Plasma fluid models, computational assignement

Hasegawa-Wakatani turbulence

The goal of this assignement is to implement the Hasegawa-Wakatani system below, in a 2D box in the plane perpendicular to the magnetic field, with one periodic direction (perpendicular to the field and the gradient).

$$\partial_t n + \{\phi, n\} + \frac{\partial_y \phi}{L_n} = C\left(\phi - n\right) \tag{1}$$

$$\partial_t \nabla_\perp^2 \phi + \left\{\phi, \nabla_\perp^2 \phi\right\} = C\left(\phi - n\right) \tag{2}$$

Since the H-W system given here describes only perturbations, the boundary condition in the direction of the gradient is a simple Dirichlet condition $n\,,\ \phi = 0\,.$

With the system implemented the following tasks can be completed:

- Perform a scan of L_n across the analytical linear threshold at fixed C (3 points is enough). You may choose to linearise the simulation by turning off the corresponding terms, or simply consider the initial phase before the nonlinearity becomes large.
- Compare the linear growth-rate and k_y linear spectrum in the simulation to the analytical calculation. Show the radial mode structure.
- Show the k_y spectrum in the nonlinear saturated phase. How is it affected by the resolution of the simulation?
- Add a background electric potential (Fourier mode k_y=0) to the dynamical equations, and compare the nonlinear result.

Advise on methods

- Because the system has a periodic direction, a Fourier decomposition in the y direction is very desirable. The linear operators act on each Fourier modes independently and take simple forms. In order to compute the nonlinear term however, it is preferable to transform back from Fourier space to real space (for this the FFTW library is the gold standard).
- A discretisation for the Poisson bracket that is consistent with 2nd order centered finite differences and conserves energy and enstrophy is given by Arakawa. At (x, y) point indices (i, j), the bracket is computed as:

$$\begin{aligned}
\left(\{f, g\}\right)_{i,j} = -\frac{1}{12dxdy} \big(& g_{i,j-1}\left(f_{i+1,j-1} + f_{i+1,j} - f_{i-1,j-1} - f_{i-1,j}\right) \\
& + g_{i,j+1}\left(f_{i-1,j} + f_{i-1,j+1} - f_{i+1,j} - f_{i+1,j+1}\right) \\
& + g_{i-1,j}\left(f_{i-1,j-1} + f_{i,j-1} - f_{i-1,j+1} - f_{i,j+1}\right) \\
& + g_{i+1,j}\left(f_{i,j+1} + f_{i+1,j+1} - f_{i,j-1} - f_{i+1,j-1}\right) \\
& + g_{i-1,j-1}\left(f_{i,j-1} - f_{i-1,j}\right) \\
& + g_{i-1,j+1}\left(f_{i-1,j} - f_{i,j+1}\right) \\
& + g_{i+1,j-1}\left(f_{i+1,j} - f_{i,j-1}\right) \\
& + g_{i+1,j+1}\left(f_{i,j+1} - f_{i+1,j}\right) \big)
\end{aligned}$$

- To avoid artifacts at the non-periodic domain boundaries, it is useful to create buffer zones where artificial dissipation is added (of the form $\nabla_\perp \cdot (D(x) \nabla_\perp g)$ in the equation for a quantity g) such that the linear mode is damped close to the boundary.

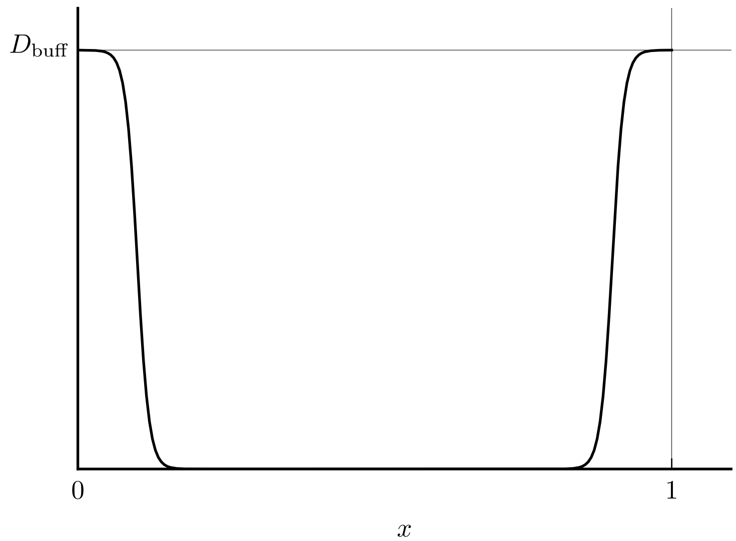

Figure 1: Example profile for the dissipative buffer zones (sum of tanh)