

$f(z)$ analytic

$$\sin z = \sin(x+jy) = \sin x \underbrace{\cos(jy)} + \cos x \underbrace{\sin(jy)}$$

$$= \underbrace{\sin x \cosh y}_u + j \underbrace{\cos x \sinh y}_v$$

$$\frac{e^{jy} + e^{-jy}}{2}$$

$$= \cosh y$$

$$\frac{e^{jy} - e^{-jy}}{2j}$$

$$= j \sinh y$$

$$\frac{\partial u}{\partial x} = \cos x \cosh y$$

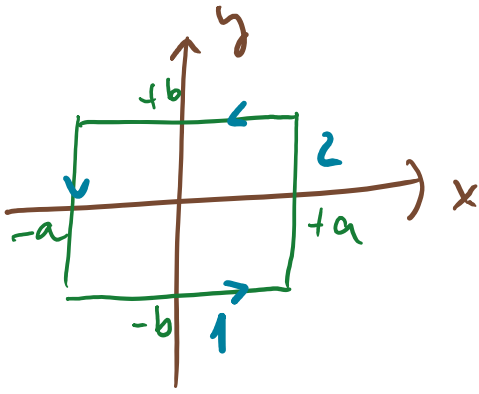
$$\frac{\partial u}{\partial y} = \sin x \sinh y$$

$$\frac{\partial v}{\partial x} = -\sin x \sinh y$$

$$\frac{\partial v}{\partial y} = \cos x \cosh y$$

$$\frac{\partial}{\partial y} \cosh y = \frac{\partial}{\partial y} \frac{e^y + e^{-y}}{2} = \sinh y$$

$$\frac{\partial}{\partial y} \sinh y = \frac{\partial}{\partial y} \frac{e^y - e^{-y}}{2} = \cosh y$$



$$z = x + jy$$

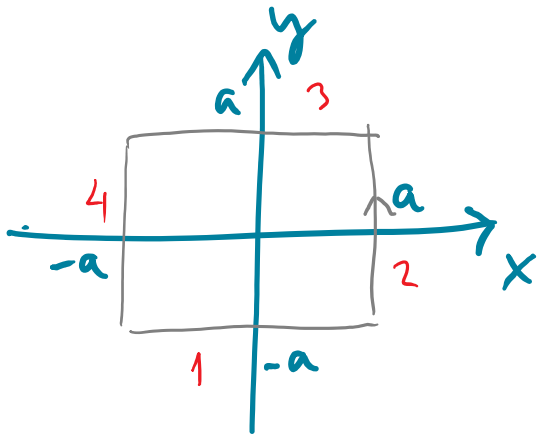
$$1: \int_{-a}^{+a} (x - jb) dx = -2jab$$

$$2: \int_{-b}^{+b} (a + jy) j dy = 2jab$$

$$3: \int_{-a}^{+a} (x + jb) dx = -2jab$$

$$4: \int_{+b}^{-b} (-a + jy) j dy = +2jab$$

$$\oint z dz = 0$$



$$f(z) = \frac{1}{z}$$

$$1: \int_{-a}^a \frac{1}{x-ja} dx$$

$$= \int_{-a}^a \ln(x-ja) = \ln \frac{a-ja}{-a-ja}$$

$$= \ln \frac{\sqrt{2}a e^{-j\pi/4}}{\sqrt{2}a e^{-j3\pi/4}} = \ln e^{j\pi/2} = j\pi/2$$

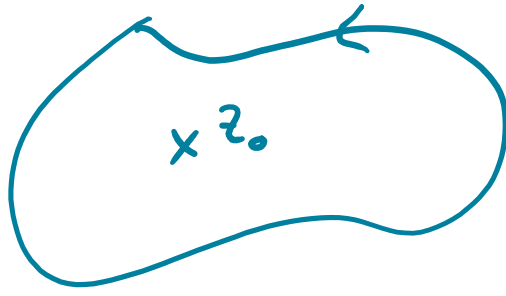
$$2: \int_{-a}^a \frac{1}{a+jy} j dy$$

$$= j \int_{-a}^a \ln(a+jy) \frac{1}{j}$$

$$= \ln \frac{a+ja}{a-ja} \rightarrow$$

$$\int_{-a}^a \frac{1}{-ja+y} dy$$

$$\oint \frac{1}{z} dz = 4 \cdot j \frac{\pi}{2} = 2\pi j$$



$$\oint \frac{f(z)}{z - z_0} dz = 2\pi j f(z_0)$$

$$\bar{A} : \bar{a}\bar{b} = 0 \quad \forall \bar{a}, \bar{b}$$

$$\Rightarrow \bar{A} = 0$$

2.14 (c)

$$\bar{A} : \bar{a}\bar{a}^* = 0 \quad \forall \bar{a}$$

$$\bar{a} = \begin{matrix} \bar{b} + \bar{c} \\ \uparrow \quad \uparrow \\ \text{real} \end{matrix}$$

$$\bar{A} : (\bar{b} + \bar{c})(\bar{b} + \bar{c}) = 0$$

$$= \underbrace{\bar{A} : \bar{b}\bar{b}}_0 + \underbrace{\bar{A} : \bar{c}\bar{c}}_0 + \bar{A} : \bar{b}\bar{c} + \bar{A} : \bar{c}\bar{b}$$

$$\bar{A} : \bar{b}\bar{c} = -\bar{A} : \bar{c}\bar{b}$$

$$\bar{A} : (\bar{b} + j\bar{c})(\bar{b} - j\bar{c}) = 0$$

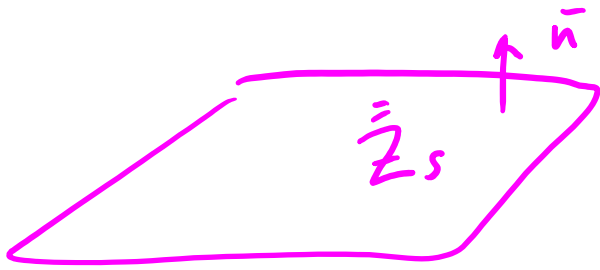
$$= \underbrace{\bar{A} : \bar{b}\bar{b}}_{\neq 0} + \underbrace{\bar{A} : \bar{c}\bar{c}}_{\neq 0} + j(\bar{A} : \bar{b}\bar{c} + \bar{A} : \bar{c}\bar{b})$$

$$j \quad 2 \bar{A} : \bar{c}\bar{b} = 0$$

$$\bar{A} : \bar{a}\bar{a} = 0 \quad \forall \bar{a}$$

$$\bar{A} = \bar{b}\bar{c} - \bar{c}\bar{b}$$

$$\bar{A} : \bar{a}\bar{a} = (\bar{b}\bar{c} - \bar{c}\bar{b}) : \bar{a}\bar{a} = (\bar{b} \cdot \bar{a})(\bar{c} \cdot \bar{a}) - (\bar{c} \cdot \bar{a})(\bar{b} \cdot \bar{a}) = 0$$



$$\vec{E}_t = \vec{Z}_s \cdot \vec{n} \times \vec{H}_t$$

$$2 \cdot 2 \operatorname{Re} \{ \vec{n} \cdot \vec{S} \} = 2 \operatorname{Re} \{ \vec{n} \cdot \vec{E} \times \vec{H}^* \}$$

$$= \vec{n} \cdot \vec{E} \times \vec{H}^* + \vec{n} \cdot \underbrace{\vec{E}^* \times \vec{H}} = 0$$

$$\vec{n} \cdot \vec{Z}_s \cdot \vec{n} \times \vec{H} \times \vec{H}^*$$

$$= -\vec{Z}_s \cdot \vec{n} \times \vec{H} \times \vec{n} \cdot \vec{H}^*$$

$$\vec{H} \cdot \vec{n} \times \vec{E}^*$$

$$= \vec{H} \cdot \vec{n} \times \vec{Z}_s^* \cdot \vec{n} \times \vec{H}^*$$

$$= \vec{H} \cdot \vec{n} \times \vec{Z}_s^* \times \vec{n} \cdot \vec{H}^*$$

$$= -\vec{n} \times \vec{H} \cdot \vec{Z}_s^T \times \vec{n} \cdot \vec{H}^*$$

$$= \vec{H} \cdot \vec{n} \times \vec{Z}_s^T \times \vec{n} \cdot \vec{H}^*$$

$$\vec{H} \cdot \vec{n} \times (\vec{Z}_s^T + \vec{Z}_s^*) \times \vec{n} \cdot \vec{H}^* = 0 \quad \forall \vec{H}$$

$$\Rightarrow \vec{n} \times (\vec{Z}_s^T + \vec{Z}_s^*) \times \vec{n} = 0$$

LOSSLESS SURFACE : $\vec{Z}_s^T = -\vec{Z}_s^*$

$$\vec{Z}_s = Z_s \vec{I}_t \rightarrow Z_s \vec{I}_t = -Z_s^* \vec{I}_t$$

$$\uparrow Z_s = jX_s$$

$$\vec{I}_t, \vec{J}, \vec{K}, \vec{L}$$

$$\vec{Z}_s = \frac{1}{n} \vec{J} \quad \text{PEMC}$$

$$\uparrow$$

$$\bar{n} \times \bar{I}_t$$

$$\bar{Z}_s = \frac{1}{M} \bar{J} \quad \text{PEMC}$$

$$\text{LOSSLESS} \rightarrow -\frac{1}{M} \bar{J} = -\frac{1}{M^*} \bar{J}$$

PEMC

$$M \bar{E} = -\bar{H}$$

$$\bar{E}_t = \bar{Z}_s \cdot \bar{n} \times \bar{H} = \frac{1}{M} \bar{n} \times (\bar{n} \times \bar{H})$$

$$\uparrow \frac{1}{M} \bar{J} = \frac{1}{M} \bar{n} \times \bar{I} \quad \underbrace{-\bar{H}_t}_{-\bar{n} \bar{n} \times \bar{I} \cdot \bar{H}}$$