FINANCIAL MARKETS THEORY / Matti Suominen

PART II: Financial markets under imperfect information

- Explicit models of securities markets:

- How is information reflected in securities prices

- Modeling demand and supply of liquidity

- Rationalizing observed deviations from continuous time arbitrage models

- Limits of arbitrage

1. Introduction

An Introduction to the Theory of Rational Expectations under Asymmetric Information:

- Hayek (1945): "We must look at the price system as...a mechanism for communicating information if we want to understand its real function... The most significant fact about this system is the economy of knowledge with which it operates, or how little the individual participants need to know in order to be able to take the right action...only the most essential information is passed on…"

- Walrasian markets: People are not aware of the mechanism that generates prices; no one learns anything from prices, people are merely constrained by the prices.

- Example of a Walrasian model: n consumers each observe a different signal y_i on the payoff to the S shares. Walrasian demand: each consumer with information y_i has a demand for shares $x_i(p, y_i)$: Market clearing requires that $\sum_{i=1}^{n} x_i(p, y_i) = S$ which gives a market clearing price $p(y)$, where $y = (y_1, y_2, \dots)$.

- RE literature: Prices contain information and affect demand (and supply) in a different way. Demand for shares takes into account the information that is contained in the fact that $p = p(y)$.

EXAMPLE

Share payoff *d* can be either *High*(100) or *Low*(0).

2 agents both observe an independent signal $s_i \in \{H, L\}$, where
$Pr[s_i = d] = q > 1/2$.

Suppose the Walrasian demand for the asset (reduced form) is given by:

$$x_i = \begin{cases} S - 2p \; if \; s_i = H \\ S - 4p \; if \; s_i = L \end{cases}$$

Suppose you observe H and the other trader L. Then market clearing implies that:

$S = S - 2p + S - 4p$

or

$$p = \frac{S}{6}$$

Your demand is: $\frac{2S}{3}$ and your friends $\frac{S}{3}$

Rational expectations under asymmetric information

- EXAMPLE continued:

In the previous example, you should realize when $P = S/6$ that the other agent must have observed $s_2 = L$. Given this you would want to reduce your demand. Likewise your friend would like to increase his demand. If you are allowed to condition on the price, the equilibrium price must be such that neither one of you wants to recontract.

Suppose that knowing both signals s_1 and s_2 your demands are:

$$x_i = \begin{cases} S - p & if \quad s_1 = s_2 = H \\ S - 3p & if \quad s_1 \neq s_2 \\ S - 5p & if \quad s_1 = s_2 = L \end{cases}$$

Suppose you observe *H* and the other trader *L*. Then there exists a fully revealing market price *p* such that market clears:

$$S = S - 3p + S - 3p$$

or

$$p = \frac{S}{6}$$

Now your demand is: $\frac{S}{2}$ and your friends $\frac{S}{2}$

-It is a co-incidence that the price is the same as before - it need not be.

-Here agents understand the model and understand that a low (high) price reflects bad (good) information of the other trader.

Milgrom and Stokey: "Information, Trade and Common Knowledge," JET, 1982.

- Central result: Regardless of the institutional structure, if the initial allocation is *ex ante* Pareto-optimal, then the receipt of private information cannot create any incentives to trade. Their result depends crucially on the assumption that it is common knowledge when a trade is carried out that it is feasible and mutually acceptable to all of the participants.

- Intuition: Groucho Marx Theorem: "I`d never join a club that would have me for a member".

Example:

- Let $\theta \in \{1, 2\}$ denote the state of the economy and x a variable that is correlated with the state of the economy. The joint probability distribution is:

	$\theta = 1$	$\theta = 2$
$x = 1$	0.2	0.05
$x = 2$	0.05	0.15
$x = 3$	0.05	0.05
$x = 4$	0.15	0.05
$x = 5$	0.05	0.2

- Traders observe some partitions that are informative of the realizations of x:

$$\widehat{P_1}: \{x = 1 \ or \ x = 2\}, \{x = 3 \ or \ x = 4\}, \{x = 5\}$$
$$\widehat{P_2}: \{x = 1\}, \{x = 2 \ or \ x = 3\}, \{x = 4 \ or \ x = 5\}$$

- Assume both agents are risk-neutral and suppose the following bet is proposed: if $\theta = 1$ agent 2 pays one dollar to agent 1, if $\theta = 2$ agent 1 pays one dollar to agent 2.

Suppose that $x = 3$ occurs. Is there trade?

- Naive behavior: both players accept the bet (as 0.2 > 0.1).

- Rational expectations: Agent 1 reasons as follows: I know it is either 3 or 4, he can think it is either 2 or 3; or alternatively, 4 or 5. If he knows it is 4 or 5, then the state must be 4 in which case I am willing to trade.

However, he must understand that if I am willing to trade, it must be that the state is not 5 and so the state is 4, in which case he is not willing to trade. So if he is willing to trade he must observe {2 or 3} and so the true state is 3.

As I am risk neutral I am indifferent to accepting the bet. Agent 2 can also figure out that the state is 3 so he is indifferent to taking the bet. He knows that if the state is 2 I will not accept the bet. The reason is that the state must be 2 given that had the state been 1 he would not have proposed the bet.

Second key result in that paper:

-Theorem 2: There exists an equilibrium with no trade where prices are fully revealing, i.e., prices reflect the information of all traders.

2. Model of investment under uncertainty

- Initial wealth W_0, gross risk free rate R = (1 + r_f), return on risky asset u = E(u) + ε

- 2 stages: Investors trade in securities markets at the beginning of period, after which payoffs are realized at the end of the period.

- E(u) > 0 and ε is normally distributed.

- E(ε) = 0, var [ε] = σ_ε^2 > 0.

- Large number of competitively behaving traders who maximize their wealth at the end of period 1.

$$\max V(W_1) = -e^{-aW_1}, a > 0$$

where

$$W_1 = RW_0 + X[u - RP]$$

here $a =$ coefficient of risk aversion and $X =$ amount invested in shares and $P =$ the share price.

Traders' problem can be written as

$$\max_{X_1} EV\,(W_1) = \; -e^{-a[E[W_1]-a/2var(W_1)]}$$

$$\max_{X_1} EV(W_1) = \; -e^{-a[RW_0+X\{E(u)-RP\}-a/2X^2var(u)]}$$

$$\max_{X_1} V(W_1) = \; -e^{-a\left[RW_0\,+\,X\{E\,(u)-RP\}-a/2X^2\sigma_\varepsilon^2\,\right]}$$

$$\Longrightarrow X = \frac{E\,(u) - \; RP}{a\sigma_\varepsilon^2}$$

- Suppose per capita supply of the security is S / m, where *m* is the number of investors in the market. Market clearing requires:

$$\frac{S}{m} = \mathrm{X} = \frac{E(u) - RP}{a\sigma_\varepsilon^2}$$

or

$$\mathrm{P} = \frac{E(u) - \frac{S}{m} a\sigma_\varepsilon^2}{R}$$

Implying that expected stock return is

$$R_S = \frac{E\ (u)}{\frac{E\,(u) - \frac{S}{m}\, a\sigma_\varepsilon^2}{R}} = \frac{R}{1 - \frac{S}{m}\, a\, \frac{\sigma_\varepsilon^2}{E\,(u)}}$$

- Expected return is a function of $\frac{S}{m}$, a and $\frac{\sigma_\varepsilon^2}{E(u)}$

- Think when could we expect changes in these variables?

3. Grossman & Stiglitz: On the Impossibility of Informationally Efficient Markets

-Walrasian markets: People are not aware of the mechanism that generates prices.

- RE literature: Prices contain information and affect demand and supply in a different way. If private information is the only reason to trade then information is perfectly revealed in equilibrium and there is no trade. There must be other reasons why people trade in reality. Other reasons to trade could be: changes in preferences, e.g. risk aversion, private investment opportunities, wealth, liquidity preferences, dynamic trading strategies,...

- Also, if information is (privately) costly information cannot be perfectly revealed as in rational expectations models. Otherwise there are no incentives to acquire information. Something must prevent uninformed agents from learning everything. Very important insight. In reality information acquisition is a large industry. What is the implications for market efficiency theorem!

The Model

-Initial wealth W_0, risk free rate R, return on risky asset $u = \theta + \varepsilon$

- 3 stages: In period 0 traders choose whether to become informed, they trade in securities markets at the beginning of period one, after which payoffs are realized at the end of the period one.

-Informed traders observe θ if they pay c.

-θ and ε are multivariate normally distributed.

-Eε = 0, E$\varepsilon\theta = 0, var[u \mid \theta] = \sigma_\varepsilon > 0$.

-Large number of competitively behaving traders, fraction λ of which acquire information (observe θ). All traders maximize the wealth at the end of period 1.

max $V(W_1) = -e^{-aW_1}, a > 0$

where

$W_1 = RW_0 + X[u - RP]$

here a = coefficient of risk aversion

X = amount invested in shares.

Informed traders

Informed traders maximize:

$$\max_{X_I} EV(W_1|\theta) = -e^{-a\left[E\left[W_{i|}\theta\right]-a/2var(W_{1i}|\theta)\right]}$$

$$\max_{X_I} EV(W_1|\theta) = -e^{-a\left[RW_0 + X_1\{E(u^*|\theta)-RP\}-a/2X_1^2 var\,(u^*|\theta)\right]}$$

$$\max_{X_I} V(W_1|\theta) = -e^{-a\left[RW_0+X_1\{\theta-RP\}-a/2X_1^2\sigma_\varepsilon^2\right]}$$

$$\Rightarrow X_I = \frac{\theta - RP}{a\sigma_\varepsilon^2}$$

So informed traders demand is linear in θ.

<u>Uninformed traders</u>

-Assume the per capita supply of the asset, x, is uncertain so uninformed cannot perfectly infer θ from $P(\theta, x)$.

-In particular assume that x is also normally distributed, independent of θ and ε.

-Trick: Assume the distribution (u| P) is normal with some mean $E\ [u \mid P\ ]$, which is linear in P, and a constant variance: var [u| P]. We will later verify that this is the case.

-It is assumed that the uninformed traders can condition on price when choosing their demand: The uninformed traders then maximize E[V (W_1) | P] and, through similar arguments as before, choose:

$$X_u = \frac{E\ [u \mid P] - \ RP}{a\ \cdot var\ [u \mid P]}$$

- These can be thought of as "Limit orders"

Equilibrium

Market clearing requires

$\lambda X_1 + (1 - \lambda) X_U = x$

$$\Longrightarrow \frac{\theta}{a\sigma_\varepsilon^2} - \frac{x}{\lambda} = \frac{-(1-\lambda)}{\lambda} X_U + \frac{RP}{a\sigma_\varepsilon^2}$$

The left hand side is independent of *P*. So the right hand side must also depend only on θ and *x*. Same is true also if we multiply by $a\sigma_\varepsilon^2$ and add $\frac{a\sigma_\varepsilon^2 Ex}{\lambda}$ to both sides.

-Define w_λ as

$$w_\lambda(\theta, x) = \theta - \frac{a\sigma_\varepsilon^2 [x - Ex]}{\lambda} = \frac{-(1-\lambda)}{\lambda} \frac{\sigma_\varepsilon^2 [E[u \mid P] - RP]}{a \cdot var[u \mid P]} + RP + \frac{a\sigma_\varepsilon^2 Ex}{\lambda}$$

-Rhs of the above eqn. is linear on *P* given our assumptions. w_λ on the other hand, is linear in θ and *x*.

-Under the above assumptions observing price is equivalent to observing $w_\lambda(\theta, x)$

-w_λ on the other hand is jointly normal with *u*.

-Given that:

$$X_u = \frac{E[u|w_\lambda] - RP}{a \cdot var\ [u|w_\lambda]}$$

where $E[u|w_\lambda]$ is linear in w_λ and $var[u|w_\lambda]$ is a constant.

-Use this in market clearing and check that $P = \alpha + \beta w_\lambda$. Verify that the conditional distribution $(u \mid P)$ is normal with some mean $E[u|P_\lambda] = E[u|w_\lambda]$, $\mathrm{var}[u|P_\lambda] = \mathrm{var}[u|w_\lambda]$.

Comparative Statics

-Note that P and w are informationally equivalent. Furthermore w is a mean preserving spread of θ, i.e.,

$$E\,[w \mid \theta] = \theta$$

and

$$var\,[w|\theta] = \frac{a^2\,\sigma_\varepsilon^4 var[x]}{\lambda^2}$$

-So we can do some comparative statics already on the informativeness of prices: What happens as we change $a, var[x], \sigma_\varepsilon^2, \lambda$?

<u>Overall Equilibrium</u>

- λ is determined so that the expected profits to the informed traders are zero.

-It can be shown that:

$$\frac{EV(W_I^{\lambda})}{EV(W_U^{\lambda})} = e^{ac}\sqrt{\frac{var\ (u|\theta)}{var\ (u|\ w)}} \ \equiv\ \gamma(\lambda)$$

-Equilibrium requires $\gamma(\lambda) = 1$.

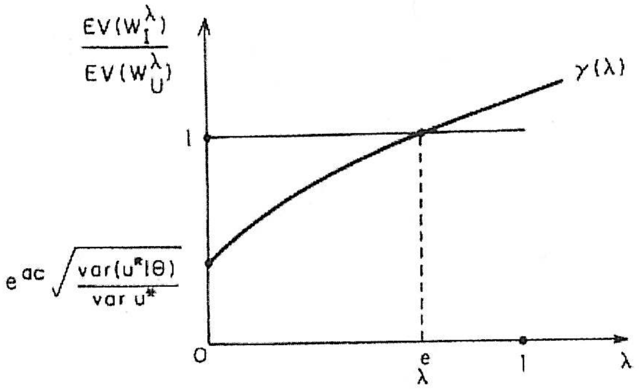

FIGURE 1

<u>Conclusion</u>

-As variance of x goes down, prices do not become more informative, but λ is reduced and trading volume goes down.

-Informativeness of prices depends only on a, c and $n = \frac{\sigma_\theta^2}{\sigma_\varepsilon^2}$.

-What are the implications for "Efficient markets"?