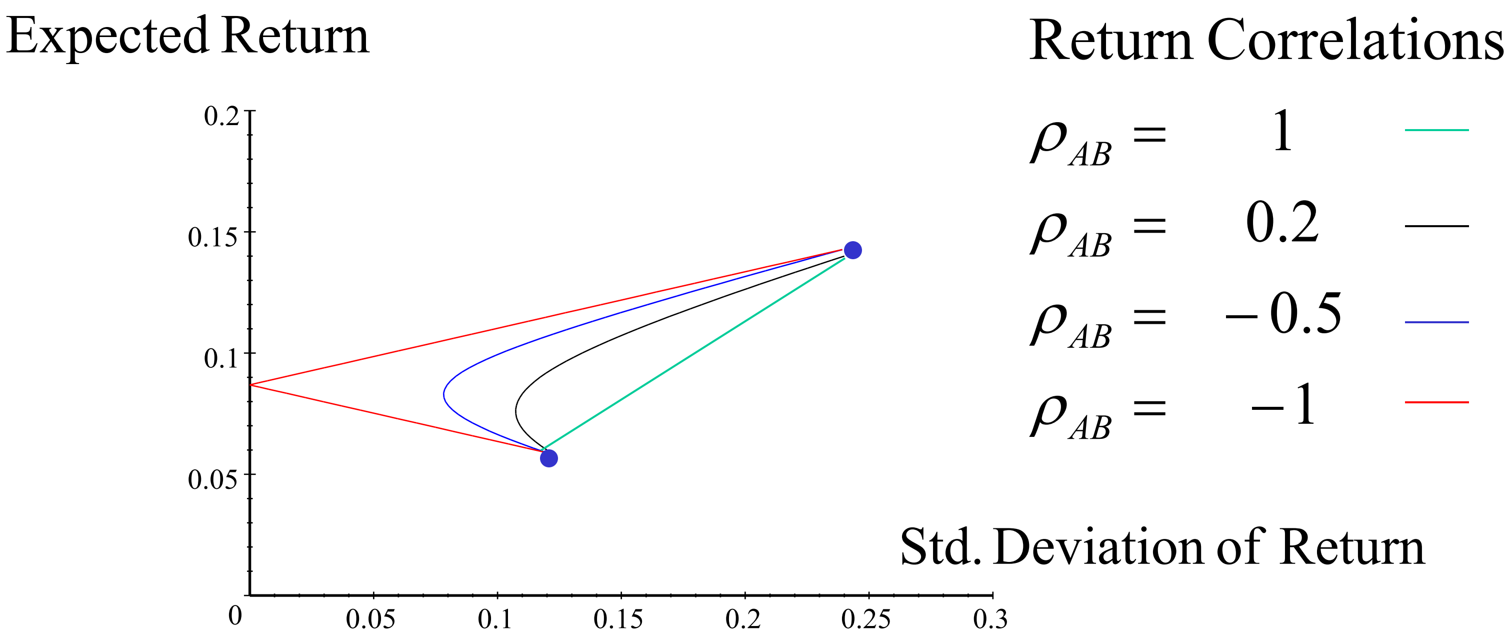
$$\sigma_p = \sqrt{(x_A)^2 \sigma_A{}^2 + (x_B)^2 \sigma_B{}^2 + 2 x_A x_B \sigma_A \sigma_B \rho_{AB}}$$

# Case II: Two Risky Assets (same Std Dev.)




- The risk return trade-off depends on the correlation of the two assets
- Negative return correlation between assets increases the diversification benefit

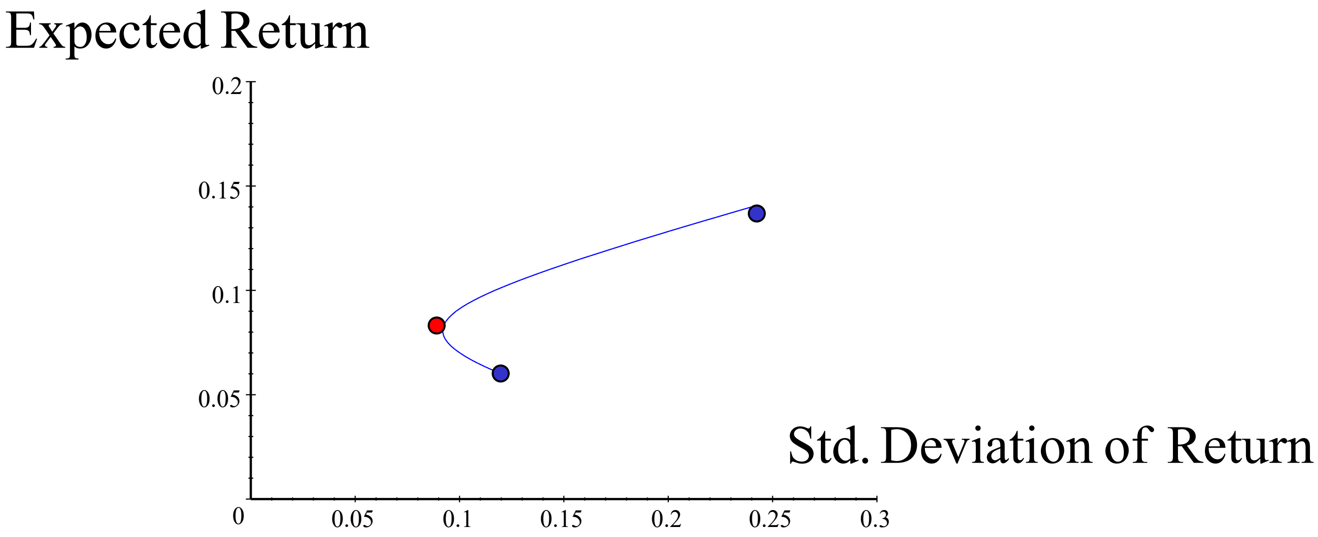
# Case III: Two Risky Assets (different Std. Dev.)



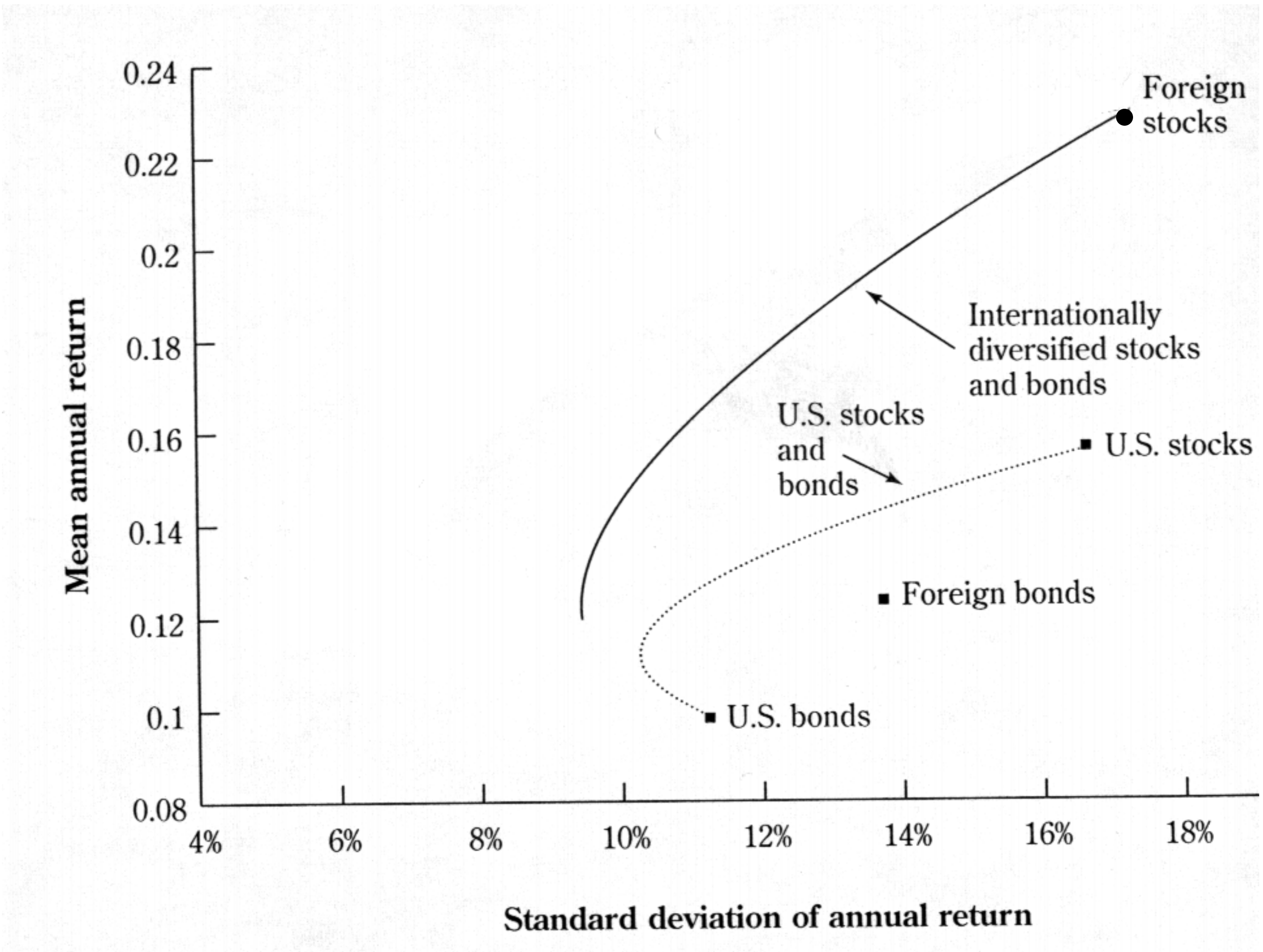

- Diversification benefits even with asset of higher std. deviation of returns
- In case of perfect negative correlation we can obtain a riskless portfolio

# Minimum Variance Portfolio

- <u>Definition:</u> The combination of portfolio weights which produces the portfolio with the lowest possible return std. deviation (or variance) constitutes the minimum variance portfolio.



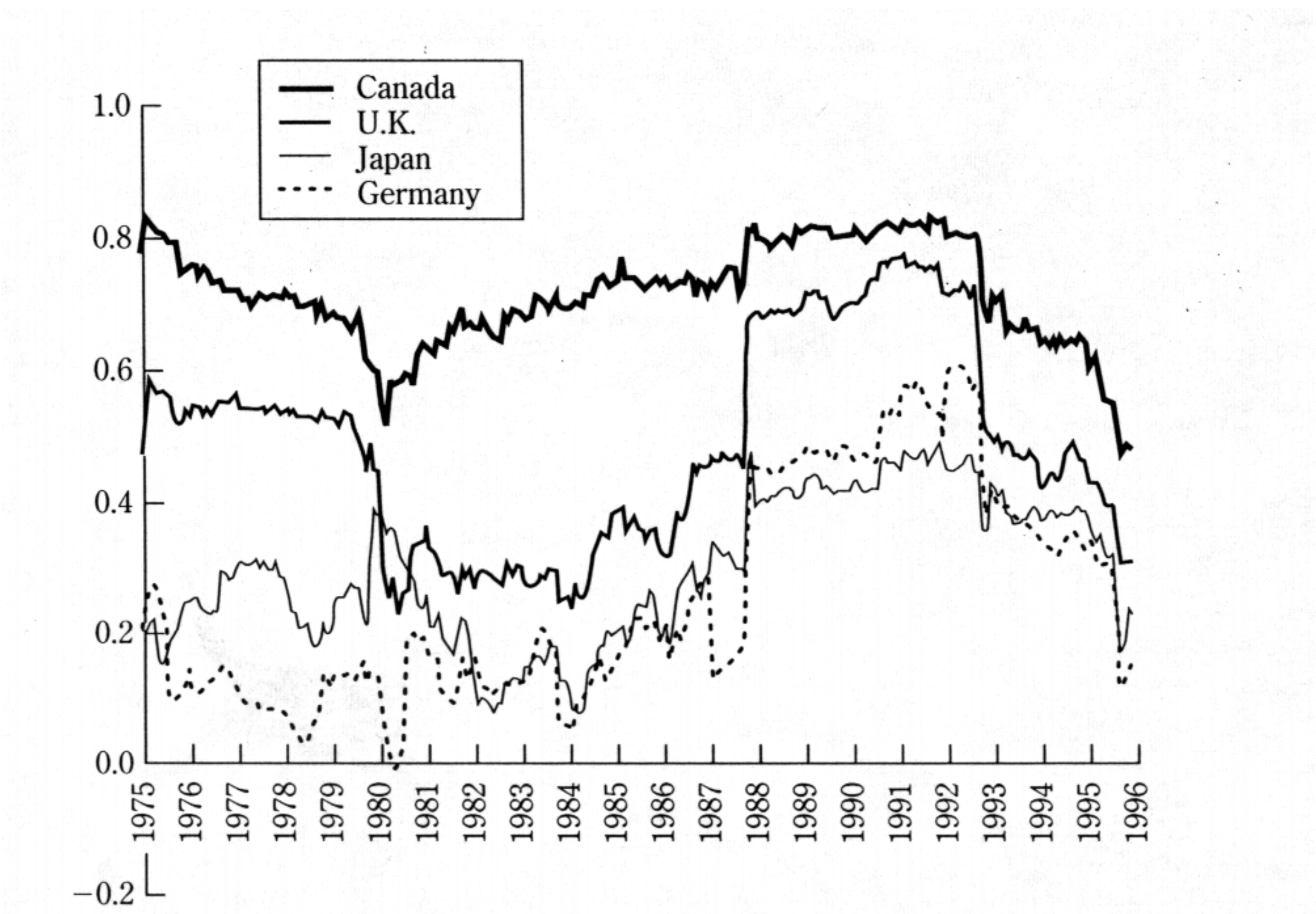
# Benefits of Globalization



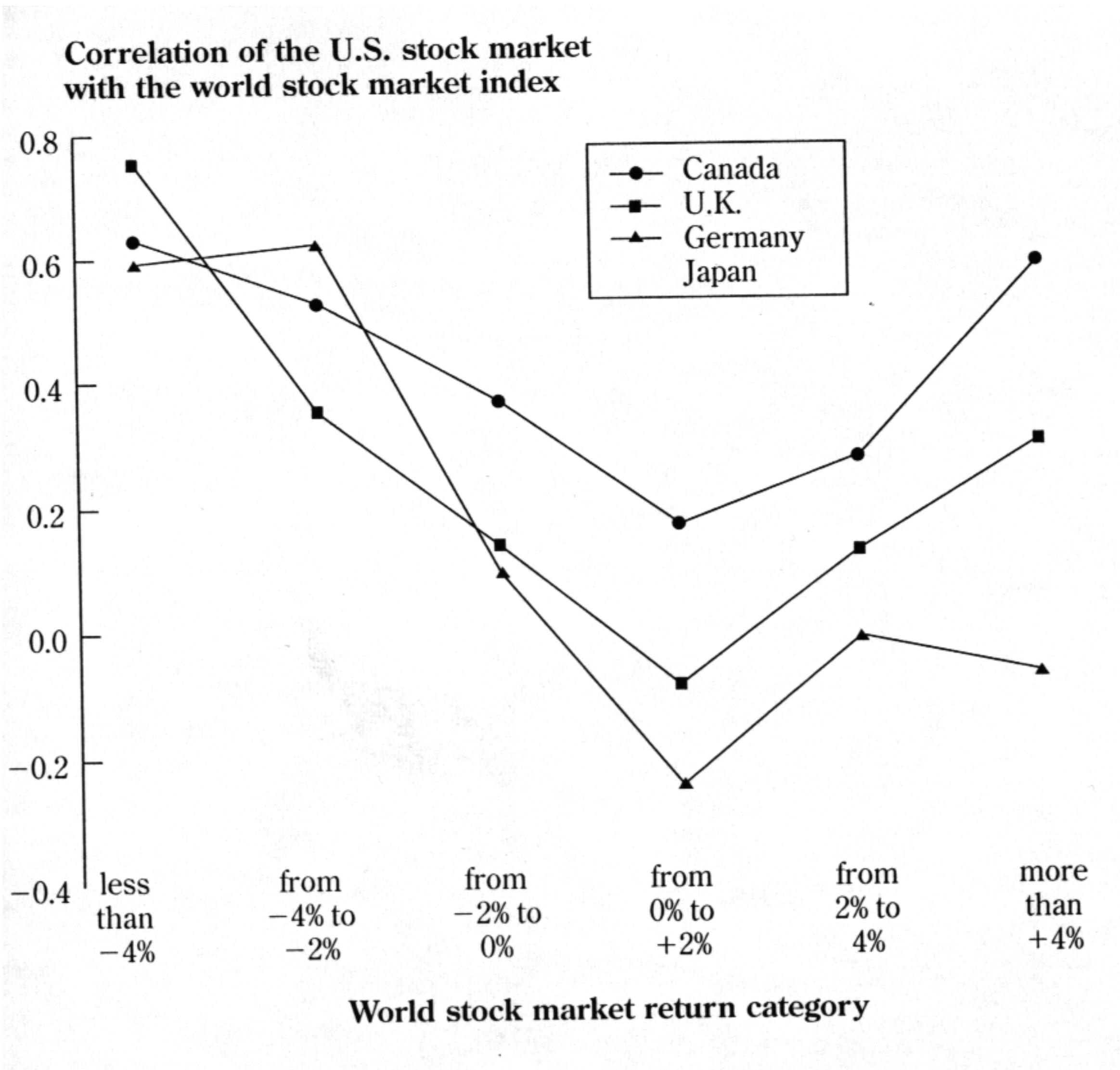
In practice correlations change over time!

# Intertemporal Correlation Structure

(Daily Return Correlation with US Based on 6 Month Moving Window)



# When is Diversification most Effective?



# Benefits of Globalization

- Stock market globalization: Securities of foreign countries become available to domestic residents
- Benefit: Globalization creates better investment opportunities
- Diversification benefits are larger if domestic and foreign returns have a low correlation
- Correlations change over time
- Correlations are lower in normal times, but (unfortunately) increase in bad times (when diversification is most needed)
  - A need to structure portfolios to avoid high correlation in periods of generally high correlations

# Summary

- Risk in a financial market depends on the risk features of security portfolios; individual asset risk may not translate into aggregate risk

- As long as the correlation is below 1, the std. deviation of a portfolio is always less than the weighted average std. deviation of its constituent securities

- The minimum variance portfolio has less risk if the asset return correlation is lower

- Access to global market improves the investor choice in the risk - expected return - space.

# Part II

- Mean-variance analysis with many assets
- Introducing a risk-free asset
- Deriving the CAPM
- Interpreting the CAPM
- General principles
- Summary

# Mean-Variance Analysis

- Can generalize portfolio theory from 2 to N securities
- Expected average (mean) portfolio return:

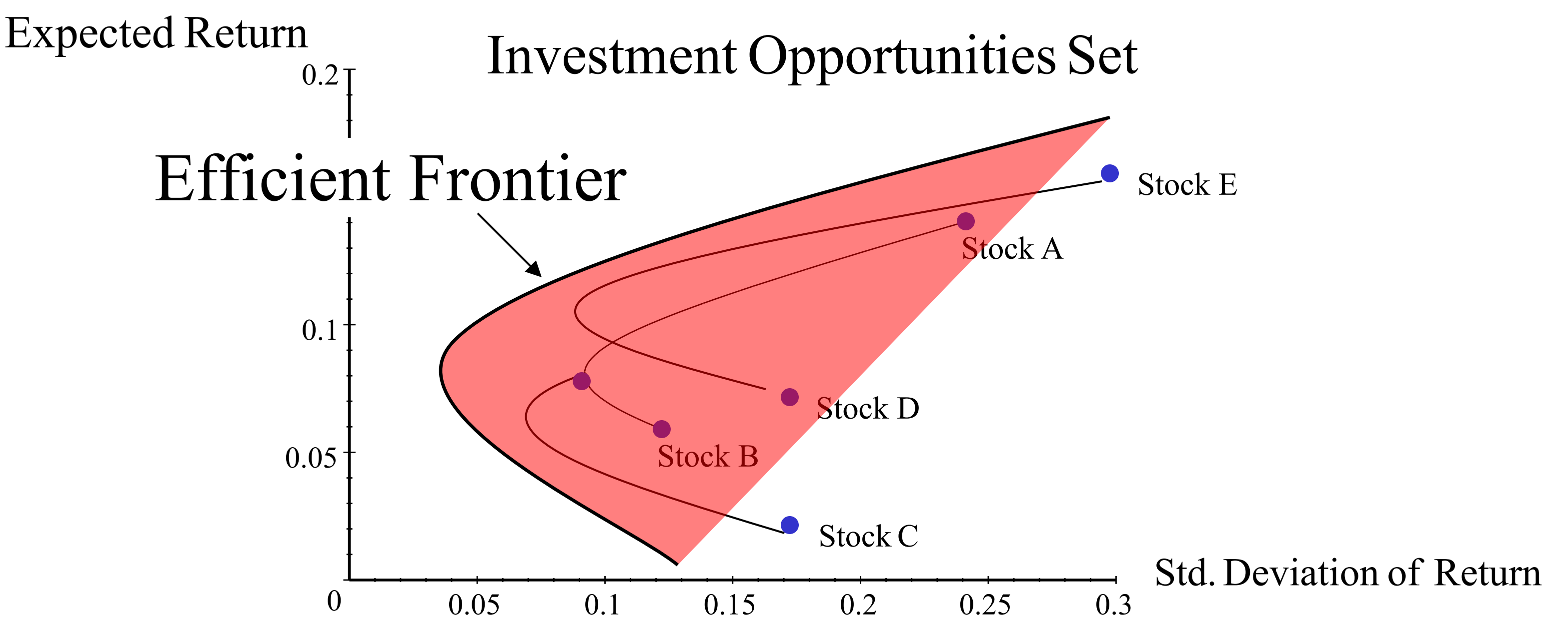
$$E(\widetilde{R}_p) = \sum_{i=1}^{N} x_i E(\widetilde{R}_i)$$

- Portfolio Risk: Variance or std. deviation of the combination of n assets

$$\text{Var}(\widetilde{R}_p) = \sum_{j=1}^{N} \sum_{i=1}^{N} x_j x_i \operatorname{cov}(\widetilde{R}_i, \widetilde{R}_j)$$

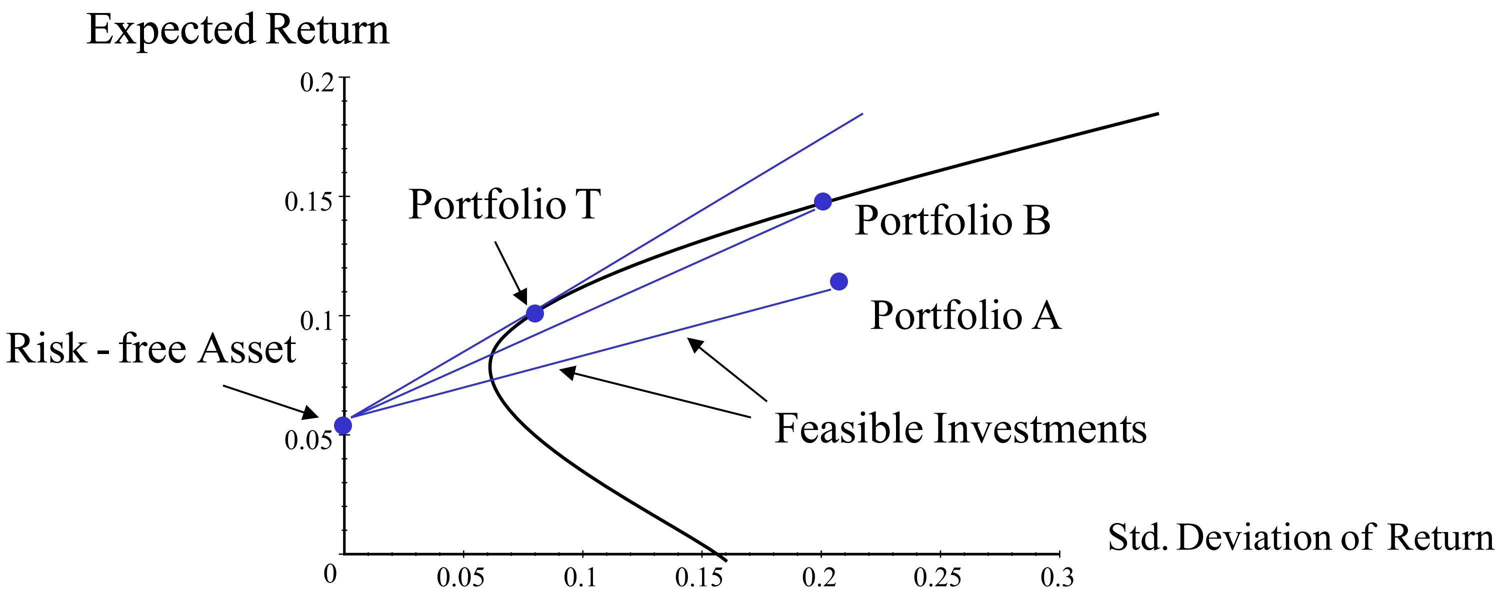
$$\sigma_P = \sqrt{\text{Var}(\widetilde{R}_p)}$$

# Mean-Variance Efficient Frontier




- Take every possible combination of risky assets (all linear combinations of portfolio weights)
- Efficient Frontier: Retains the best mean variance portfolio

# Introducing a Riskless Asset




- Combine riskless asset with feasible portfolios (A, B, etc.) in the investment opportunity set
- Points on blue line are new feasible portfolios
- What is the best portfolio to be combined with riskless asset?
- Answer: The <u>tangency portfolio T</u> is optimal (pushes the blue line furthest to the north-west)

# Towards the CAPM

Nobel Price 1990 for Sharpe

- First Insight: The portfolio T is the optimal one for every investor regardless of his risk preferences;

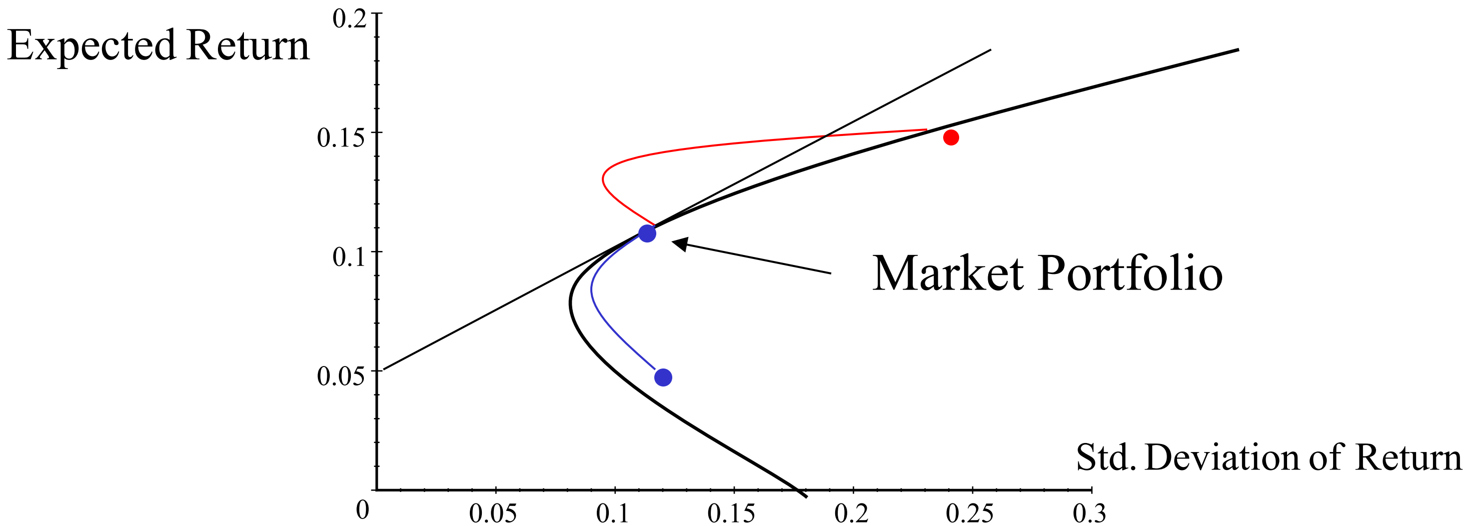
  Why? Because can control risk by choosing more or less riskless asset.
- Second Insight: If portfolio T is optimal, then every investor should hold varying proportions of portfolio T; but no other risky assets
- Third Insight: All assets should have returns high enough so that someone is willing to hold them (the market clears);

  Therefore; if all want to hold portfolio T, the market can only clear if portfolio T is equal to the entire market (market portfolio)

# Towards the CAPM

Nobel Price 1990 for Sharpe

- We can use these 3 insights to derive the CAPM model which tells us the price of risk in a financial market
- Assume we are in equilibrium and the market portfolio is the tangency portfolio
- If we combine the market portfolio with any other risky security we make a sub-optimal return-risk trade-off, but we cannot improve tangency limit (like for red line)



# Towards the CAPM

Nobel Price 1990 for Sharpe

- Assume we have *a*% in some risky asset *i* and (*1-a*)% in the market portfolio,

$$E(\tilde{R}_p) = aE(\tilde{R}_i) + (1-a)E(\tilde{R}_m)$$

$$\sigma(\tilde{R}_p) = \sqrt{a^2\sigma_i^2 + (1-a)^2\sigma_m^2 + 2a(1-a)\sigma_{im}}$$

- We can determine the return-risk trade-off for this new portfolio:

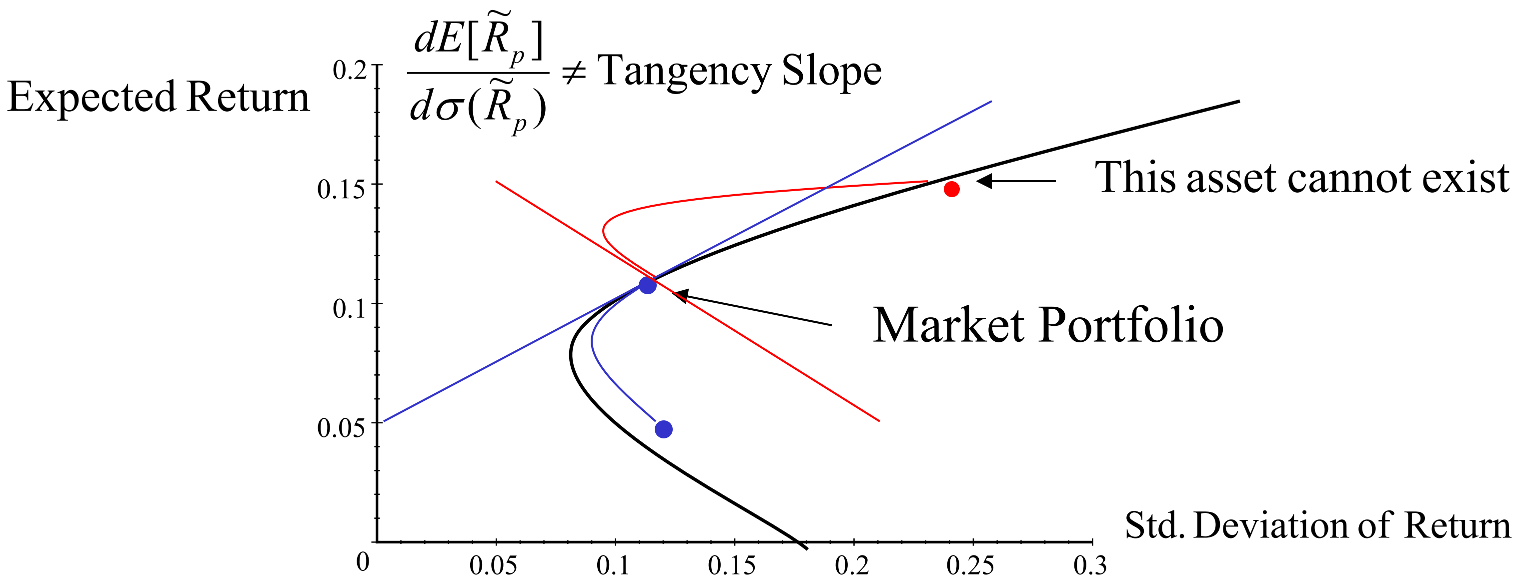
$$\frac{dE[\tilde{R}_p]}{d\sigma(\tilde{R}_p)} = \frac{\dfrac{\partial E[\tilde{R}_p]}{\partial a}}{\dfrac{\partial \sigma(\tilde{R}_p)}{\partial a}}$$

<u>Interpretation:</u> How much does the expected return change relative to the standard deviation as we alter the portfolio weight *a*.

# Towards the CAPM

Nobel Price 1990 for Sharpe

- The risk-return trade-off cannot be different from the tangency slope as we leave the optimal market (*a=0)*
- Otherwise the market portfolio would not have been the tangency portfolio on the efficient frontier



# Towards the CAPM

Nobel Price 1990 for Sharpe

- Derive risk-return trade-off:

HW: Please Check!

$$\frac{\partial E(R_p)}{\partial a} = E(R_i) - E(R_m)$$

$$\frac{\partial \sigma(R_p)}{\partial a} = \frac{1}{\sigma(R_p)}\left[a\sigma_i^2 - (1-a)\sigma_m^2 + (1-2a)\sigma_{mi}\right]$$

$$\left.\frac{dE(\tilde{R}_p)}{d\sigma(\tilde{R}_p)}\right|_{a=0} = \left.\frac{\partial E(\tilde{R}_p)}{\partial a} \Big/ \frac{\partial \sigma(\tilde{R}_p)}{\partial a}\right|_{a=0} = \frac{E(\tilde{R}_i) - E(\tilde{R}_m)}{\frac{\sigma_{mi} - \sigma_m^2}{\sigma_m}}$$

- Equilibrium condition for riskless rate *r*:

$$\text{Risk - return trade - off} \quad = \quad \text{Tangency Slope}$$

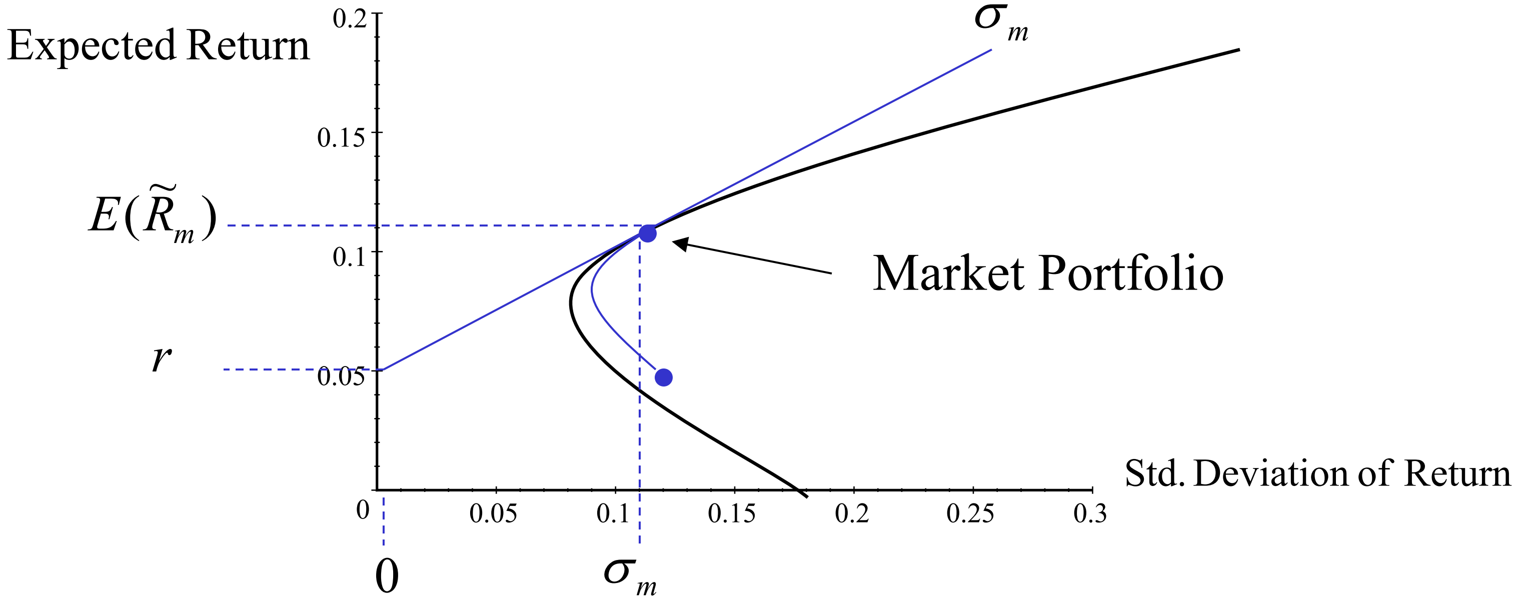
$$\frac{E(\tilde{R}_i) - E(\tilde{R}_m)}{\frac{\sigma_{mi} - \sigma_m^2}{\sigma_m}} \qquad = \qquad ?$$

# Towards the CAPM

Nobel Price 1990 for Sharpe

- The tangency slope is determined by the characteristics of the market portfolio ($E(R_m)$, $\sigma_m$) and the risk-free rate $r$

$$\text{Tangency Slope} = \frac{E(\tilde{R}_m) - r}{\sigma_m} \quad \text{for a} = 0$$



# Towards the CAPM

Nobel Price 1990 for Sharpe

- Equilibrium condition:

$$\text{Risk - return trade - off} = \text{Tangency Slope}$$

$$\frac{E(\tilde{R}_i) - E(\tilde{R}_m)}{\dfrac{\sigma_{mi} - \sigma_m^2}{\sigma_m}} = \frac{E(\tilde{R}_m) - r}{\sigma_m}$$

$$\Leftrightarrow \quad E(\tilde{R}_i)\sigma_m^2 = \sigma_{mi}\left[E(\tilde{R}_m) - \mathrm{r}\right] + \sigma_m^2 r$$

⇨ Capital Asset Pricing Model (CAPM):

$$E(\tilde{R}_i) - \mathrm{r} = \frac{\sigma_{mi}}{\sigma_m^2}\left[E(\tilde{R}_m) - \mathrm{r}\right]$$

# Interpreting the CAPM

- We define as the <u>price of risk</u> of asset *i* the following ratio (beta):

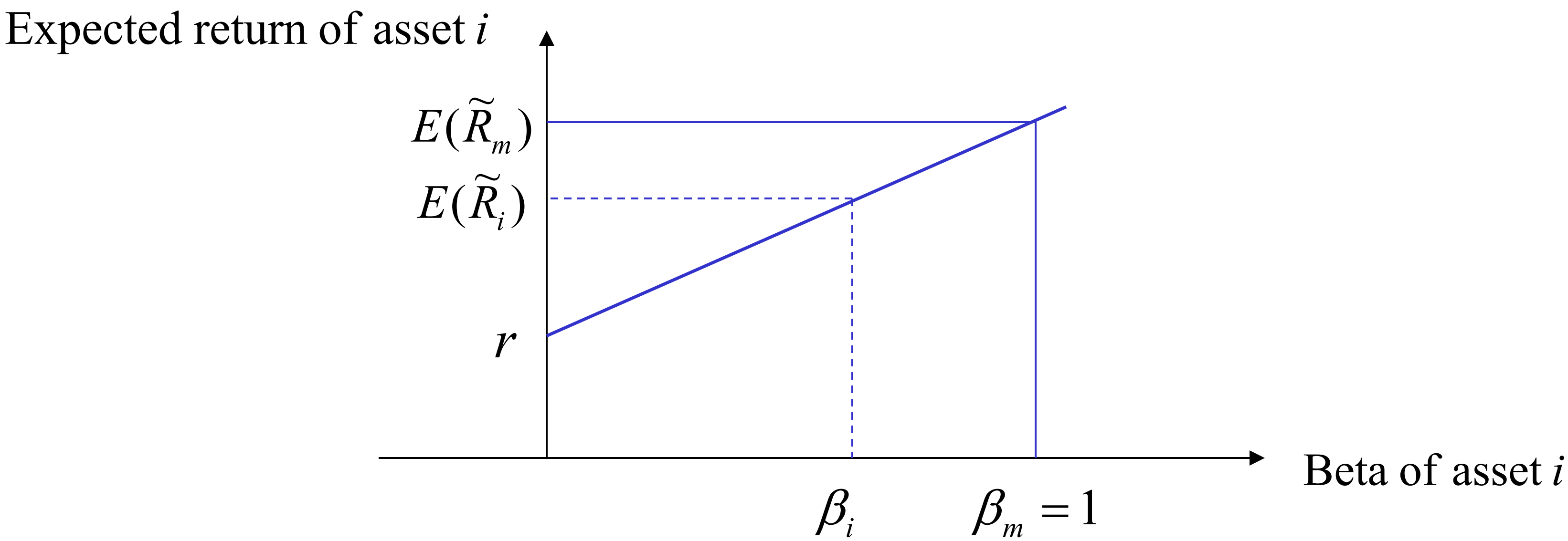
$$\beta_i = \frac{\sigma_{mi}}{\sigma_m^2} \qquad E(\tilde{R}_i) - \mathrm{r} = \beta_i \left[ E(\tilde{R}_m) - \mathrm{r} \right]$$

- The beta tells us how much expected return $E(R_i)$ asset *i* must yield relative to the expected market return $E(R_m)$
- What is beta exactly? It is the covariance of the asset return with the market return relative to the variance of the market return
- <u>Message:</u> Return risk in a financial market has a price (cost) to the extent it is covariance with the market return

Risk is Covariance with the Market

Risk is <u>not</u> Variance

# The Price of Risk

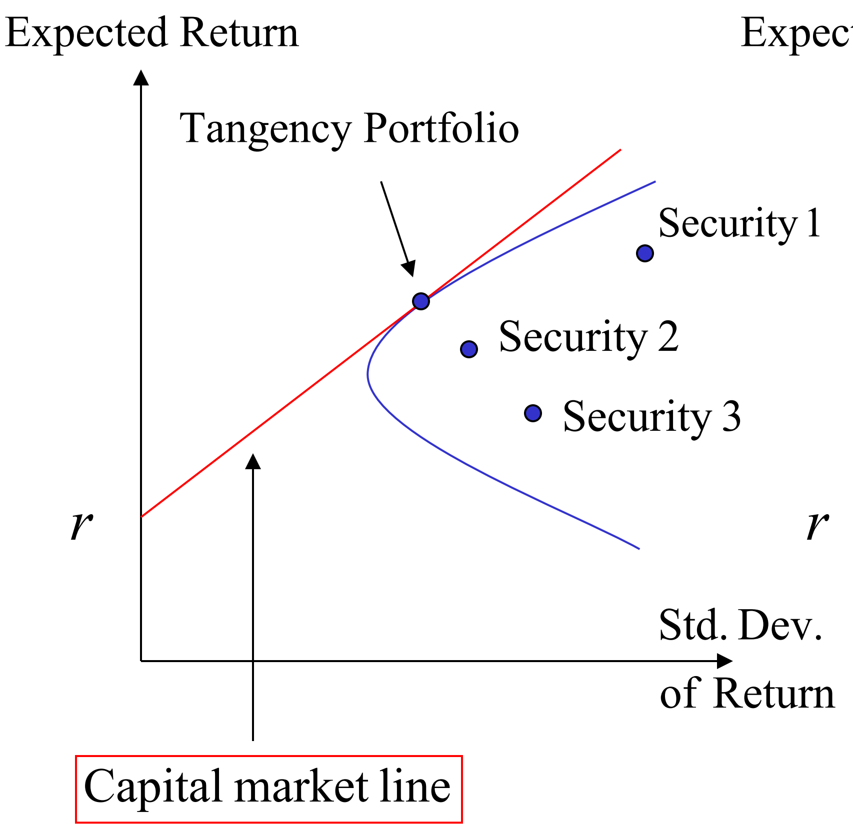


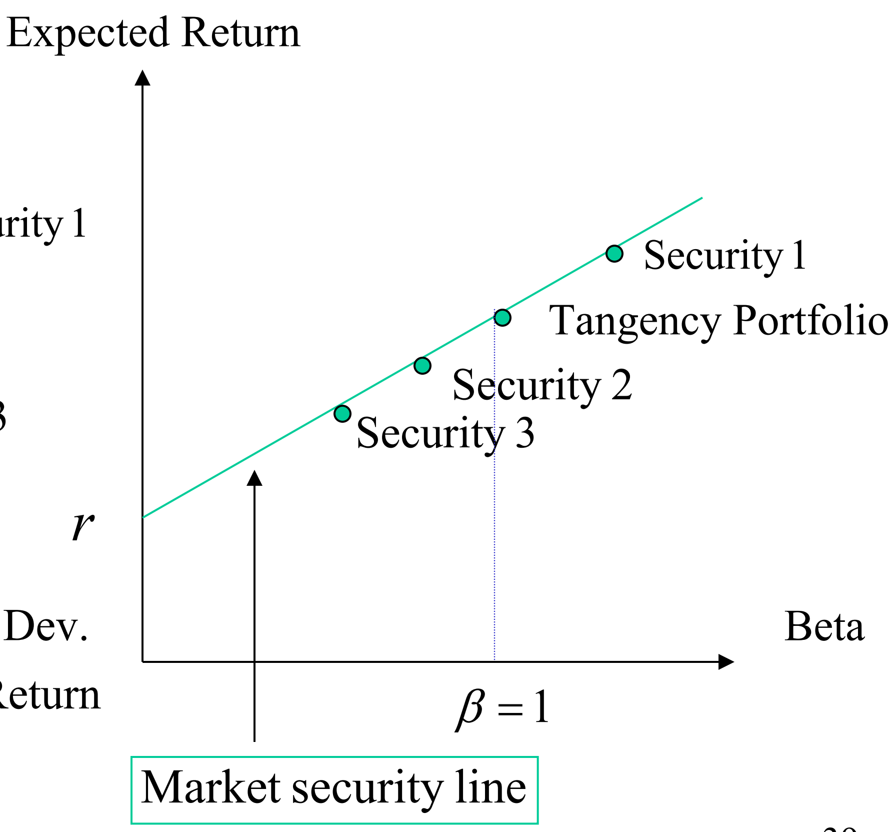

- Expected return of an asset is proportional to beta
- The beta of the whole market is 1
- Securities with $\beta_i < 1$ require an expected return lower than the market; these securities have below average risk
- Securities with $\beta_i > 1$ require an expected return higher than the market; these securities have above average risk
- Securities with $\beta_i < 0$ require an expected return lower than the risk-free rate; these securities provide insurance benefits

# Two Different Diagrams

Mean - Std. Dev. Diagram

Mean - Beta Diagram



# General Principles

- Investors only pay for risk that is economy wide (systematic, meaning correlated with market)
- The reason why non-systematic risk has no influence on expected return is that such risk can be eliminated by fair trade among all investors (risk trading)
- In equilibrium, all investors are left with is risk which cannot be diminished by further trading; that is the systematic risk
- The equilibrium is obtained if each security has a price which makes the expected return proportional to its beta

# Summary

- Mean-variance analysis extends to the multiple asset case; optimal portfolio weights bring us to the efficient frontier
- If a risk-free asset exists, the optimal portfolio on the efficient frontier is the tangency portfolio; which has to be the market portfolio
- The expected return on a risky asset is given by beta measuring covariance of these returns with the market
- The market has a beta of 1; assets with higher betas are expected to beat the market; assets with with lower betas require lower expected returns

# Exercises:
# Problem 1

In an article in the *Financial Analysts Journal*, Ibbotson, Carr and Robinson report the following statistics, for the U.S. and British (U.K.) stock markets, based on the US dollar performance of stock market indexes for the two countries.

The standard deviation (in percent per year) are 17.7 and 33.6 for the U.S. and the U.K., respectively. The correlation of the two markets was 0.617

Assuming that these statistics remain stable and that expected US dollar rates of return to a U.S. investor now are 14 percent in the U.S. market and 20 percent in the U.K. market. U.S. Treasury Bills yield 6 percent.

1. What is the expected rate of return of a portfolio 50 percent invested in the U.S. stock market and 50 percent in the U.K. market?
2. What is the annual standard deviation of a portfolio 50 percent invested in the U.S. stock market and 50 percent in the U.K. market?

# Problem 1 (answer to 1 and 2)

1. Expected return of a portfolio with 50 percent weights in both markets:

$$
\begin{aligned}
E(R_p) &= 0.5E(R_{US}) + 0.5E(R_{UK}) \\
&= 0.5 \times 14 + 0.5 \times 20 \\
&= 17\%
\end{aligned}
$$

2. Standard deviation:

$$\sigma_p^2 = (0.5)^2 \sigma_{US}^2 + (0.5)^2 \sigma_{UK}^2 + 2(0.5)^2 \rho(R_{US} R_{UK}) \sigma_{US} \sigma_{US} = 0.0543$$

$$\sigma_p = 0.233 = 23.3\%$$

# Problem 1 continued

3. Suppose your stock portfolio is now invested entirely in the U.S. market but you wonder to what extent international diversification makes sense. More practically, you could invest in any one of the four following more or less diversified portfolios :

| | | |
|---|---|---|
| | $x_{US} + x_{UK}$ | = 1.00 |
| 1) | 0.8 + 0.2 | = 1.00 |
| 2) | 0.7 + 0.3 | = 1.00 |
| 3) | 0.6 + 0.4 | = 1.00 |
| 4) | 0.5 + 0.5 | = 1.00 |

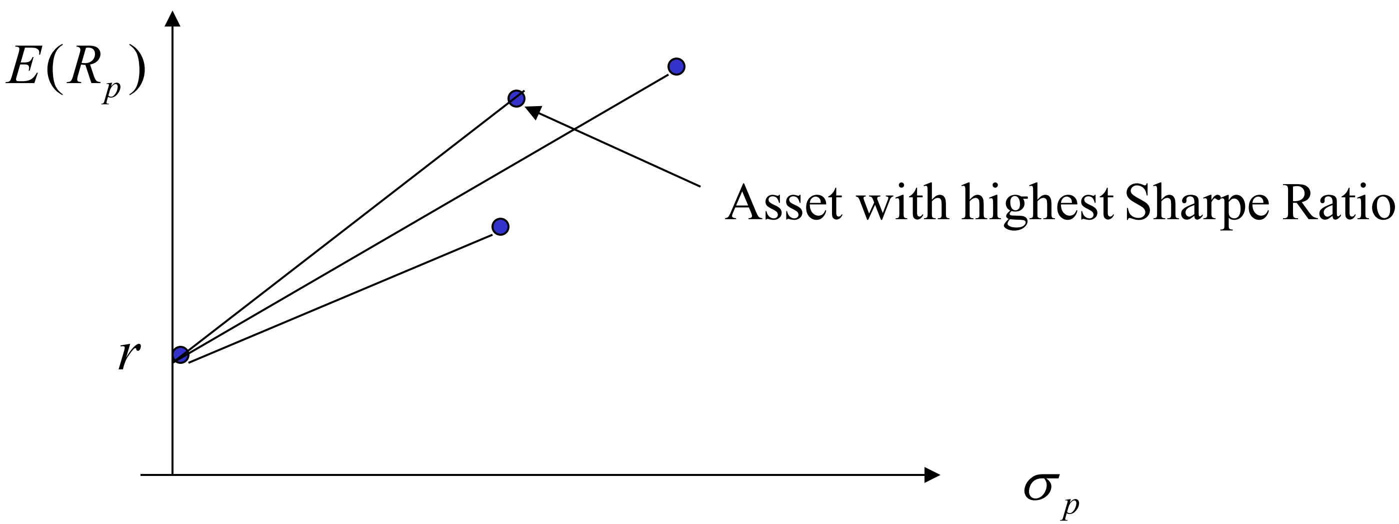
Is there any one portfolio that dominates the others?

# Answer to Problem 1 part 3

3. We can calculate so-called Sharpe ratios:

$$\text{Sharpe Ratio} = \frac{E(R_p) - r}{\sigma_p}$$

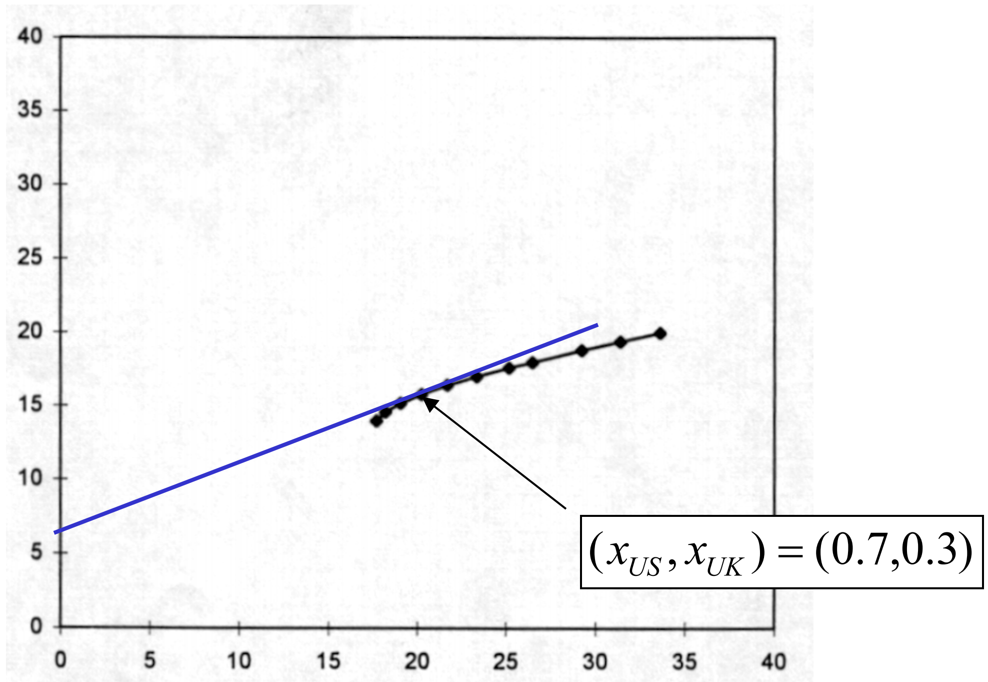
The Sharpe ratio measures the risk premium per unit of risk (measured in standard deviations)



# Answer to part 3 continued

| $x_{\text{USA}}$ | $E(R_p)$ | $\sigma_p$ | $[E(R_p) - r]/\sigma_p$ |
|---|---|---|---|
| 1.0 | 14.0 | 17.7 | 0.452 |
| 0.9 | 14.6 | 18.2 | 0.473 |
| 0.8 | 15.2 | 19.1 | 0.483 |
| 0.7 | 15.8 | 20.2 | 0.484 |
| 0.6 | 16.4 | 21.7 | 0.480 |
| 0.5 | 17.0 | 23.3 | 0.472 |
| 0.4 | 17.6 | 25.2 | 0.461 |
| 0.3 | 18.2 | 27.1 | 0.450 |
| 0.2 | 18.8 | 29.2 | 0.438 |
| 0.1 | 19.4 | 31.4 | 0.427 |
| 0.0 | 20.0 | 33.6 | 0.417 |

# Answer to part 3 continued




The portfolio with portfolio a weight $x_{\mathrm{US}} = .7$ has the highest Sharpe ratio.

General optimization problem:

$$\max \left[ \frac{E(R_p) - r}{\sigma_R} \right]$$

$$s.t \quad \sum_i x_i = 1$$

# Part III

- Insights from CAPM
  - Assumptions
  - Intuition
- Betas of Portfolios
- Estimating Betas
- Testing the CAPM
  - Testability?
  - Fama and MacBeth
  - Interpreting the Evidence
- Multifactor Models
- Summary

# CAPM

Price of risk (excess return over risk-free asset) is measured by the beta of an asset:

$$E(R_i)\text{-}r = \beta_i\left[E(R_m)\text{-}r\right] \quad \text{with} \quad \beta_i = \frac{\sigma_{mi}}{\sigma_m^2}$$

| | | |
|---|---|---|
| $\sigma_{mi}$ | : | Covariance of asset return with market return |
| $\sigma_m^2$ | : | Variance of market return |
| $E(R_i)$ | : | Expected return on asset |
| $E(R_m)$ | : | Expected return on market |
| $r$ | : | Risk - free rate |

# CAPM Assumptions

What assumption did we use to derive CAPM?

1. Investors only care about the mean and the variance of their portfolio
2. There are no transaction costs like bid-ask spreads, taxes (e.g., capital gains tax), commissions, etc
3. Investors have homogenous beliefs about the mean, standard deviation and the correlation of every security

HW:

- Think which assumptions are unlikely to hold.
- What could be the implications?

# CAPM Intuition

Why is covariance the price of risk in a financial market?

<u>Answer:</u> Covariance measures the marginal contribution of each asset to the overall risk.

Decompose market variance into covariances:

$$\sigma_m^2 = \sum_{i=1}^{N}\sum_{j=1}^{N} x_i x_j \operatorname{cov}(R_i, R_j) = \sum_{i=1}^{N} x_i \operatorname{cov}\left(R_i, \sum_{j=1}^{N} x_j R_j\right) =$$

$$= \sum_{i=1}^{N} x_i \operatorname{cov}\left(R_i, R_m\right)$$

Percentage risk contribution of asset i $= x_i \dfrac{\operatorname{cov}(R_i, R_m)}{\sigma_m^2} = x_i \beta_i$

Marginal risk contribution of asset i $= \beta_i$

# Betas of Portfolios

A portfolio manager combines three securities A, B and C to with betas $\beta_A$ , $\beta_B$ , $\beta_C$ in a portfolio. What is the beta of the portfolio for portfolio weights $x_A$ , $x_B$ , $x_C$ ?

$$\beta_p = \frac{\text{cov}(R_p, R_m)}{Var(R_m)} = \frac{\text{cov}(x_A R_A + x_B R_B + x_C R_C, R_m)}{Var(R_m)} =$$

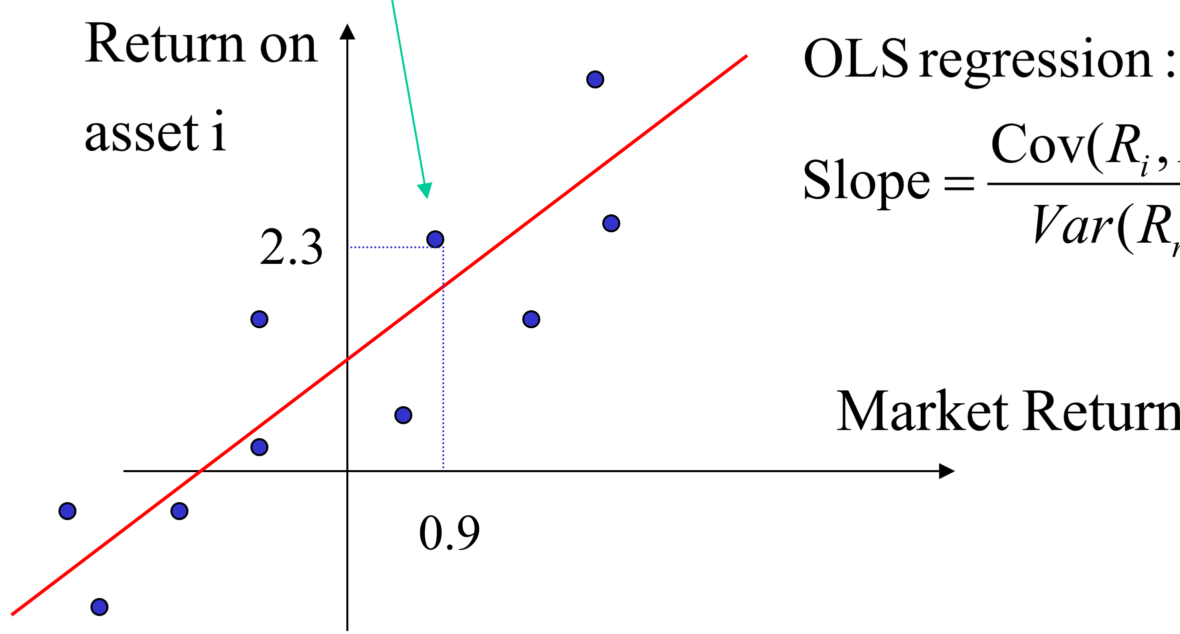
$$= x_A \frac{\text{cov}(R_A, R_m)}{Var(R_m)} + x_B \frac{\text{cov}(R_B, R_m)}{Var(R_m)} + x_C \frac{\text{cov}(R_C, R_m)}{Var(R_m)} =$$

$$= x_A \beta_A + x_B \beta_B + x_C \beta_C$$

The beta of a portfolio is the value weighted average of the individual security betas.

# Estimating Betas

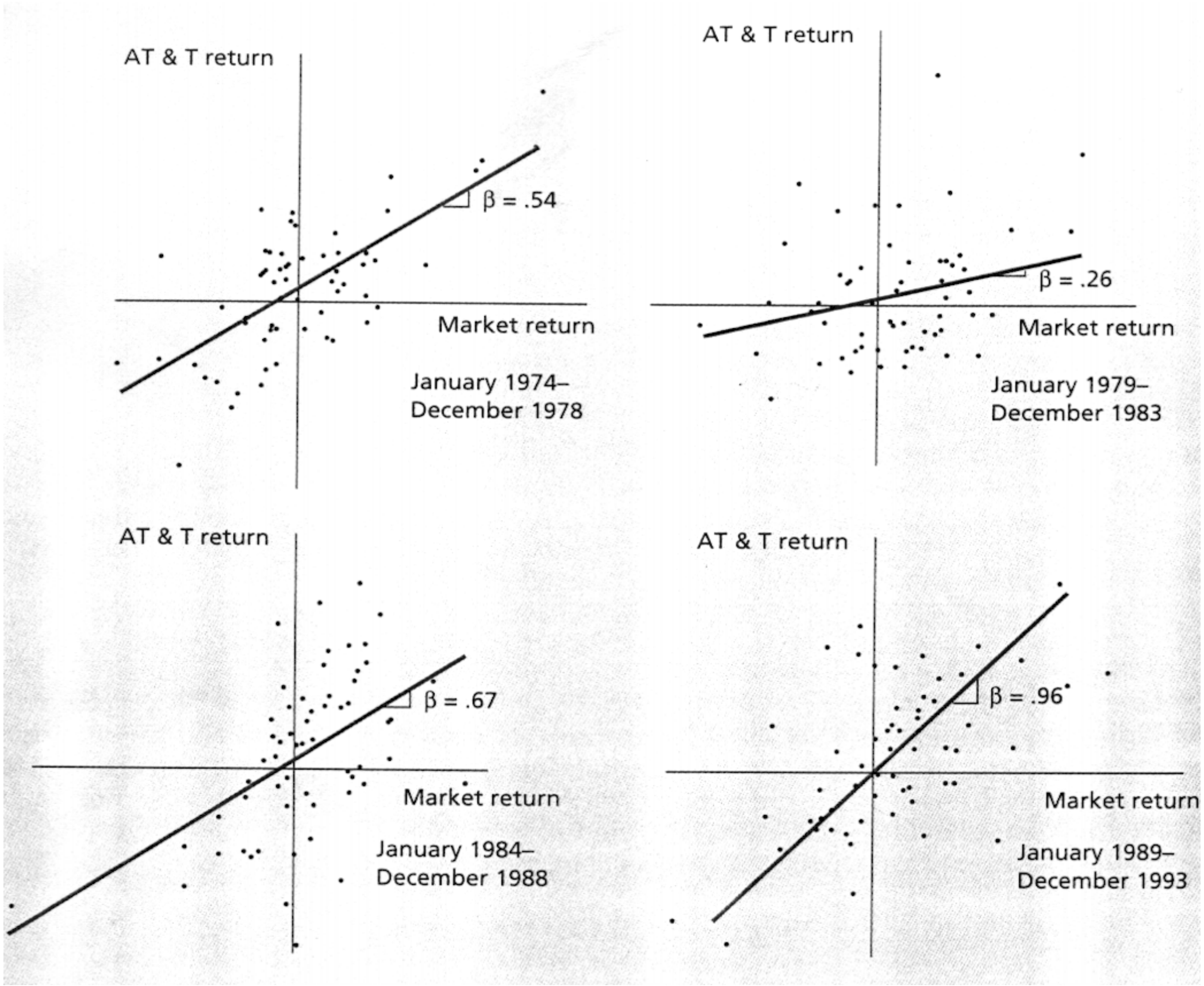
| Time (Month) | 1/88 | 2/88 | 3/88 | … | 12/88 |
|---|---|---|---|---|---|
| Return on asset i | 2.3 | 3.4 | – 0.3 | … | 2.8 |
| Market Return (S&P500) | 0.9 | 3.3 | 0.5 | … | 3.1 |




OLS regression :

$$\text{Slope} = \frac{\text{Cov}(R_i, R_m)}{Var(R_m)} = \beta_i$$

# Estimating Betas



# Estimation with few Observations

Example: 5 Soccer players compete over one season to have the highest number of goals scored per game. The season has a total of 26 games.

| Player | Average number of goals after 3 games | Average number of goals after 26 games |
|---|---|---|
| 1 | 2.33 | ? |
| 2 | 1.33 | ? |
| 3 | 1.00 | ? |
| 4 | 0.67 | ? |
| 5 | 0.00 | ? |

# Estimating Betas

- Estimation problem: Beta estimates with few return observations tend to overestimate the true beta for large estimates and underestimate the true beta for low estimates.
- Improve estimation with a beta adjustment
- Example: Bloomberg beta adjustment

$$\text{Adjusted beta} = 0.66 \times \text{Unadjusted beta} + 0.34$$

- Estimate beta for entire industry portfolio

# Estimating Portfolio Betas

Estimated betas and costs of (equity) capital ($r$) for large pharmaceutical companies and for a portfolio of these companies. The precision of the portfolio beta is much better than that of betas for individual companies—note the lower standard error for the portfolio.

| | $\beta_{equity}$ | Standard Error | Cost of Capital |
|---|---|---|---|
| Abbot Laboratories | 1.01 | .13 | 15.6% |
| American Home Products | .89 | .11 | 14.6 |
| Bristol-Myers | .81 | .10 | 13.9 |
| Johnson & Johnson | .93 | .11 | 14.9 |
| Lilly, Eli, & Company | 1.24 | .12 | 17.6 |
| Merck & Company | .85 | .12 | 14.2 |
| Pfizer | 1.02 | .14 | 15.7 |
| Rorer Group | 1.18 | .23 | 17.1 |
| Schering-Plough | .84 | .11 | 14.1 |
| Smith Kline-Beecham | .93 | .16 | 14.9 |
| Squibb | 1.18 | .20 | 17.1 |
| Syntex | 1.41 | .15 | 19.1 |
| Upjohn | 1.19 | .18 | 17.2 |
| Warner-Lambert | 1.05 | .13 | 16.0 |
| Market value–weighted industry portfolio | .98 | .07 | 15.4 |

*Source:* S. C. Myers and L. Shyam-Sunder, "Cost of Capital Estimates for Investment in Pharmaceutical Research and Development," in R. B. Helms (ed.), *Competitive Strategies in the Pharmaceutical Industry*, American Enterprise Institute, Washington, D.C., 1995.

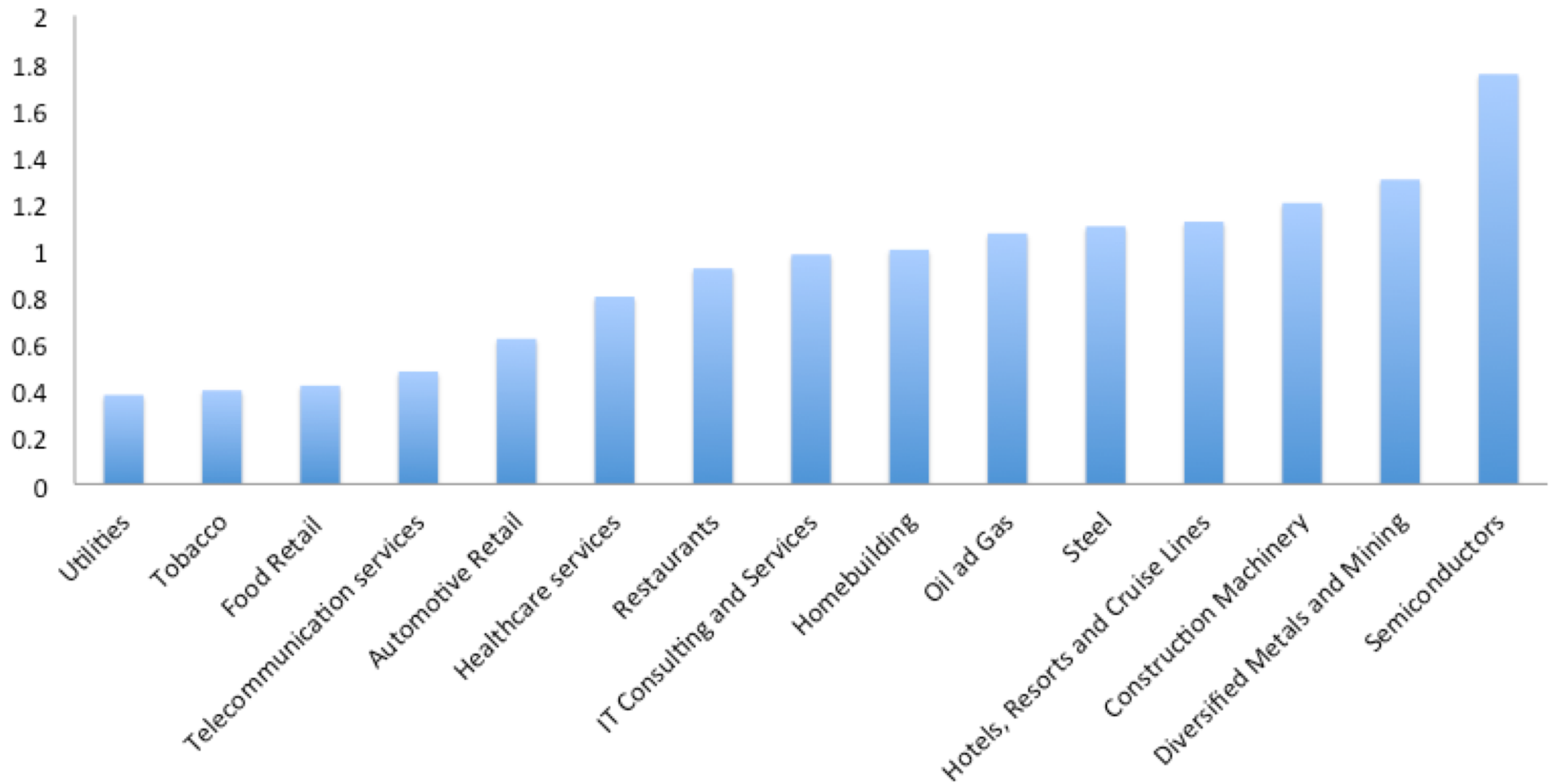
# Estimating Portfolio Betas

Betas for 17 large utilities, 1990–1994. The average beta was calculated from the monthly rates of return on a portfolio of the 17 companies.

| | Beta | Standard Error |
|---|---|---|
| Allegheny Power Systems | .38 | .11 |
| Central Main Power | .43 | .14 |
| Duke Power | .48 | .11 |
| Houston Industries | .25 | .13 |
| IPALCO Enterprises | .63 | .16 |
| Minnesota Power & Light | .49 | .13 |
| Northeast Utilities | .37 | .13 |
| Northern States Power | .64 | .13 |
| Pacific Gas & Electric | .46 | .11 |
| Potomac Electric Power | .37 | .10 |
| Public Service of Colorado | .57 | .11 |
| Puget Sound Power & Light | .43 | .13 |
| SCE | .60 | .12 |
| Southern | .41 | .13 |
| Southwestern Public Service | .56 | .12 |
| Texas Utilities | .35 | .10 |
| Wisconsin Energy | .53 | .14 |
| Portfolio | .47 | .09 |

Betas by Industry in US, 2012

(source: CapitalIQ)



# Testing the CAPM

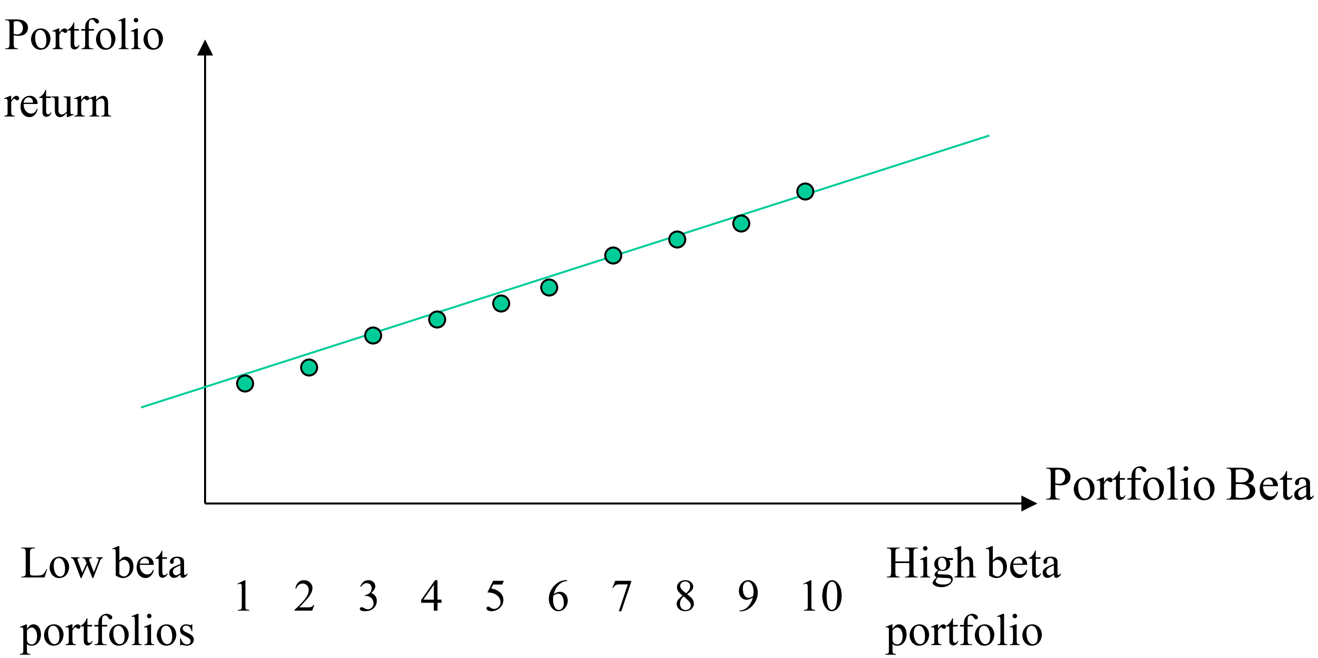
- Roll (1977): CAPM cannot be tested because we do not know the market portfolio!
  - S&P500
  - All traded stocks and bonds
  - Also real estate
  - International securities included

  } What is the market?

  There always exist a hypothetical market portfolio with the property that the respective betas are linearly related to the excess return
- CAPM defense against falsification: You mismeasured the market portfolio!
- <u>Conclusion:</u> Any test of the CAPM is a joint test of the theory and the conjectured market portfolio

# Testing the CAPM

- CAPM predicts: High beta portfolios have higher returns
- Fama and MacBeth (1973):
  - Step 1: Estimate the betas of all stocks over 5 year periods
  - Step 2: Sort the stocks into 10 portfolios according to their beta
  - Step 3: Verify if the 10 portfolios are located on the market security line



# CAPM Evidence

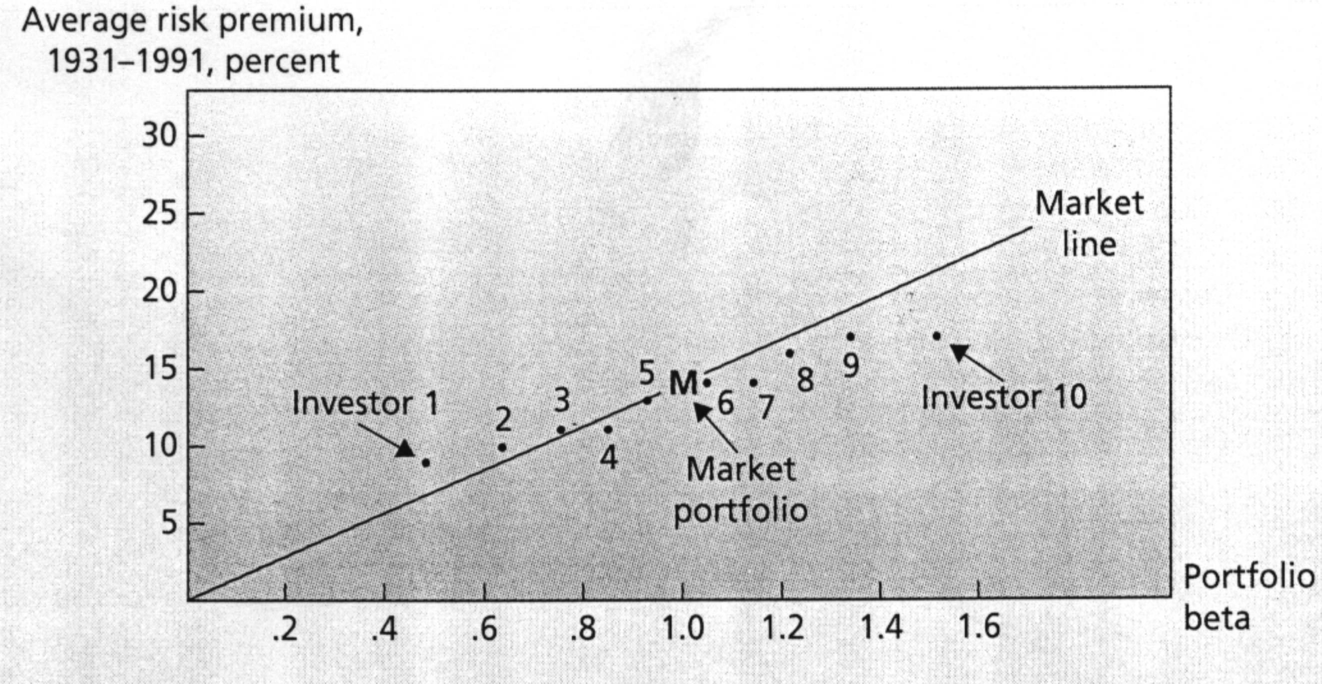
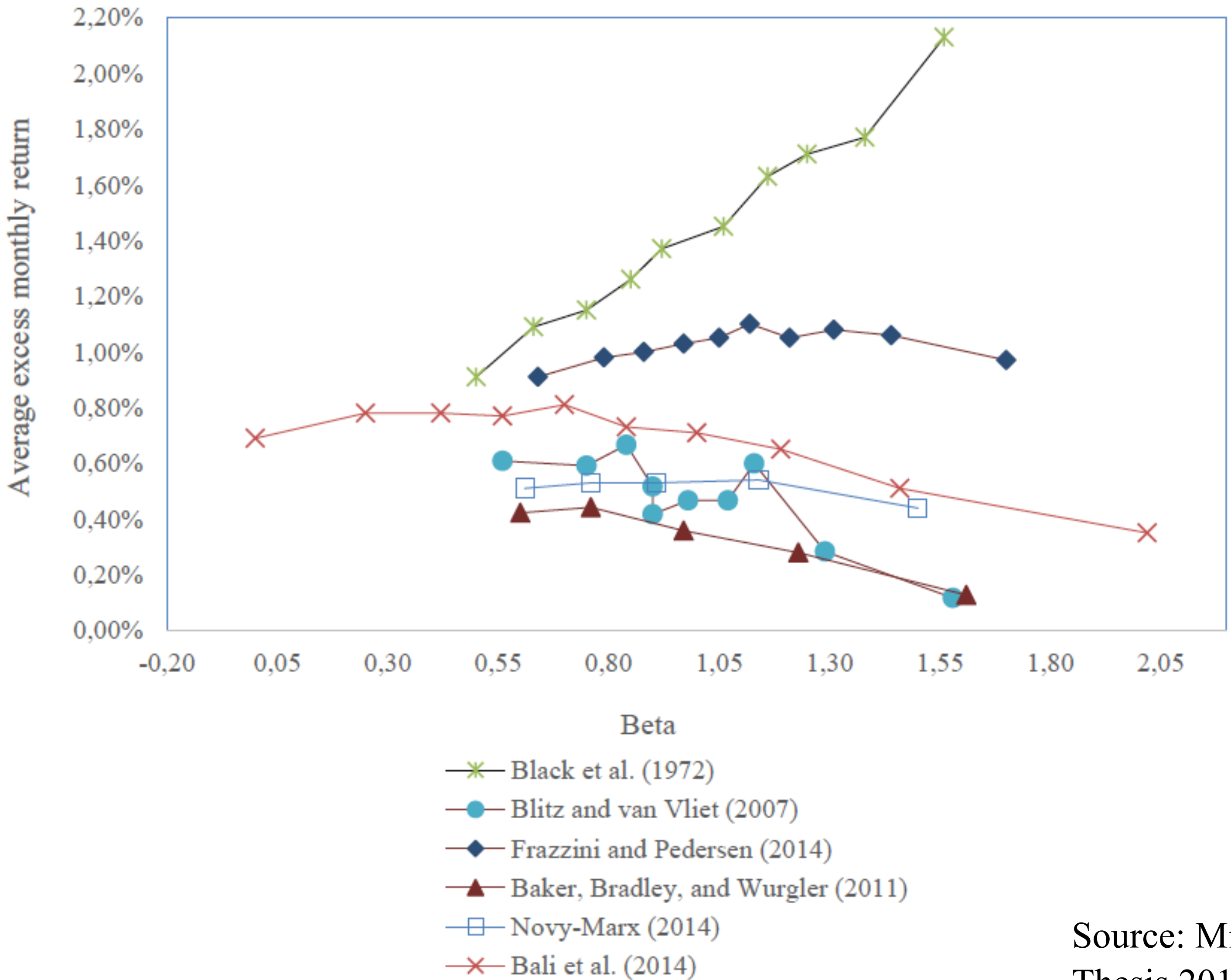



**Figure 8-10** The capital asset pricing model states that the expected risk premium from any investment should lie on the market line. The dots show the actual average risk premiums from portfolios with different betas. The high-beta portfolios generated higher average returns, just as predicted by the CAPM. But the high-beta portfolios plotted below the market line, and four of the five low-beta portfolios plotted above. A line fitted to the 10 portfolio returns would be "flatter" than the market line. [*Source:* F. Black, "Beta and Return," *Journal of Portfolio Management,* **20**:8–18 (Fall 1993).]

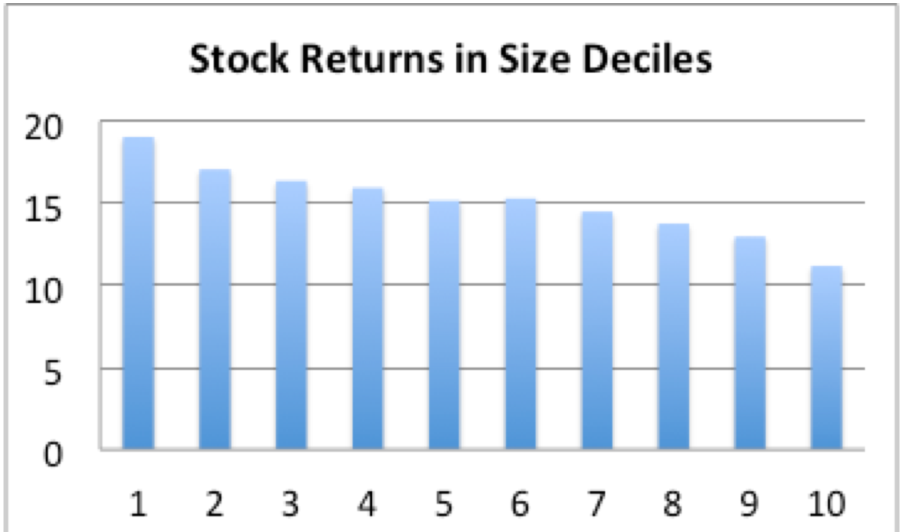
# CAPM Evidence

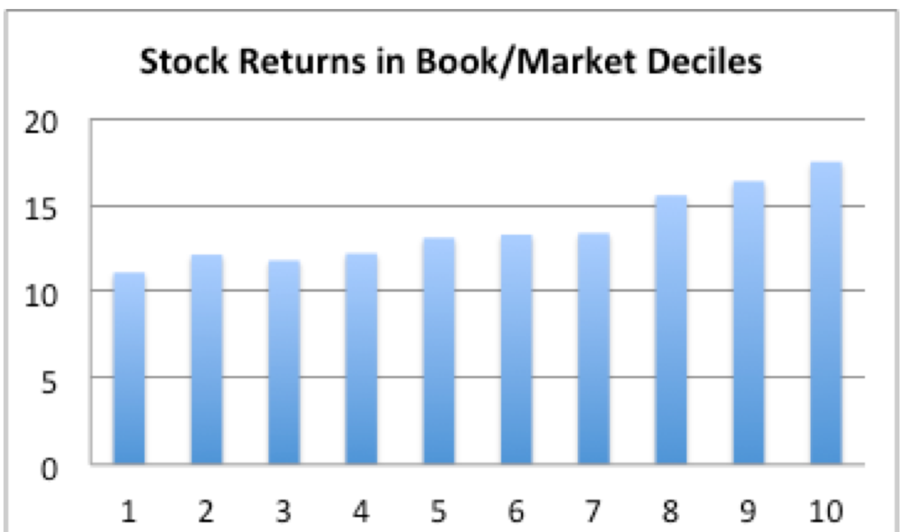



Source: Miikka Hakala Thesis 2015, Aalto Univ.

# Missing Risk Factors?

US Stock Returns 1928-2014



# Interpreting the Evidence

- The relationship between beta and mean return is weaker than the theory suggests
- More recent data shows no relationship between beta and mean returns
- Small stocks seem to outperform large stocks

<u>How to explain this empirical failure</u>

- Market risk is not correctly identified with the market portfolio of traded securities (include labor market risk?)
- Betas are not constant but change over time?
- Behavioral factors are missing?
- Transaction costs and control issues matter for investment decisions
- Beta estimates are systematically biased

# Multifactor Models

$$\text{Stock i}: \quad \widetilde{R}_i = \alpha_i + \beta_i \widetilde{R}_m + \varepsilon_i$$

$$\text{Stock j}: \quad \widetilde{R}_j = \alpha_j + \beta_j \widetilde{R}_m + \varepsilon_j$$

- Residual correlation: The error terms $\varepsilon_i$ and $\varepsilon_j$ are typically correlated; a common factor (other than the market return) may therefore influence both error terms.
- Generalize CAPM to k-factor model:

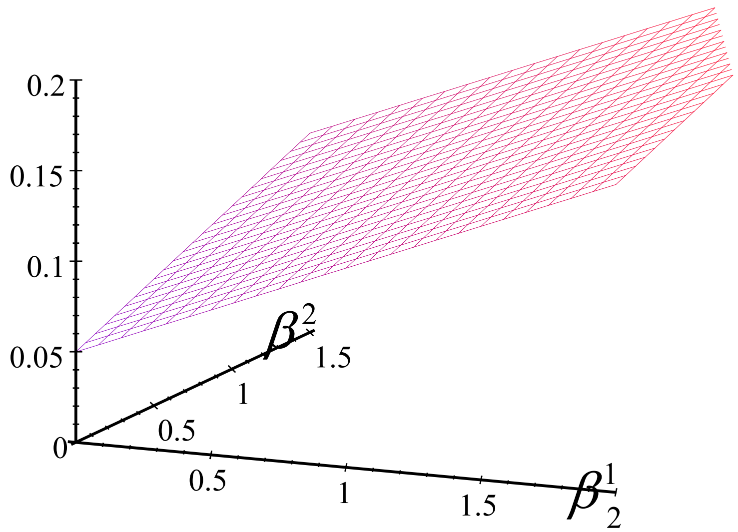
$$\widetilde{R}_i = \alpha_i + \beta_i^1 \widetilde{F}^1 + \beta_i^2 \widetilde{F}^2 + \ldots + \beta_i^K \widetilde{F}^K$$

- This is called Arbitrage Pricing Theory (APT)

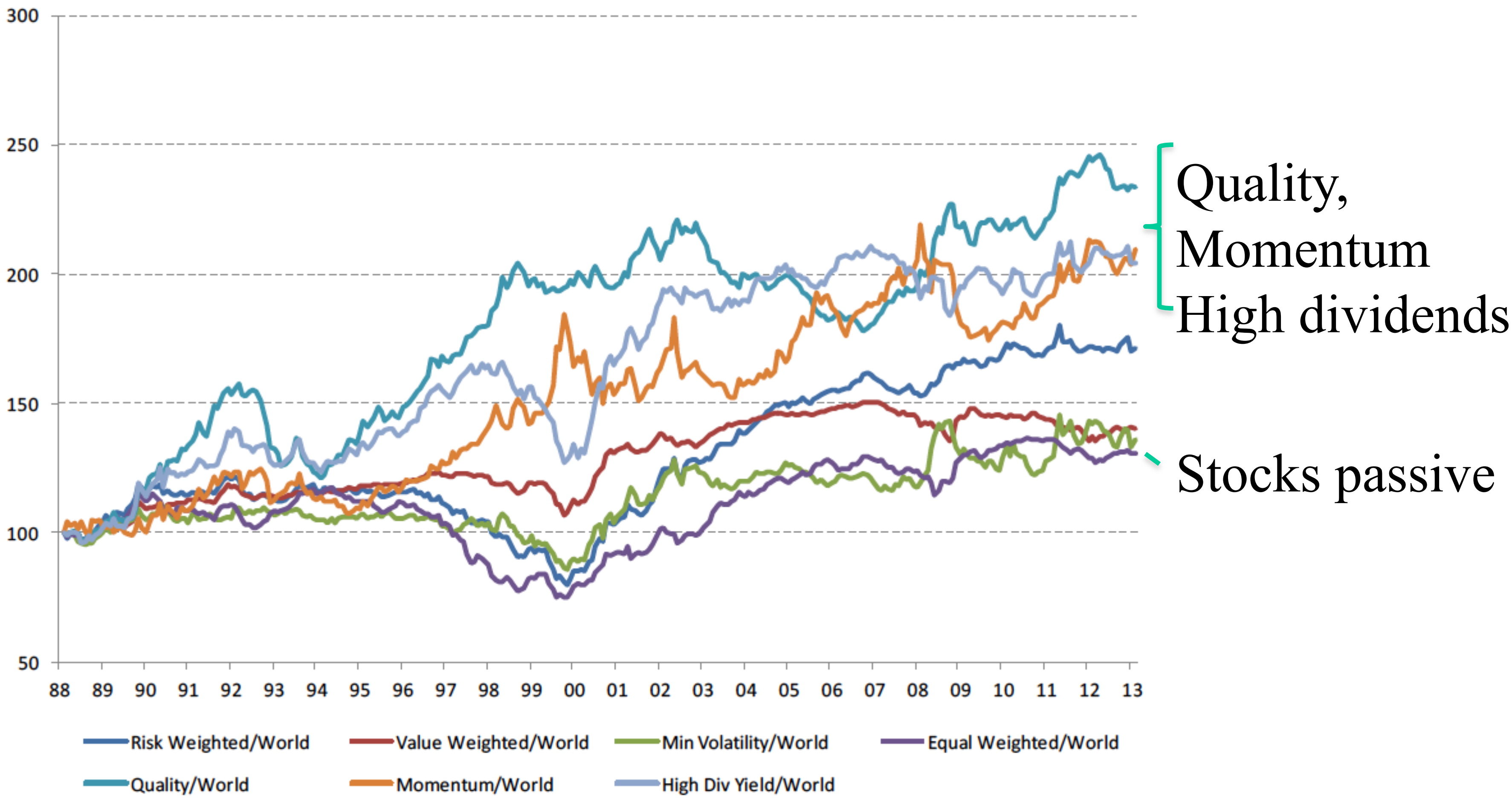
# Two Factor Model

- Market security line becomes market security hyper plane
- Betas $\beta^1$ and $\beta^2$ measure different dimensions of the risk premium

Return on asset i

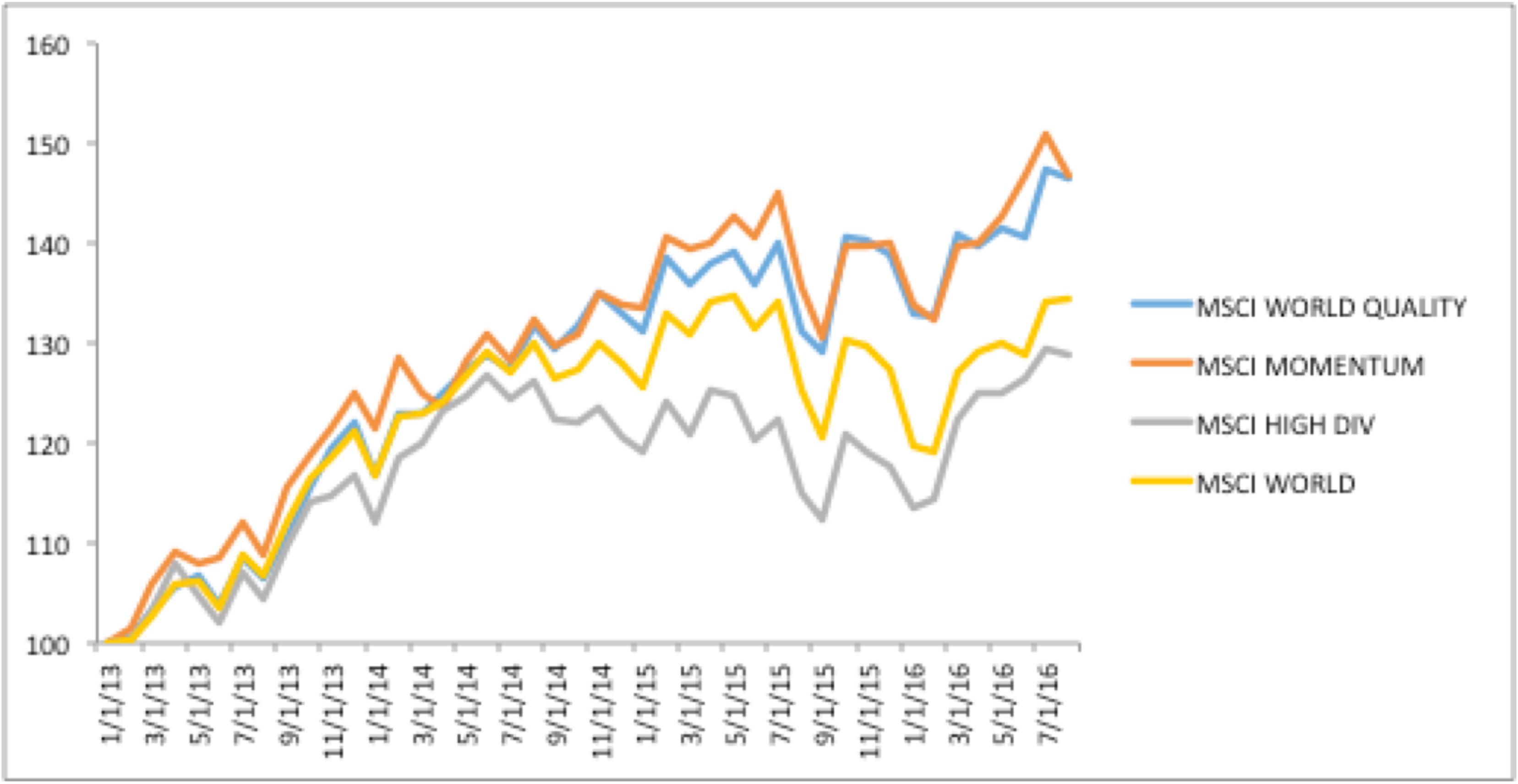


# Smart beta long only indexes – investing through ETFS?
## Source: Foundations of factor Investing MSCI (2013)




Source: Morgan Stanley

# Equity Long-only Smart beta 2013-2016




Source: Morgan Stanley

# AQR: Introduction to multi-asset class and multi-strategy investing

## A portfolio benefits from diversification between multiple strategies

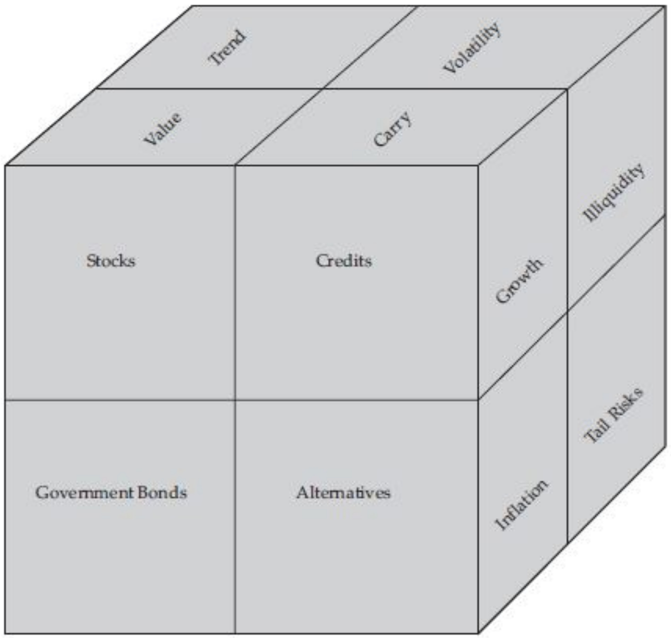
***"First key investment implication: Harvest market rewards from multiple sources to achieve more effective portfolio diversification and superior risk-adjusted returns. [...]
Second key investment implication: Humbly attempt to exploit time-varying expected returns"***

- Antti Ilmanen, AQR Capital Management

## Principle

- "There are many ways to improve investment practices to enhance long-run returns. The most important is to collect risk premia from diverse sources."
- "Investors can try to boost returns by exploiting value, carry and momentum tilts."
- The Cube - Perspectives on investments:
  - Asset Class (front)
  - Strategy Style (top)
  - Risk Factor (side)

- Antti Ilmanen, AQR Capital Management





Source: Ilmanen, 2011, Expected Returns: An Investor's Guide to Harvesting Market Rewards

# Introduction to multi-asset class and multi-strategy investing

**A portfolio benefits from diversification via low correlations between asset classes**

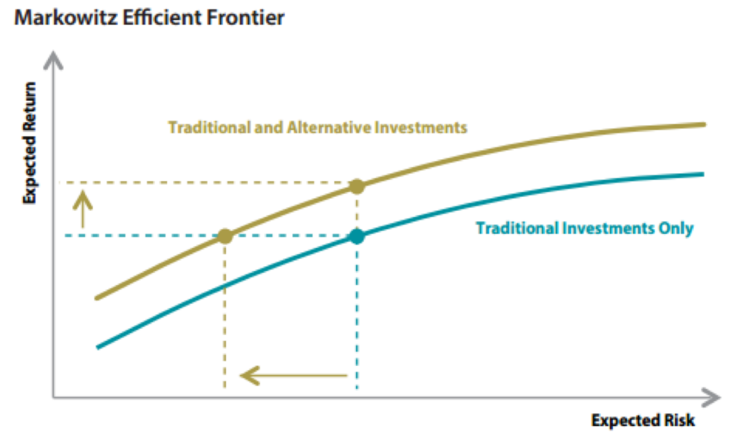
***" The US University Endowment Funds, such as Harvard and Yale, have been leaders in diversified multi-asset class investing for over two decades. Through this approach to investing and with a large exposure to alternative asset classes, they have consistently achieved attractive annual returns with moderate risk."***

\- Gottex Asset Management, London, 2015

**Principle**

- By combining multiple asset classes, i.e. stocks, bonds and alternative investments with low or negative correlations, an investor may achieve both a higher portfolio return and a reduction in risk.
- Alternative investments include hedge funds, smart beta products, private equity and real assets such as real estate, commodities, natural resources and infrastructure.
- Liquidity and liquidity risk bring about a third dimension, allowing higher expected returns with same price risk.

**Graphical illustration**




https://www.hedgecovest.com/article/using-efficient-frontier-alternatives

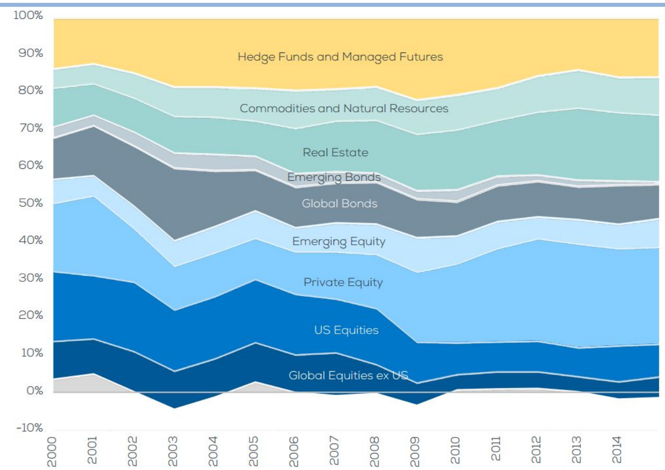
# Static portfolio asset allocation

[1] Gottex Asset Management, London, 2015, http://www.frontierim.com/files/file/download/id/1220

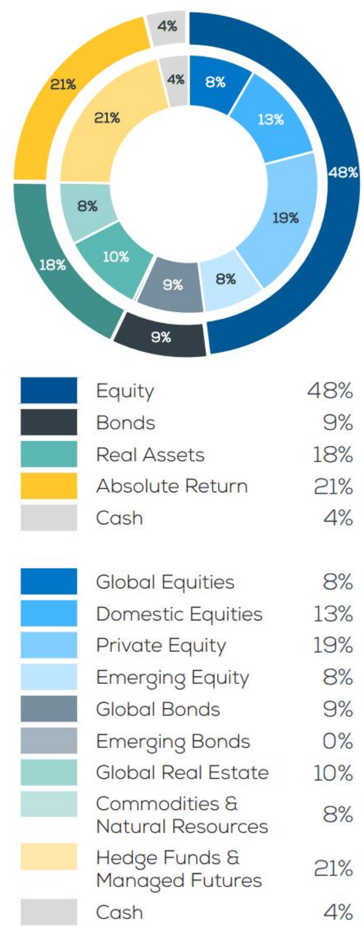
## Policy portfolios

- Traditional portfolio allocation decisions boil down to expected returns, volatilities and correlations between asset classes. An investor can with a model minimize the riskiness or maximize the Sharpe ratio of his or her portfolio and reach the optimal point, in terms of risk-adjusted returns, on the efficient frontier.
- The abovementioned approach is highly dependent on a number of strict assumptions. In reality the future might not be similar to the past and future returns, volatilities and correlations likely vary over time. This method should only be employed when defining the very long term policy portfolio, since it does not tell anything about the actual relative valuations of the asset classes.

## Harvard / Yale Endowment Asset Allocation over time [1]




## Asset Allocation of the top 20 US Endowment Funds 2014 [1]

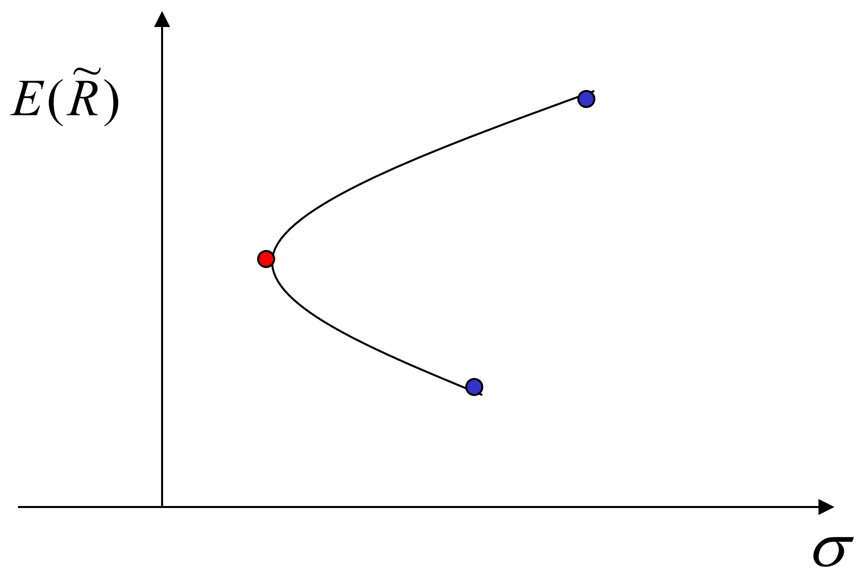


# Summary

- Central prediction of the CAPM is that assets are priced according to their marginal contribution to the overall market risk
- Any test of the CAPM model requires us to identify the market portfolio (joint testing)
- Empirical tests of the CAPM contradict the prediction that mean returns should increase in stock beta; beta risk is underpriced
- Other variables like firm size explain mean returns
- The CAPM model can be generalized (APT) to allow for a risk premium for factors other than the market return

# Exercise: Finding the Minimum Variance Portfolio

- Problem: You are given the expected return, standard deviation and the correlation of two assets. How do you find the minimum variance portfolio?




What are the portfolio weights which minimize the standard deviation of the portfolio?

# Answer to the Exercise: Finding the Minimum Variance Portfolio

- Minimize the standard deviation given by

$$\sigma(\widetilde{R}_p) = \sqrt{{x_1}^2{\sigma_1}^2 + (1-x_1)^2\sigma_2 + 2x_1(1-x_1)\sigma_{12}} = \sqrt{z(x_1)}$$

- Set the first derivative equal to 0:

$$\frac{d\sigma(\widetilde{R}_p)}{dx_1} = \frac{1}{2\sqrt{z(x_1)}} \times \frac{dz(x_1)}{dx_1} =$$

$$= \frac{2x_1{\sigma_1}^2 - 2(1-x_1)\sigma_2^2 + 2(1-2x_1)\sigma_{12}}{2\sqrt{{x_1}^2{\sigma_1}^2 + (1-x_1)^2\sigma_2^2 + 2x_1(1-x_1)\sigma_{12}}} = 0$$

$$\Rightarrow \quad x_1 = \frac{\sigma_2^2 - \sigma_{12}}{\sigma_1^2 + \sigma_2^2 - 2\sigma_{12}} \quad \text{where} \quad \sigma_{12} = \sigma_1\,\sigma_1\,\rho_{12}$$