FIXED INCOME

TERM STRUCTURE OF INTEREST RATES

17 April

Traditional approach

Empirical models of bond prices and bond derivatives

1. Merton $dr = \alpha dt + \sigma dZ$
2. Vasicek $dr = (\alpha + \beta r)dt + \sigma dZ$
3. CIR (SR) $dr = (\alpha + \beta r)dt + \sigma r^{(1/2)} dZ$
4. Brennan and Swartz $dr = (\alpha + \beta r)dt + \sigma r dZ$
5. CIR (VR) $dr = \sigma r^{(3/2)} dZ$

More general form in Chan, Karolyi, Longstaff and Sanders (1992):

$dr = (\alpha + \beta r)dt + \sigma r^{\gamma} dZ$

Previous models and others come from using restrictions on the model

- These models are applied to valuation of interest rate contingent claims in
  - Gibson and Schwartz (1990)
  - Jamshidian (1989)
  - Ramaswany and Sundaresan (1986)
  - Sundaresan (1989)

Chan, Karolyi, Longstaff and Sanders (1992) estimate short rate process using GMM

$r_{t+1} – r_t = \alpha + \beta r_t + \varepsilon_{t+1}$

$E(\varepsilon_{t+1}) = 0$ $E(\varepsilon_{t+1}^2) = \sigma^2 r^{2\gamma}$

Tests suggest Merton, Vasicek, and CIR SR models mispecified (γ low)

Unrestricted estimates:

α = 0.0174 (0.87)
β = -0.2213 (-0.53)
γ = 1.48 (3.83)
$R2$ = 2.59% $R2(var)$ = 20.46%

Restricted estimates:

E.g. CIR (SR)

α = 0.0189 (0.94)
β = -0.2339 (-0.66)
γ = 0.5

$R2$ = 1.64%

$R2$ (for var) = 5.46%

- Valuing calls and bonds (under expectation hypothesis, i.e. assuming risk neutral probabilities are same as real probabilities)

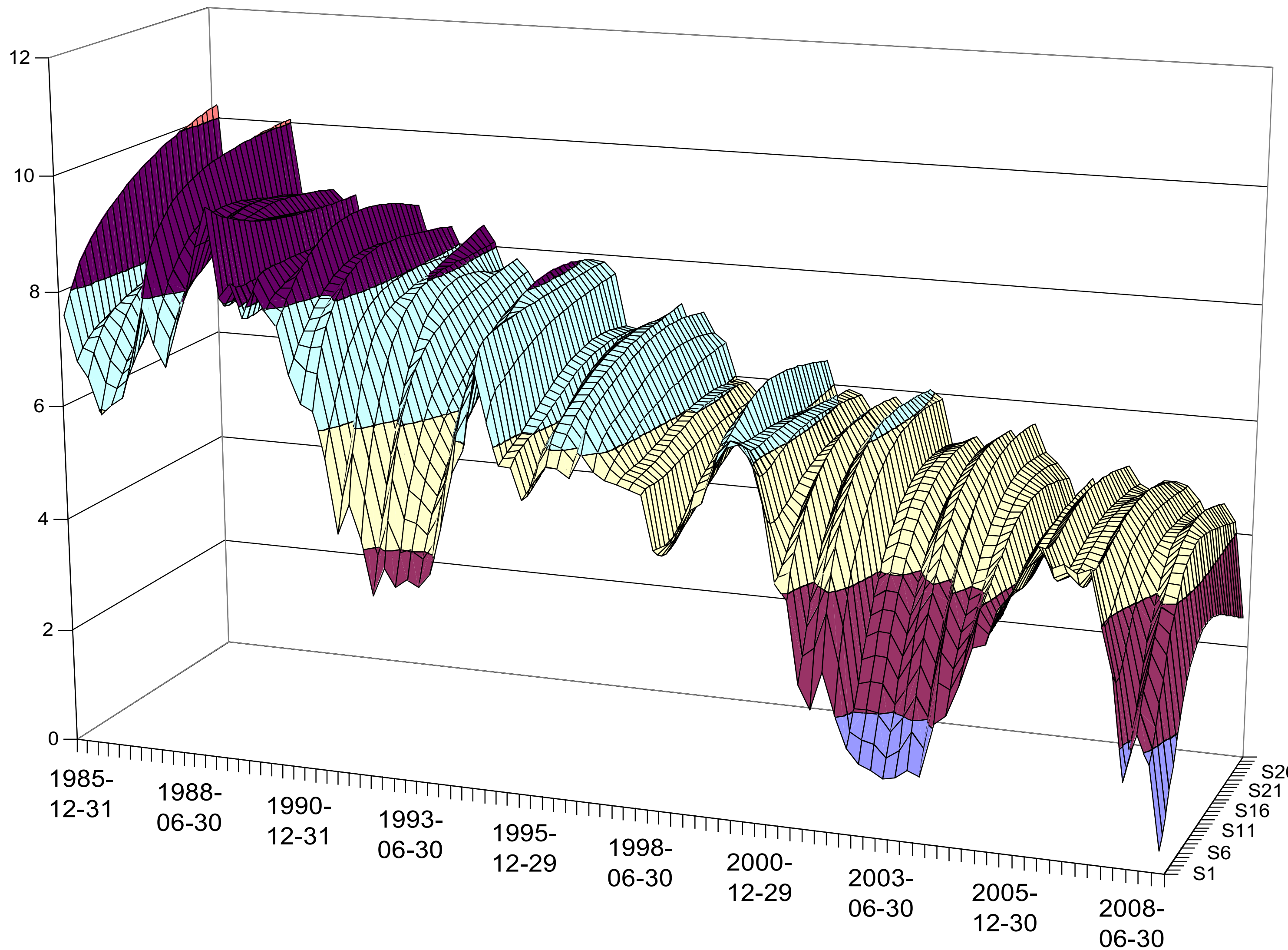
r_0 = 6%	CIR	Unrestricted
• 30year yield	8.11%	6.89%
• 2-year Option EX=95	5.79	5.17
• 2-year Option EX=100	2.56	1.03

This analysis can be further continued by estimating risk neutral probabilities and using those in bond and option valuation.

Problems with traditional approach and EVIDENCE OF LIMITS OF ARBITRAGE

- Duarte, Longstaff, Yu, Review of Financial Studies, 2011
- Greenwood and Vayanos, Review of Financial Studies, 2014
- Greenwood and Hanson, Review of Financial Studies, 2013
- Cochrane and Piazzesi, American Economic Review, 2005

US Zero-coupon Yield Curve (1-30Y) – 1985-2008



Yield curve arbitrage

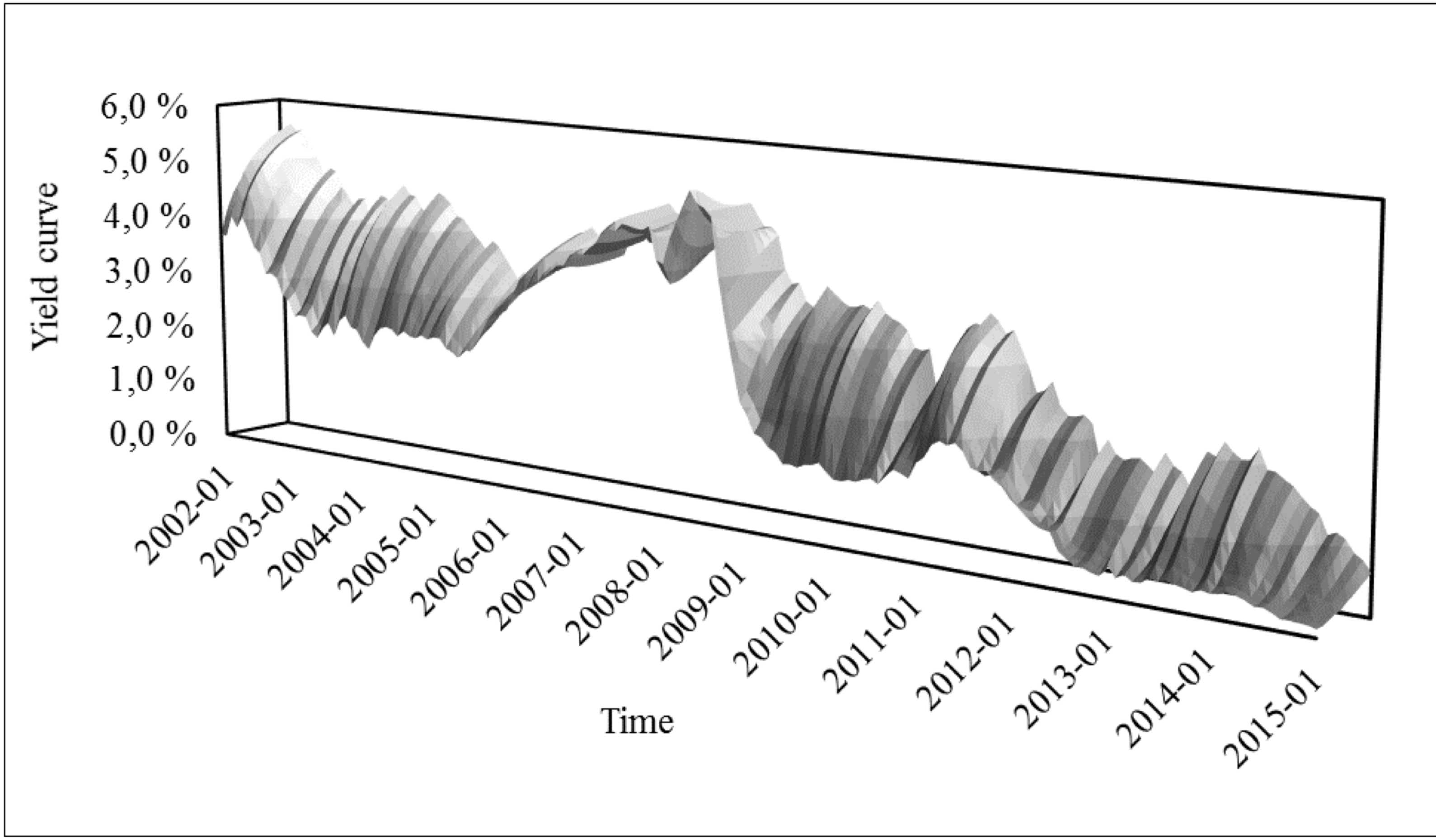
Another major type of fixed income arbitrage involves taking long and short positions at different points along the yield curve. These yield curve arbitrage strategies often take the form of a "butterfly" trade, where, for example, an investor may go long five-year bonds, and short two- and ten-year bonds in a way that zeros out the exposure to the level and slope of the term structure in the portfolio.

Duarte, Longstaff and Yu (2011) show that using a measure of fair value for yields (from a term structure model), and trading the deviations from fair value, they obtain monthly returns of 0.5%-0.6%.

EMPIRICAL APPLICATION (Lassi Karsimus, thesis 2015)

- Some hedge fund styles outperform others in the fixed income space, yield curve arbitrage among the top performers (Duarte, Longstaff and Yu (2007))
- Assumption: yield curve is not arbitrage free at all times
- Goal: locate and trade mispricings until they disappear
- Analytical framework:
  – Use a two-factor short-rate model to produce a yield curve (swap curve)
  – Compare the model curve to the market curve
  – Locate ‘mispricings’
  – Hedge the misprincings with a market-neutral portfolio (compatible with the model)

Data (for modeling)

- 

Methodology – General

- Yield curve is to be modeled by the Cox-Ingresoll-Ross (CIR) 2-factor model, as well as the Longstaff-Schwartz 2-factor model
- 1y and 10y rates are assumed to be most efficiently prices (liquid rates)
- Models have 2 factors and 6 parameters
- Fitting methodology (calibration)
  - For a given month, fit the model exactly to the liquid rates by iterating through *factors*
  - For the sample period, minimize the market-to-model differences of illiquid rates (RSS) by iterating over *parameters*
  - Going forward (trading begins), calibrate again to liquid rates month-by-month by iterating over factors

→ Model may imply mispricings in the illiquid rates

Methodology – CIR2F model

$$P(t,T) = A_1(t,T)A_2(t,T)e^{-x_1(t)B_1(t,T)-x_2(t)B_2(t,T)}$$

Zero-coupon bond prices

$$\mathrm{d}x_1(t) = k_1(\theta_1 - x_1(t))\mathrm{d}t + \sigma_1\sqrt{x_1(t)}\mathrm{d}W_1(t)$$

$$\mathrm{d}x_2(t) = k_2(\theta_2 - x_2(t))\mathrm{d}t + \sigma_2\sqrt{x_2(t)}\mathrm{d}W_2(t),$$

Factor processes, r = x1+ x2

k, *theta* and *sigma* (for x1 and x2) are mean-reversion speed, long-run mean and short-rate volatility, respectively

$$A(t,T) = \left(\frac{2he^{(h+k)(T-t)/2}}{2h + (h+k)(e^{h(T-t)} - 1)}\right)^{2k\theta/\sigma^2}$$

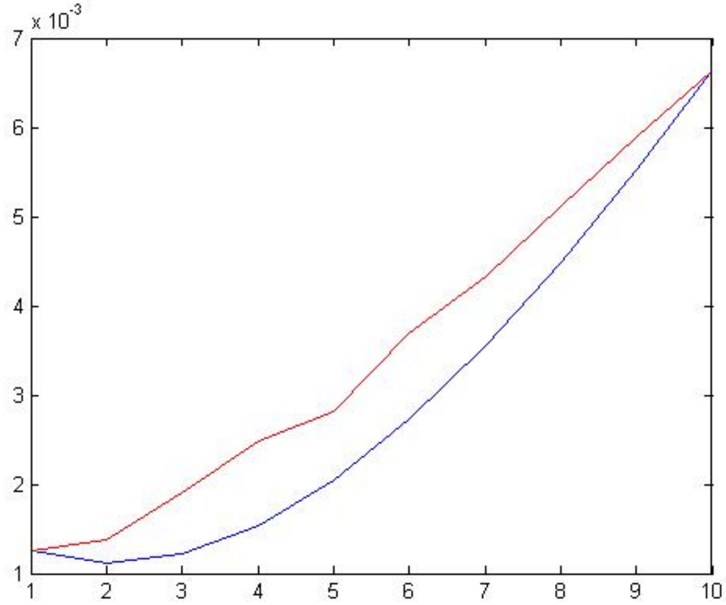
$$B(t,T) = \frac{2(e^{h(T-t)} - 1)}{2h + (h+k)(e^{h(T-t)} - 1)}$$

$$h = \sqrt{k^2 + 2\sigma^2}.$$

x1 and x2 are fitted exactly to **liquid rates;**
k, theta and sigma are calibrated wrt. **illiquid rates,** out-of-the-sample

Methodology – Trading decisions

- The largest difference between the market and the model rates will be traded so that it is assumed to converge (somewhat) to the market curve (e.g., the 7-year rate)
  - If the model rate is higher than the market, then one will go long the rate in question (and vice versa, short)
  - 50% convergence assumed to mean disappearance of mispricings
  - Time limit of 1 year is used to cut a trade if there is no convergence

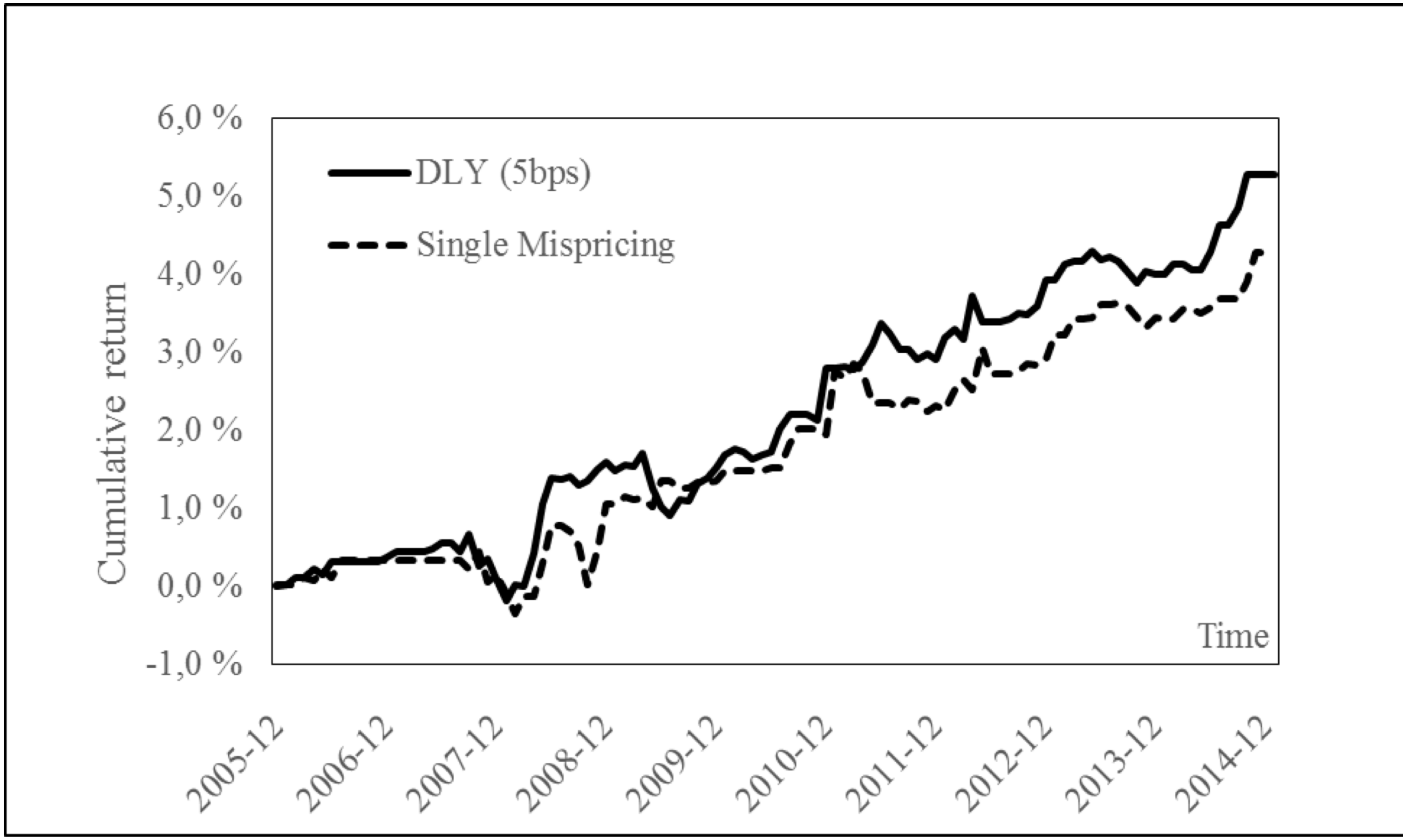


Methodology – Hedging the trades

- ‘Mispriced’ position is hedged by taking opposite (long/short) positions in the liquid rates
  - Differentiate model with respect to the two factors to attain sensitivities (Greeks)
  - Find hedge ratios (w1 and w2) that eliminate the factor risks from the portfolio of the 3 swaps
  - This butterfly trade hedges both duration (parallel shifts) and twists

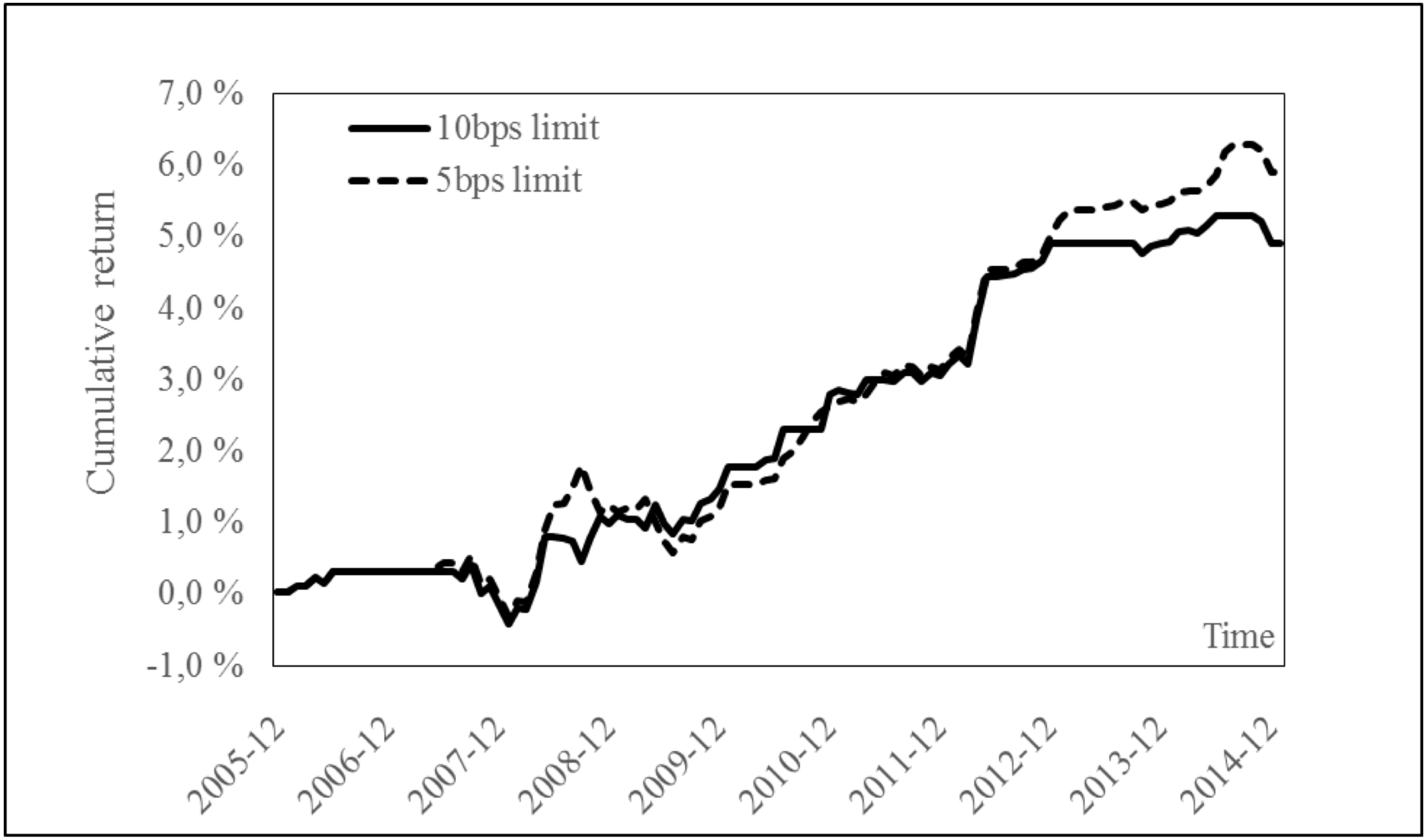
Results – YC arb cumulative returns (1)

- DLY strategy, out-of-the-sample with 4y sample period

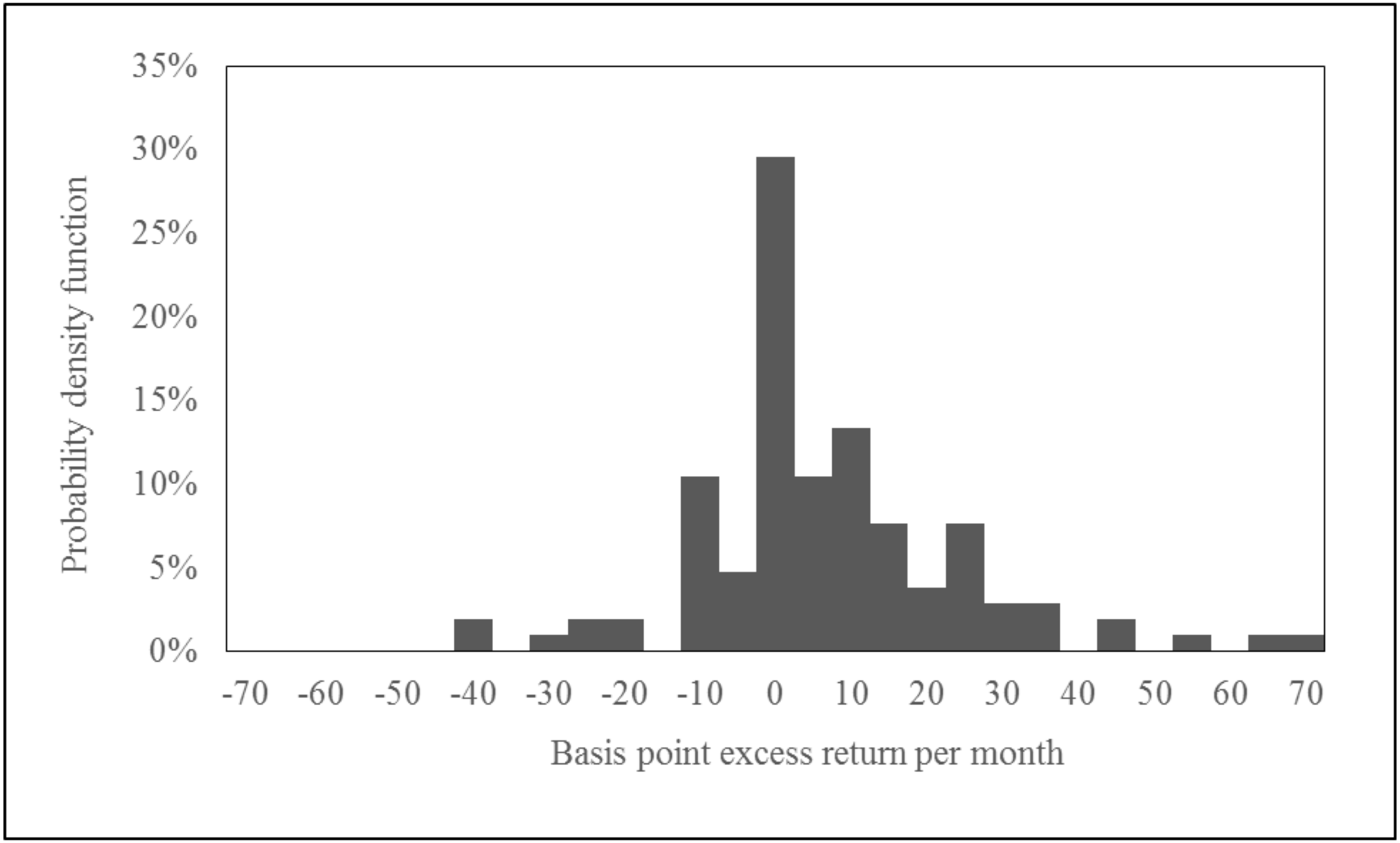


Results – YC arb cumulative returns (2)

- DLY strategy, out-of-the-sample with 4y rolling sample period



Results – YC arb return distribution



Summary conclusions

- Yield curve arbitrage continues to generate attractive risk-adjsuted returns in the EUR swap space (confirming the results in Duarte et al)
- Replicated strategy yields basically pure multifactor alpha
  - Returns statistically significant when controlled by common risk factors
  - R^2 is at 20%, implying limited explanatory power wrt. variations in returns
- Possible explanations for returns and correlations:
  - Strategy is sophisticated enough to face limited competition; the hedging procedure all but guarantees low volatility