4. Kyle (1985) and Admati and Pfleiderer (1988)

Kyle (1985)

Kyle (1985) model starts from a very different assumption compared to Grossmann and Stiglitz (1980) that there is just one informed trader who possesses private information about the value of the asset, which is not revealed through public sources. The model describes how the informed trader divides his trades across periods to maximize his profits and studies the resulting financial market equilibrium. Traders are risk neutral.

Single auction equilibrium:

The ex post liquidation value of the asset is v where v is normally distributed with mean p_0 and variance Σ_0. Noise trading is $u \sim N\,(0, \sigma_u^2)$. u (number of shares) and v are independent. Informed trader observes v but not u. He submits order x (number of shares) to the market maker who observes only the net demand $y = x + u$. Market maker is willing to trade any quantity y at price $P = E(v \mid y)$ (earns zero profit in equilibrium).

There exists a linear equilibrium

$$X = \beta(v - p_0) \text{ and } P(y) = p_0 + \lambda y.$$

Proof: Suppose the market maker follows such linear pricing rule then informed trader maximizes:

$$\max_x E\,\{[v - P(x+u)]x|v\} = \max_x E\left\{\left[v - \left(p_0 + \lambda(x+u)\right)\right]x|v\right\} = (v - p_0 - \lambda x)x$$

First order condition gives

$$x = \frac{(v - p_0)}{2\lambda}$$

Thus $\beta = \frac{1}{2\lambda}$.

Now we have to check that the market makers pricing rule is consistent with zero profits. For this we have to check what is $E(v|y) = E\left(v|\frac{(v-p_0)}{2\lambda} + u = y\right)$.
Observing y is equivalent to observing the following noisy signal

$$s = p_0 + \frac{y}{\beta} = v + \frac{u}{\beta}.$$

Now

$$E(v|y) = E(v|s) = \frac{p_0/\Sigma_0 + s\beta^2/\sigma_u^2}{1/\Sigma_0 + \beta^2/\sigma_u^2} = p_0 + \frac{y\beta/\sigma_u^2}{1/\Sigma_0 + \beta^2/\sigma_u^2}$$

Equilibrium prevails when

$$\frac{\beta/\sigma_u^2}{1/\Sigma_0 + \beta^2/\sigma_u^2} = \frac{\frac{1}{2\lambda}\Sigma_0}{\frac{1}{4\lambda^2}\Sigma_0 + \sigma_u^2} = \lambda \Rightarrow \lambda = \frac{1}{2}\left(\frac{\Sigma_0}{\sigma_u^2}\right)^{1/2}.$$

Therefore $\beta = \left(\frac{\Sigma_0}{\sigma_u^2}\right)^{-1/2} = \sigma_u/{\Sigma_0}^{1/2}$.

Now

$$\Sigma_1 = \frac{1}{1/\Sigma_0 + \beta^2/\sigma_u^2} = \Sigma_0/2$$

so that in case of one trading period half of insider´s information can be said to be revealed to the markets.

Try to characterize a two period Kyle model.

Admati and Pfleiderer (1988)

All traders are risk neutral. Assets termination value:

$$F_T = \overline{F} + \sum_{t=1}^{T} \delta_t$$

$\delta_t \sim N\,(0, 1)$ and is publicly announced at the beginning of period t. n_t informed traders obtain access to a costly signal $\delta_{t+1}+\ \varepsilon_t$, where $\varepsilon_t \sim N\,(0, \phi_t)$ in the beginning of period t and submit market orders x_t^i to the market maker. n_t is determined endogenously in the model. Denote by φ_t the total amount of liquidity trading in period *t*, where $\varphi_t \sim N\,(0, \Psi_t)$. As in Kyle model a competitive market maker observes only

$$w_t\ =\ \sum_{i=1}^{n_t} x_t^i\ +\ \varphi_t$$

and trades the net demand for the asset at a price

$$P\ =\ E[F_T\mid(w_1, \dots, w_t), \textstyle\sum_{\tau=1}^{t} \delta_\tau].$$

The paper show that there exists an equilibrium where

$$P\ =\ \overline{F} + \sum_{\tau=1}^{t} \delta_\tau +\ \lambda_t(n_t, \Psi_{\mathrm{t}}\,)w_t$$

and

$$x_t^i\ =\ \beta_t\,(\delta_{t+1} + \varepsilon_t)$$

Proof: Informed trader maximizes:

$$\max_{x_t^i} E\left(x_t^i(F_T - P_t)|\, \delta_{t+1} + \varepsilon_t, \textstyle\sum_{\tau=1}^{t} \delta_\tau\right)$$

$$= \max_{x_t^i} E\left(x_t^i \left(\delta_{t+1} - \lambda_t\left(n_{t,}\Psi_t\right)w_t\right)|\, \delta_{t+1} + \varepsilon_t\right)$$

$$= \max_{x_t^i} E\left(x_t^i\left(\delta_{t+1} - \lambda_t\left(n_{t,}\Psi_t\right)\left[x_t^i + (n_t - 1)\beta_t(\delta_{t+1} + \varepsilon_t) + \varphi_t\right]\right)|\, \delta_{t+1} + \varepsilon_t\right)$$

$$= \max_{x_t^i}\left(x_t^i \frac{var(\delta_{t+1})}{var(\delta_{t+1}) + \phi_t}(\delta_{t+1} + \varepsilon_t) - x_t^i\lambda_t\left(n_{t,}\Psi_t\right)\left[x_t^i + (n_t - 1)\beta_t(\delta_{t+1} + \varepsilon_t)\right]\right)$$

First order condition gives:

$$\beta_t = \frac{var(\delta_{t+1})}{(n_t + 1)\lambda_t(var(\delta_{t+1}) + \phi_t)}$$

Zero profit condition for the market maker implies that

$$\lambda_t = \frac{cov\left(\delta_{t+1,}w_t\right)}{var(w_t)} = \frac{n_t\beta_t var(\delta_{t+1})}{(n_t\beta_t)^2(var(\delta_{t+1}) + \emptyset_t) + \Psi_t}$$

Substituting for β gives:

$$\lambda_t = \frac{var\ (\delta_{t+1})}{(n_t + 1)} \sqrt{\frac{n_t}{(var(\delta_{t+1}) + \emptyset_t)\Psi_t}}$$

Note: λ_t is decreasing in Ψ_t and n_t. Note also that liquidity traders' expected losses are λ_t Ψ_t and therefore increase in $\lambda_{t.}$ Implication: Liquidity traders have an incentive to trade amongst themselves. That is, to trade in periods when other liquidity trades are trading and when Ψ_t is as large as possible.

This will attract more informed traders and n_t will be larger. Because of competition among informed traders this will however only benefit the liquidity traders, as it reduces λ_t, and further increases their incentives to concentrate their trading it the same period.

Admati and Pfleiderer argue that this kind of behavior leads to more concentrated trading in mornings and afternoons in the stock markets.

3.2 Other related papers

Foster and Viswanathan (1990)

A Theory of the Interday Variations in Volume, Variance, and Trading Costs in Securities Markets

In an adverse selection model of a securities market with one informed trader and several liquidity traders, they study the implications of the assumption that the informed trader has more information on Monday than on other days. They examine the interday variations in volume, variance, and adverse selection costs, and find that on Monday the trading costs and the variance of price changes are highest, and the volume is lower than on Tuesday. These effects are stronger for firms with better public reporting and for firms with more discretionary liquidity trading.

Idea: Information accumulates on weekends, hence there is more asymmetric info on Mondays (high volatility).

Uninformed trades move their trading to other days when they can trade against less informed traders (due to public information arrival the informed traders´ informational advantage declines over time), hence the volume on Mondays is the lowest.

Comment
- Markets have been developed for liquidity trader type of institutions (e.g. pension funds) to trade amongst themselves to avoid trading against informed traders. What are the implications of this?

Empirical findings on trading volume and volatility

-**Karpof** (1988): strong contemporaneous correlation between volatility and trading volume.

-Informed trading?

-Large literature on conditional volatility models: GARCH

-**Andersen** (1996): "Mixture of Distribution Hypothesis" models information arrival to markets to privately informed traders whose trading reveals information to market participants. Views volume and volatility as related mainly to information arrival. Conditional volatility changes as the probability of information arrival changes over time. Information arrival causes increases in volume as traders try to profit from their information prior to this being reflected in prices.

-**Lamoreux and Lastrapes,** (1991): Past Trading volume helps predict futurevolatility.

-**Easley, Kiefer, O`Hara and Paperman** (1996); **Easley, Hvidkjaer and O`Hara** (2002): Time varying probability of informed trading "PIN measure"

-**Suominen** (2004) shows that due to uninformed traders behavior, as the probability of informed trading increases volume can in fact decrease.

-Perhaps the correlation between volume and volatility is caused by something else, than private information!