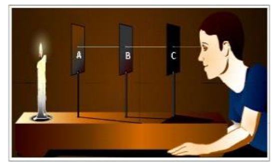
Ray optics & Optical beams

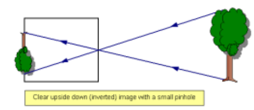
Photonics
(ELEC-E3240)

Zhipei Sun

Photonics Group
Department of Electronics and Nanoengineering
Aalto University

Rectilinear Propagation of Light





Pinhole camera

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Lots of videos on Youtube to show you how to build your own pinhole camera.

The first long-distance (>km) optical data transmission system

The Beacon Tower (~1000BC)

Capacity, Distance, Speed, Working condition

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Reflection of Light

- Refractive index $n = \frac{c_{vacuum}}{c_{material}}$
  - Values for n :
    - $n_{air} = 1$
    - $n_{H_2O} \approx 1{,}3$
    - $n_{SiO_2} \approx 1{,}5$
    - $n_{Si} \approx 3{,}5$

At the interfaces, light is refracted according to the *Snell's law*

$$n_1 \cdot \sin \alpha_1 = n_2 \cdot \sin \alpha_2$$

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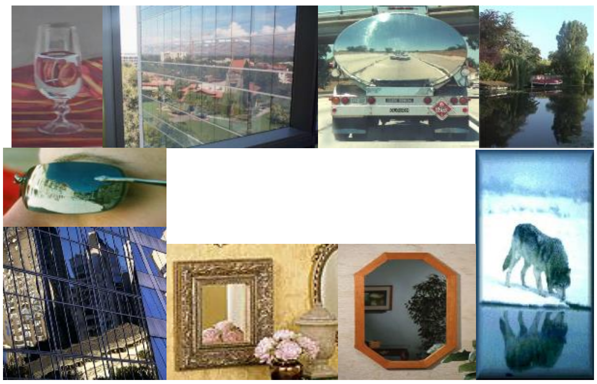
Reflections

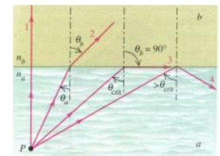


Total Internal Reflection

- Total reflection when entering low-*n* medium

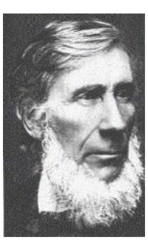
  Critical angle $\sin \alpha_c = \frac{n_2}{n_1}$

  For glass/air, α_c=41.8^o



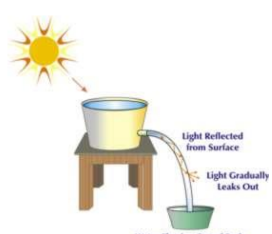
- Total reflection can bind light into the high-*n* material
  - Optical fibers and planar waveguides

Bending light?



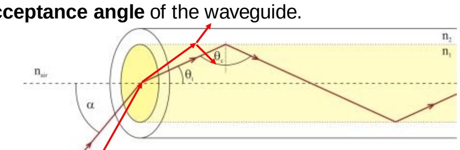
John Tyndall

(1820-1893)



Ray Optical Picture of Light Confinement in Optical Fiber

To successfully launch light into a waveguide, the angle of incidence to the end facet must be less than or equal to the **acceptance angle** of the waveguide.



Apply Snell's Law to light refracting into waveguide:

$\sin \alpha = n_1 \sin \theta_t$

$= \sqrt{n_1^2 - n_2^2}$ Assume critical angle TIR

The refractive indices of the core and cladding therefore determine a **cone of acceptance**.

Numerical Aperture

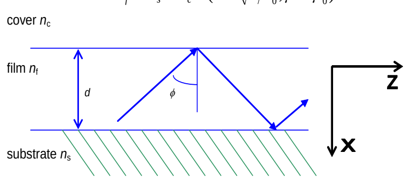
$$NA = n_{air} \sin \alpha = \sqrt{n_1^2 - n_2^2} = n_1\sqrt{2\Delta}$$

Index difference

$$\Delta = \frac{n_1^2 - n_2^2}{2n_1^2} \cong \frac{n_1 - n_2}{n_1}$$

A 3-layer Slab Waveguide: The Ray-Optics Description

$n_f > n_s \geq n_c \quad (n = \sqrt{\varepsilon/\varepsilon_0}, \mu = \mu_0)$



Total internal reflection rule: sinϕ>n_c/n_f

Ray Optics

Rays are used to model the propagation of light through an optical system, by dividing the real light field up into discrete rays that can be computationally propagated through the system by the techniques of ray tracing.

Pierre de Fermat (1601-1665)

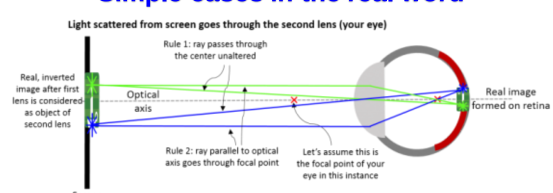
Sir Isaac Newton (1642-1726)

Wiki

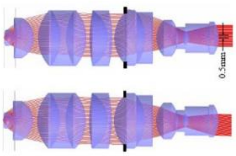
Further Reading: Ray Optics (& Matrix Optics)

- Chapter 2: A. Yariv, Optical Electronics in Modern Communications, 5th edition, Oxford University Press, 1997.
- Chapter 1: B.E.A. Saleh, M.C. Teich, Fundamentals of Photonics, Wiley, 2007
- Chapters 5-6, E. Hecht, Optics, Addison Wesley, 2002
- R. K. Verma, Ray Optics, Discovery Publishing, 2006.

Ray tracing for design Simple cases in the real word



Ray tracing for design
Complicated cases in the real word



Lens design

Microsoft HoloLens

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Is light a wave or particle?

Wave-particle duality

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This question has puzzled scientist for >200 years

Light is a particle! **Light is a wave!**

Isaac Newton (1643-1727)

Christiaan Huygens (1629-1695)
Francesco Maria Grimaldi (1618-1663)
Augustin-Jean Fresnel (1788 to 1827)
Thomas Young (1773 to 1829)
James Clerk Maxwell (1831 to 1879)

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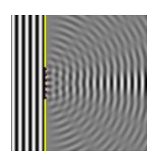
Albert Einstein (1879-1955)

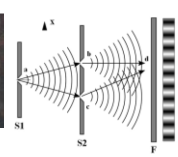
Wave Optics

Wave optics (Electromagnetic theory) studies interference, diffraction, polarization, and other phenomena (e.g., light-matter interaction) for which the ray approximation of geometric optics is not valid.

Christiaan Huygens (1629-1695)



Thomas Young (1773-1829)



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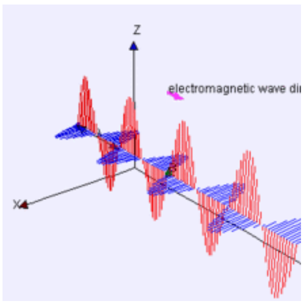
3D Diagram of Optical Wave



Wiki

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Electromagnetic Model of Light Propagation

Maxwell's Equations

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$
$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$$
$$\nabla \cdot \mathbf{B} = 0$$
$$\nabla \cdot \mathbf{D} = \rho$$

James Clerk Maxwell (1831–1879)

$\mathbf{E}(x, y, z, t)$, $\mathbf{H}(x, y, z, t)$: Electric and magnetic field amplitudes are functions of *x, y, z* and *t*.

Electromagnetic Model of Light Propagation

Maxwell's equations

$$\begin{aligned} \nabla \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t} && (1) \\ \nabla \times \mathbf{H} &= \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} && (2) \\ \nabla \cdot \mathbf{B} &= 0 && (3) \\ \nabla \cdot \mathbf{D} &= \rho && (4) \end{aligned}$$

Constitutive relations: $\mathbf{B} = \mu \mathbf{H},\ \mathbf{D} = \varepsilon \mathbf{E}$ ($\mathbf{J}$ = 0) (ρ = 0) $\Longrightarrow$

$$\boxed{\begin{aligned} \nabla \times \mathbf{E} &= -\frac{\partial (\mu \mathbf{H})}{\partial t} \\ \nabla \times \mathbf{H} &= \frac{\partial (\varepsilon \mathbf{E})}{\partial t} \\ \nabla \cdot \mathbf{H} &= 0 \\ \nabla \cdot (\varepsilon \mathbf{E}) &= 0 \end{aligned}}$$

For waveguides (including fibres), we assume operation in a source free medium: ρ = 0, **J**=0. For now, we assume a linear and isotropic medium: permittivity, ε, and permeability, μ, are independent of **E** and its orientation.

Electromagnetic Model of Light Propagation

$$\begin{aligned} \nabla \times \mathbf{E} &= -\frac{\partial (\mu \mathbf{H})}{\partial t} && (1) \\ \nabla \times \mathbf{H} &= \frac{\partial (\varepsilon \mathbf{E})}{\partial t} && (2) \\ \nabla \cdot \mathbf{H} &= 0 && (3) \\ \nabla \cdot (\varepsilon \mathbf{E}) &= 0 && (4) \end{aligned}$$

Take the curl of both sides of Equation 1: $\nabla \times (\nabla \times \mathbf{E}) = \nabla \times (-\frac{\partial \mu \mathbf{H}}{\partial t})$

With μ(r,t) independent of time and position, we get: $\nabla \times \nabla \times \mathbf{E} = -\mu(\nabla \times \frac{\partial \mathbf{H}}{\partial t})$

Reversing the order of the curl and time derivative operators and assuming that ε is time invariant: $\nabla \times \nabla \times \mathbf{E} = -\mu \frac{\partial}{\partial t}(\nabla \times \mathbf{H})$

$$= -\mu\varepsilon \frac{\partial}{\partial t}\left(\frac{\partial \mathbf{E}}{\partial \mathbf{t}}\right) = -\mu\varepsilon\left(\frac{\partial^2 \mathbf{E}}{\partial \mathbf{t}^2}\right).$$

Using a vector identity $\nabla \times \nabla \times \mathbf{E} = \nabla(\nabla \cdot \mathbf{E}) - \nabla^2 \mathbf{E} \overset{(*)}{=} \nabla(-\mathbf{E} \cdot \frac{\nabla \varepsilon}{\varepsilon}) - \nabla^2 \mathbf{E}$, we get:

(*) $\nabla \cdot \mathbf{D} = 0$
$\rightarrow \nabla \cdot \varepsilon \mathbf{E} = \nabla \varepsilon \cdot \mathbf{E} + \varepsilon \nabla \cdot \mathbf{E} = 0$
$\rightarrow \nabla \cdot \mathbf{E} = -\mathbf{E} \cdot \frac{\nabla \varepsilon}{\varepsilon}$

$$\boxed{\nabla^2 \mathbf{E} - \mu\varepsilon \frac{\partial^2 \mathbf{E}}{\partial t^2} = -\nabla\left(\mathbf{E} \cdot \frac{\nabla \varepsilon}{\varepsilon}\right)} \quad (5)$$

Wave Equations

$$\boxed{\nabla^2\mathbf{E} - \mu\varepsilon\frac{\partial^2\mathbf{E}}{\partial t^2} = -\nabla\left(\mathbf{E}\cdot\frac{\nabla\varepsilon}{\varepsilon}\right)} \quad (5)$$

For most structures in guided-wave optics the term $\nabla\varepsilon/\varepsilon$ is negligible, and the wave equation (5) reduces to its homogeneous form:

$$\nabla^2\mathbf{E} - \mu\varepsilon\frac{\partial^2\mathbf{E}}{\partial t^2} = 0$$

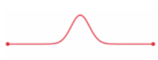
Similarly for **H:** $\nabla^2\mathbf{H} - \mu\varepsilon\frac{\partial^2\mathbf{H}}{\partial t^2} = 0$

In Cartesian coordinates: $\nabla^2\mathbf{E} = \nabla^2 E_x\hat{x} + \nabla^2 E_y\hat{y} + \nabla^2 E_z\hat{z},$

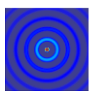
and we get scalar wave equations:

$$\boxed{\nabla^2 E_i - \mu\varepsilon\frac{\partial^2 E_i}{\partial t^2} = 0 \quad (E_i = E_x, E_y, E_z)} \quad (6)$$

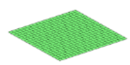
Solutions of this equation describe propagation of disturbances out from the region at a fixed speed in one or in all spatial directions, as do physical waves (e.g., sound / light / wave waves) from plane or localized sources.



A pulse traveling through a string with fixed endpoints as modeled by the wave equation.



Spherical waves coming from a point source.



A solution to the 2D wave equation

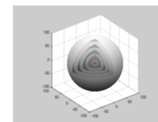
Helmholtz Equation

$$\boxed{\nabla^2 E_i - \mu\varepsilon\frac{\partial^2 E_i}{\partial t^2} = 0 \quad (E_i = E_x, E_y, E_z)} \quad (6)$$

Monochromatic waves: $E_i(\mathbf{r},t) = \mathrm{Re}[\phi(\mathbf{r})e^{i\omega t}]$ $\quad (E_i = E_x, E_y, E_z)$

Then from Equation (6) $\nabla^2 E_i - \mu\varepsilon\frac{\partial^2 E_i}{\partial t^2} = 0$

we get: $\nabla^2\phi + \omega^2\mu\varepsilon\phi = 0$



Cut-away of spherical wavefronts

With $k = \frac{2\pi}{\lambda} = \frac{2\pi n}{\lambda_0} = \frac{2\pi\upsilon n}{c} = \omega\frac{\mathrm{n}}{\mathrm{c}} = \omega\sqrt{\mu\varepsilon}$ we get:

$$\nabla^2\phi + k^2\phi = 0 \quad \text{The Helmholtz Equation}$$

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Electromagnetic Model of Light Propagation

Maxwell's Equations

$$\nabla\times\mathbf{E} = -\mu_o\frac{\partial\mathbf{H}}{\partial t} \qquad \nabla\cdot\mathbf{E} = \frac{\rho}{\varepsilon_o} = 0$$

$$\nabla\times\mathbf{H} = \varepsilon_o n^2\frac{\partial\mathbf{E}}{\partial t} \qquad \nabla\cdot\mathbf{H} = 0$$

$\mathbf{E}(x,y,z,t)$
$\mathbf{H}(x,y,z,t)$

Electric and magnetic field amplitudes are functions of *x, y, z* and *t*.

$$\mathbf{E}(x,y,z,t) = \mathbf{E}'(x,y)e^{j(\omega t-\beta z)}$$

$$\mathbf{H}(x,y,z,t) = \underbrace{\mathbf{H}'(x,y)}_{\text{Stationary wave in } x,\,y}\underbrace{e^{j(\omega t-\beta z)}}_{\text{Propagating wave in } z,\,t}$$

Electromagnetic Model of Light Propagation

Maxwell's equations: ∇ x **E** in Cartesian coordinates

$$-\mu_o\frac{\partial\mathbf{H}}{\partial t} = \nabla\times\mathbf{E}$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ E_x & E_y & E_z \end{vmatrix}$$

$$= \mathbf{i}\begin{vmatrix} \partial/\partial y & \partial/\partial z \\ E_y & E_z \end{vmatrix} - \mathbf{j}\begin{vmatrix} \partial/\partial x & \partial/\partial z \\ E_x & E_z \end{vmatrix} + \mathbf{k}\begin{vmatrix} \partial/\partial x & \partial/\partial y \\ E_x & E_y \end{vmatrix}$$

$$= \mathbf{i}\left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right) - \mathbf{j}\left(\frac{\partial E_z}{\partial x} - \frac{\partial E_x}{\partial z}\right) + \mathbf{k}\left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right)$$

$E = E'(x,y)e^{j(\omega t-\beta z)}$ $\qquad$ $H = H'(x,y)e^{j(\omega t-\beta z)}$

$\frac{\partial E_{x,y,z}}{\partial z} = -j\beta E_{x,y,z}$ $\qquad$ $\frac{\partial H_{x,y,z}}{\partial z} = -j\beta H_{x,y,z}$

$\frac{\partial E_{x,y,z}}{\partial t} = j\omega E_{x,y,z}$ $\qquad$ $\frac{\partial H_{x,y,z}}{\partial t} = j\omega H_{x,y,z}$

x-components:

$$-\mu_o\frac{\partial H_x}{\partial t} = \frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}$$

$$-\mu_o j\omega H_x = \frac{\partial E_z}{\partial y} + j\beta E_y \quad (7)$$

y-components:

$$-\mu_o\frac{\partial H_y}{\partial t} = -\left(\frac{\partial E_z}{\partial x} - \frac{\partial E_x}{\partial z}\right)$$

$$-\mu_o j\omega H_y = -\frac{\partial E_z}{\partial x} - j\beta E_x \quad (8)$$

z-components:

$$-\mu_o\frac{\partial H_z}{\partial t} = \frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}$$

$$-\mu_o j\omega H_z = \frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} \quad (9)$$

Electromagnetic Model of Light Propagation

Maxwell's equations: ∇ x **H** in Cartesian coordinates

$$\varepsilon_o n^2 \frac{\partial \mathbf{E}}{\partial t} = \nabla \times \mathbf{H}$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ H_x & H_y & H_z \end{vmatrix}$$

$$= \mathbf{i}\begin{vmatrix} \partial/\partial y & \partial/\partial z \\ H_y & H_z \end{vmatrix} - \mathbf{j}\begin{vmatrix} \partial/\partial x & \partial/\partial z \\ H_x & H_z \end{vmatrix} + \mathbf{k}\begin{vmatrix} \partial/\partial x & \partial/\partial y \\ H_x & H_y \end{vmatrix}$$

$$= \mathbf{i}\left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right) - \mathbf{j}\left(\frac{\partial H_z}{\partial x} - \frac{\partial H_x}{\partial z}\right) + \mathbf{k}\left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right)$$

$$E = E'(x,y)e^{j(\omega t - \beta z)} \qquad H = H'(x,y)e^{j(\omega t - \beta z)}$$

$$\frac{\partial E_{x,y,z}}{\partial z} = -j\beta E_{x,y,z} \qquad \frac{\partial H_{x,y,z}}{\partial z} = -j\beta H_{x,y,z}$$

$$\frac{\partial E_{x,y,z}}{\partial t} = j\omega E_{x,y,z} \qquad \frac{\partial H_{x,y,z}}{\partial t} = j\omega H_{x,y,z}$$

x-components:	*y*-components:	*z*-components:
$\varepsilon_o n^2 \frac{\partial E_x}{\partial t} = \frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}$	$\varepsilon_o n^2 \frac{\partial E_y}{\partial t} = -\left(\frac{\partial H_z}{\partial x} - \frac{\partial H_x}{\partial z}\right)$	$\varepsilon_o n^2 \frac{\partial E_z}{\partial t} = \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}$
$\varepsilon_o n^2 j\omega E_x = \frac{\partial H_z}{\partial y} + j\beta H_y$	$\varepsilon_o n^2 j\omega E_y = -\frac{\partial H_z}{\partial x} - j\beta H_x$	$\varepsilon_o n^2 j\omega E_z = \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}$
(10)	(11)	(12)

Electromagnetic Model of Light Propagation

$$\frac{\partial E_z}{\partial y} + j\beta E_y = -j\omega\mu_o H_x \tag{7}$$

$$\frac{\partial H_z}{\partial y} + j\beta H_y = j\omega\varepsilon_o n^2 E_x \tag{10}$$

$$-j\beta E_x - \frac{\partial E_z}{\partial x} = -j\omega\mu_o H_y \tag{8}$$

$$-j\beta H_x - \frac{\partial H_z}{\partial x} = j\omega\varepsilon_o n^2 E_y \tag{11}$$

Eliminate H_y

$$E_x\left(\mu_o\varepsilon_o\omega^2 n^2 - \beta^2\right) = -j\omega\mu_o \frac{\partial H_z}{\partial y} - j\beta \frac{\partial E_z}{\partial x} \tag{13}$$

Eliminate E_x

$$H_y\left(\mu_o\varepsilon_o\omega^2 n^2 - \beta^2\right) = -j\omega\varepsilon_o n^2 \frac{\partial E_z}{\partial x} - j\beta \frac{\partial H_z}{\partial y} \tag{14}$$

Eliminate E_y

$$H_x\left(\mu_o\varepsilon_o\omega^2 n^2 - \beta^2\right) = j\omega\varepsilon_o n^2 \frac{\partial E_z}{\partial y} - j\beta \frac{\partial H_z}{\partial x} \tag{15}$$

Eliminate H_x

$$E_y\left(\mu_o\varepsilon_o\omega^2 n^2 - \beta^2\right) = -j\omega\mu_o \frac{\partial H_z}{\partial x} - j\beta \frac{\partial E_z}{\partial y} \tag{16}$$

Key parameters: Ez, Hz

Electromagnetic Model of Light Propagation

$$H_y\left(\mu_o\varepsilon_o\omega^2 n^2 - \beta^2\right) = -j\omega\varepsilon_o n^2 \frac{\partial E_z}{\partial x} - j\beta \frac{\partial H_z}{\partial y} \tag{14}$$

$$H_x\left(\mu_o\varepsilon_o\omega^2 n^2 - \beta^2\right) = j\omega\varepsilon_o n^2 \frac{\partial E_z}{\partial y} - j\beta \frac{\partial H_z}{\partial x} \tag{15}$$

$$\varepsilon_o n^2 j\omega E_z = \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} \tag{12}$$

$$\Rightarrow \quad \frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + \left(\mu_o\varepsilon_o\omega^2 n^2 - \beta^2\right)E_z = 0$$

Electromagnetic Model of Light Propagation

$$E_x\left(\mu_o\varepsilon_o\omega^2 n^2 - \beta^2\right) = -j\omega\mu_o \frac{\partial H_z}{\partial y} - j\beta \frac{\partial E_z}{\partial x} \tag{13}$$

$$E_y\left(\mu_o\varepsilon_o\omega^2 n^2 - \beta^2\right) = -j\omega\mu_o \frac{\partial H_z}{\partial x} - j\beta \frac{\partial E_z}{\partial y} \tag{16}$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -j\omega\mu_o H_z \tag{9}$$

$$\Rightarrow \quad \frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + \left(\mu_o\varepsilon_o\omega^2 n^2 - \beta^2\right)H_z = 0$$

Electromagnetic Model of Light Propagation

$$\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + \left(\mu_o \varepsilon_o \omega^2 n^2 - \beta^2\right) E_z = 0$$

$$\frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + \left(\mu_o \varepsilon_o \omega^2 n^2 - \beta^2\right) H_z = 0$$

2nd order differential equations in E_z and H_z
Solutions are **Eigenfunctions / Eigenmodes**

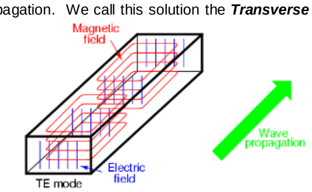
Naming Modes: TE, TM, HE and HM modes

$$\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + \left(\mu_o \varepsilon_o \omega^2 n^2 - \beta^2\right) E_z = 0$$

$$\frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + \left(\mu_o \varepsilon_o \omega^2 n^2 - \beta^2\right) H_z = 0$$

Case 1: E_z and H_z are both zero. A trivial solution (no wave propagates).

Case 2, **TE mode:** $E_z = 0$ and $H_z \neq 0$ In this type of wave, the electric field is transverse to the direction of propagation and the magnetic field is parallel to the direction of propagation. We call this solution the ***Transverse Electric (TE) mode***.

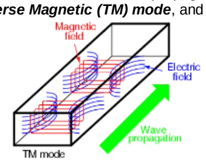


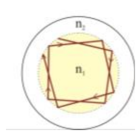
Naming Modes: TE, TM, HE and HM modes

$$\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + \left(\mu_o \varepsilon_o \omega^2 n^2 - \beta^2\right) E_z = 0$$

$$\frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + \left(\mu_o \varepsilon_o \omega^2 n^2 - \beta^2\right) H_z = 0$$

Case 3, **TM mode**: $H_z = 0$ and $E_z \neq 0$ In this type of wave, the magnetic field is transverse to the direction of propagation and the electric field is parallel to the direction of propagation. We call this solution the ***Transverse Magnetic (TM) mode***, and it is an example of a meridional ray.

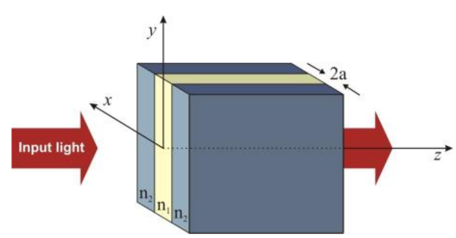




Case 4, **HE/HM mode**: $E_z \neq 0$ and $H_z \neq 0$ These types of waves are called ***hybrid-mode*** waves, and are denoted as ***HE*** or ***HM***, depending upon if the transverse electric or magnetic field is larger respectively. These waves are examples of helical (skew) rays.

EM model solution for 2D infinite slab waveguide

We will now solve the previous wave equations for the special case of a 2D slab waveguide, infinite in the *y-z* plane, thickness 2*a* in the *x*-direction, and with light propagation along the *z*-axis.



EM model solution for 2D infinite slab waveguide

A 2D slab waveguide, infinite in *y*, has EM wave solutions with no *y*-dependency.

$$\frac{\partial^2 E_z}{\partial x^2}+\cancelto{0}{\frac{\partial^2 E_z}{\partial y^2}}+\left(\mu_o\varepsilon_o\omega^2 n^2-\beta^2\right)E_z=0 \qquad \frac{\partial^2 H_z}{\partial x^2}+\cancelto{0}{\frac{\partial^2 H_z}{\partial y^2}}+\left(\mu_o\varepsilon_o\omega^2 n^2-\beta^2\right)H_z=0$$

If we now just consider only a TM wave solution ($H_z = 0$), then the wave equation we need to solve is just:

$$\frac{\partial^2 E_z}{\partial x^2}+\left(\mu_o\varepsilon_o\omega^2 n^2-\beta^2\right)E_z=0$$

where $n = n_1$ for $-a < x < a$ (core)
$n = n_2$ for $|x| > a$ (cladding)

The general solution of this can be expressed in two parts, depending upon whether the wave is in the core or the cladding.

We also apply boundary conditions:
E_z and its derivative must be continuous at the core/cladding interface ($x = \pm a$).
E_z must tend to zero at $x = \pm\infty$.

EM model solution for 2D infinite slab waveguide

General solutions :

$$E_z(x,z)=A\sin(\kappa_1 x-C)\exp(-j\beta z) \qquad -a\le x\le a \qquad \text{(i.e. in the core)}$$

$$E_z(x,z)=B\exp(-\kappa_2 x)\exp(-j\beta z) \qquad x\ge a \qquad \text{(i.e. in the cladding)}$$

$$E_z(x,z)=B\exp(\kappa_2 x)\exp(-j\beta z) \qquad x\le -a \qquad \text{(i.e. in the cladding)}$$

$$\kappa_1=\sqrt{\mu_o\varepsilon_o\omega^2 n^2-\beta^2}=\sqrt{k^2 n_1^2-\beta^2} \qquad \text{(i.e. } \kappa_1 \text{ real in core)}$$

$$\kappa_2=\sqrt{\beta^2-\mu_o\varepsilon_o\omega^2 n^2}=\sqrt{\beta^2-k^2 n_2^2} \qquad \text{(i.e. } \kappa_2 \text{ imaginary in cladding)}$$

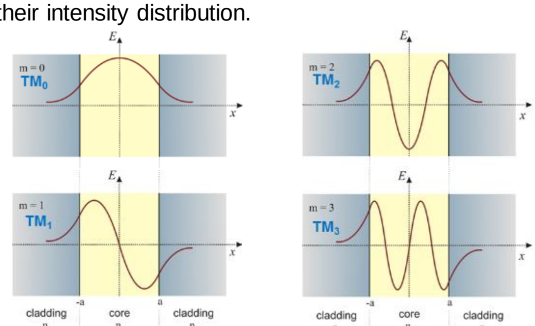
Therefore, within the core we have a propagating wave in the *z*-direction, with standing wave solutions in the *x*-direction.

Within the cladding, we also have a propagating wave in the *z*-direction, but with an exponentially decaying (evanescent) wave in the *x*-direction.

Cyclic nature of sinusoidal solution implies discrete wavefunctions (modes) .

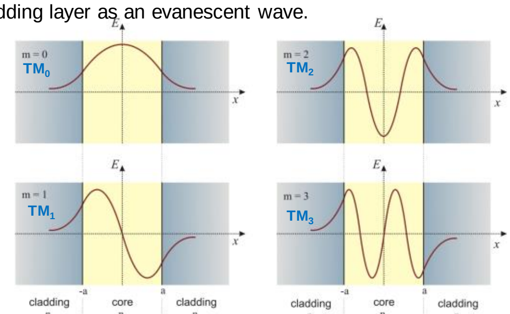
EM model solution for 2D infinite slab waveguide

Mode numbering: Numbers correspond to # of nulls in their intensity distribution.



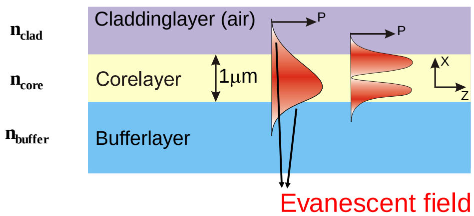
Evanescent Wave

Note: A significant fraction of the energy propagates within the cladding layer as an evanescent wave.



Evanescent wave coupling is commonly used in photonic and nanophotonic devices as waveguide sensors

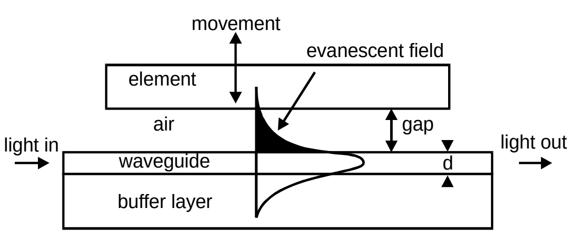
Evanescent wave



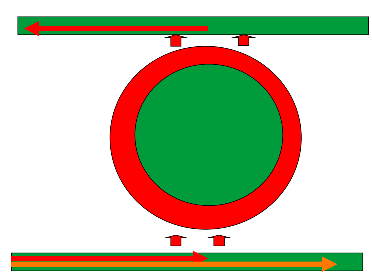
Evanescent wave: Useful or Useless

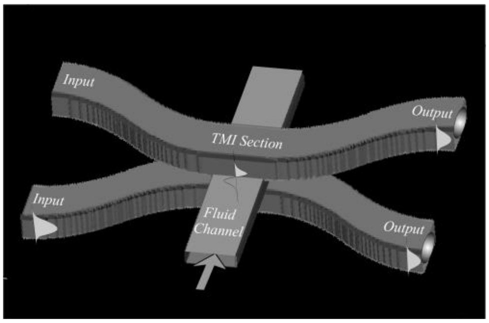
- Evanescent waves are electromagnetic waves that penetrate tens of nm through a surface and propagate along the surface.
- They many be used to sample volume "outside" a waveguide for sensing
- They may be used to locally excite a sample on the other side of an interface for imaging

Evanescent wave



Disc resonator as add/drop filter





G.J.M.Krijnen et.al., J MM, **9**, 203-205 (1999).

Aalto University
School of Electrical
Engineering

Waveguide modes

•In optical waveguide, the radiation is restricted to travel in certain "modes" (i.e., Only a finite number of modes will be guided. The spectrum of β for guided modes is discrete)
•Every eigenvalue β corresponds to a distinct mode of the system and every mode has a unique field distribution (profile).
•Most modes will not be guided and the values of β will lead to unguided modes (radiation modes). The spectrum of β for unguided modes is continuous (i.e. there are infinite number of unguided modes).
•All modes are orthogonal. Each mode is unique and cannot be described in terms of other modes.
•The modes of a given system form a complete set. Any continuous distribution of field can be described as a superposition of the appropriately weighted modes of the waveguide.

Mode Characteristics

The field amplitude is related to the optical power carried in the waveguide. The power is calculated by integrating the z component of the Poynting vector over the cross sectional area:

$$S_z = \frac{1}{2}\mathrm{Re}(\mathbf{E}\times\mathbf{H}\cdot\hat{z})$$

The time-averaged power in a TE-mode is:

$$P_z = \frac{1}{2}\int_{-\infty}^{\infty} E_y H_x dx = \left(\frac{\beta}{2\omega\mu_0}\right)\int_{-\infty}^{\infty}\left|E_y^2\right|dx$$

Optical mode confinement:

Fraction of the power contained in the core is $\frac{P_{core}}{P_{total}} = \frac{\int_{-a}^{a} E_y(x)H_x^*(x)dx}{\int_{-\infty}^{\infty} E_y(x)H_x^*(x)dx}$