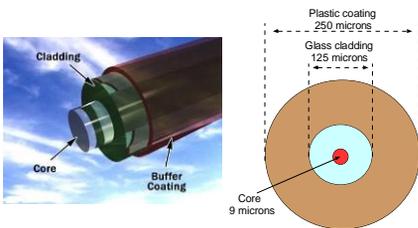


- ### Lecture Topics
- Course introduction
  - Ray optics & optical beams
  - Waveguides / optical fibers
  - Optical amplifiers
  - Fiber optics and applications
  - Lab work
  - Silicon photonics
  - Structural coloration
  - Plasmonics
  - Poster Presentation & discussion

## Optical Fibers

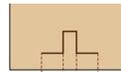


Charles Kuen Kao

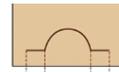
## Refractive index in Fiber



- **Step-index fibre:** The refractive index changes radically between the core and the cladding  
e.g., Water based "fibre": core - water ( $n=1.333$ ); cladding - air ( $n=1$ )
- **Graded-index fibre:** The refractive index gradually decreases away from the center of the core.

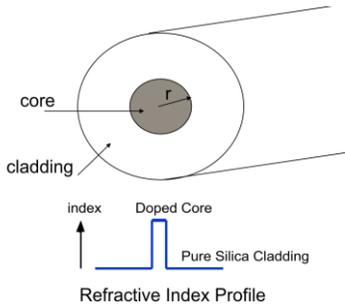


Step-index fibre



Graded-index fibre

### Step-index Optical Fiber

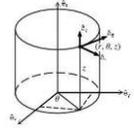


### Wave Equation in Cylindrical Coordinates

$$\nabla^2 E_t - \mu\epsilon \frac{\partial^2 E_t}{\partial t^2} = 0$$

With  $\vec{E}(\vec{r}, t) = \vec{E}(\vec{r})e^{i\omega t}$  wave equation takes the form:

$$\nabla^2 \mathbf{E} + k_0^2 n^2 \mathbf{E} = 0$$

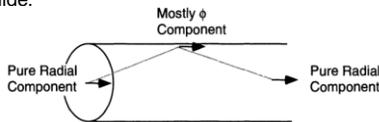


Electric field in cylindrical coordinates:

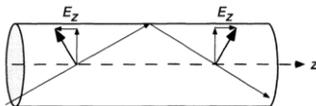
$$\mathbf{E}(r, \phi, z) = \hat{r}E_r(r, \phi, z) + \hat{\phi}E_\phi(r, \phi, z) + \hat{z}E_z(r, \phi, z)$$

### Light Propagation in Optical Fiber

There is coupling between a radial field component and an azimuthal field component while traveling in a cylindrical waveguide:



The longitudinal component does not couple to the other two components as it propagates, even after a reflection at a cylindrical surf:



### Light Propagation in Optical Fiber

$E_z$  does not couple to  $E_r$  or  $E_\phi$ . Therefore it is possible to write the scalar wave equation for  $E_z$  directly in cylindrical coordinates.

Wave equation for  $E_z$  :

$$\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial E_z}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 E_z}{\partial \phi^2} + \frac{\partial^2 E_z}{\partial z^2} + k_0^2 n^2 E_z = 0$$

Similarly, We can obtain  $E_r$ ,  $E_\phi$ ,  $H_r$  and  $H_\phi$  from  $E_z$  and  $H_z$ .

## Light Propagation in Optical Fiber

Separation of variables:

$$E_z(r, \phi, z) = R(r)\Phi(\phi)Z(z)$$

Substituting to the wave equation gives:

$$R''\Phi Z + \frac{1}{r}R'\Phi Z + \frac{1}{r^2}R\Phi''Z + R\Phi Z'' + R\Phi Z'' + k_0^2 n^2 R\Phi Z = 0$$

Multiplying by  $r^2/R\Phi Z$  and with  $Z(z) = e^{-i\beta z}$  we get:

$$r^2 \frac{R''}{R} + r \frac{R'}{R} + \frac{\Phi''}{\Phi} - r^2 \beta^2 + k_0^2 n^2 r^2 = 0$$

## Light Propagation in Optical Fiber

We use a standard separation technique:

$$r^2 \frac{R''}{R} + r \frac{R'}{R} - r^2 \beta^2 + k_0^2 n^2 r^2 = -\frac{\Phi''}{\Phi} = \nu^2 \quad (8)$$

( $\nu$  is the separation constant)

Equation 8 can be solved directly for  $\Phi(\phi)$ :

$\Phi''(\phi) = -\nu^2 \Phi$  has the solution  $Ae^{-i\nu\phi}$  (A constant,  $\nu$  integer)

Substituting to equation 8 gives:

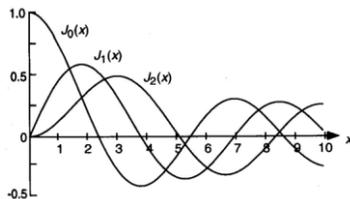
$$r^2 \frac{\partial^2 R}{\partial r^2} + r \frac{\partial R}{\partial r} + r^2 (k_0^2 n^2 - \beta^2 - \frac{\nu^2}{r^2}) R = 0$$

This is the **Bessel differential equation**, and the solutions are called Bessel functions of order  $\nu$ .

## Light Propagation in Fiber Core: Bessel Functions

With  $(k_0^2 n^2 - \beta^2)$  positive, the solution is:

$$R(r) = J_\nu(\kappa r) \quad (\kappa^2 = k_0^2 n^2 - \beta^2) \quad (\text{in the core})$$



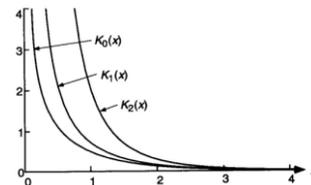
Bessel functions of the first kind

$$R(r) = J_\nu(\kappa r)$$

## Light Propagation in Fiber Cladding: Bessel Functions

With  $(k_0^2 n^2 - \beta^2)$  negative, the solution is:

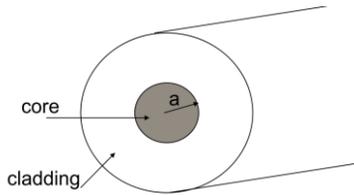
$$R(r) = K_\nu(\gamma r) \quad (\gamma^2 = \beta^2 - k_0^2 n^2) \quad (\text{in the cladding})$$



Modified Bessel functions of the second kind

$$R(r) = K_\nu(\gamma r)$$

## Field Distribution in the Step-index Fiber



for  $r < a$ :

$$\begin{aligned} E_z(r, \phi, z) &= AJ_v(\kappa r) e^{i\nu\phi} e^{-i\beta z} \\ H_z(r, \phi, z) &= BJ_v(\kappa r) e^{i\nu\phi} e^{-i\beta z} \end{aligned} \quad (9)$$

for  $r > a$ :

$$\begin{aligned} E_z(r, \phi, z) &= CK_v(\gamma r) e^{i\nu\phi} e^{-i\beta z} \\ H_z(r, \phi, z) &= DK_v(\gamma r) e^{i\nu\phi} e^{-i\beta z} \end{aligned}$$

A, B, C and D are determined from the boundary conditions

## Field Distribution in the Step-index Fiber

To match the boundary conditions at the core/cladding interface, azimuthal field components  $E_\phi$  and  $H_\phi$  need to be known, in addition to the longitudinal components. We use Maxwell's equations:

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} = -\mu \frac{\partial \mathbf{H}}{\partial t} = -i\omega\mu \mathbf{H}$$

$$\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} = \varepsilon \frac{\partial \mathbf{E}}{\partial t} = i\omega\varepsilon \mathbf{E}$$

and we get:

$$\begin{aligned} E_\phi &= \frac{-i}{\alpha^2} \left( \frac{\beta}{r} \frac{\partial E_z}{\partial \phi} - \omega\mu \frac{\partial H_z}{\partial r} \right) \\ E_r &= \frac{-i}{\alpha^2} \left( \frac{\mu\omega}{r} \frac{\partial H_z}{\partial \phi} + \beta \frac{\partial E_z}{\partial r} \right) \\ H_\phi &= \frac{-i}{\alpha^2} \left( \omega\varepsilon \frac{\partial E_z}{\partial r} + \frac{\beta}{r} \frac{\partial H_z}{\partial \phi} \right) \\ H_r &= \frac{-i}{\alpha^2} \left( \beta \frac{\partial H_z}{\partial r} - \frac{\omega\varepsilon}{r} \frac{\partial E_z}{\partial \phi} \right) \end{aligned} \quad (10) \quad \alpha^2 = k_0^2 n^2 - \beta^2$$

- Positive in the core
- Negative in the cladding (for allowed values of  $\beta$ )

## Field Distribution in the Step-index Fiber

Insert the longitudinal fields in equations 9 to equations 10:

In the core region ( $r < a$ ) we get:

$$\begin{aligned} E_r &= \frac{-i\beta}{\kappa^2} \left( A\kappa J'_v(\kappa r) + \frac{i\omega\mu\nu}{\beta r} BJ_v(\kappa r) \right) e^{i\nu\phi} e^{-i\beta z} \\ E_\phi &= \frac{-i\beta}{\kappa^2} \left( \frac{i\nu}{r} AJ_v(\kappa r) - \frac{\omega\mu}{\beta} B\kappa J'_v(\kappa r) \right) e^{i\nu\phi} e^{-i\beta z} \\ H_r &= \frac{-i\beta}{\kappa^2} \left( B\kappa J'_v(\kappa r) - \frac{i\omega\varepsilon_{core}\nu}{\beta r} AJ_v(\kappa r) \right) e^{i\nu\phi} e^{-i\beta z} \\ H_\phi &= \frac{-i\beta}{\kappa^2} \left( \frac{i\nu}{r} BJ_v(\kappa r) + \frac{\omega\varepsilon_{core}}{\beta} A\kappa J'_v(\kappa r) \right) e^{i\nu\phi} e^{-i\beta z} \end{aligned}$$

Where  $J'_v(\kappa r) = dJ_v(\kappa r) / d(\kappa r)$

## Field Distribution in the Step-index Fiber

Insert the longitudinal fields in equations 9 to equations 10:

In the cladding region ( $r > a$ ) we get:

$$\begin{aligned} E_r &= \frac{i\beta}{\gamma^2} \left( C\gamma K'_v(\gamma r) + \frac{i\omega\mu\nu}{\beta r} DK_v(\gamma r) \right) e^{i\nu\phi} e^{-i\beta z} \\ E_\phi &= \frac{i\beta}{\gamma^2} \left( \frac{i\nu}{r} CK_v(\gamma r) - \frac{\omega\mu}{\beta} D\gamma K'_v(\gamma r) \right) e^{i\nu\phi} e^{-i\beta z} \\ H_r &= \frac{i\beta}{\gamma^2} \left( D\gamma K'_v(\gamma r) - \frac{i\omega\varepsilon_{clad}\nu}{\beta r} CK_v(\gamma r) \right) e^{i\nu\phi} e^{-i\beta z} \\ H_\phi &= \frac{i\beta}{\gamma^2} \left( \frac{i\nu}{r} DK_v(\gamma r) + \frac{\omega\varepsilon_{clad}}{\beta} C\gamma K'_v(\gamma r) \right) e^{i\nu\phi} e^{-i\beta z} \end{aligned}$$

Where  $K'_v(\gamma r) = dK_v(\gamma r) / d(\gamma r)$

### Field Distribution in the Step-index Fiber

In order to determine the propagation constant  $\beta$  and the amplitude coefficients A, B, C and D, we apply the boundary conditions:

- At  $r=a$  the four tangential components  $E_z$ ,  $E_\phi$ ,  $H_z$  and  $H_\phi$  must be continuous. We write the four boundary-value equations in matrix form and then set the determinant of the matrix equal to zero:

$$\begin{bmatrix} J_v(\kappa a) & 0 & -K_v(\gamma a) & 0 \\ 0 & J_v(\kappa a) & 0 & -K_v(\gamma a) \\ \frac{\beta v}{a\kappa^2} J_v(\kappa a) & i \frac{\omega\mu}{\kappa} J_v'(\kappa a) & \frac{\beta v}{a\gamma^2} K_v(\gamma a) & i \frac{\omega\mu}{\gamma} K_v'(\gamma a) \\ -i \frac{\omega\epsilon_{core}}{\kappa} J_v'(\kappa a) & \frac{\beta v}{a\kappa^2} J_v(\kappa a) & -i \frac{\omega\epsilon_{clad}}{\gamma} K_v'(\gamma a) & \frac{\beta v}{a\gamma^2} K_v(\gamma a) \end{bmatrix} \begin{bmatrix} A \\ B \\ C \\ D \end{bmatrix} = 0$$

### Field Distribution in the Step-index Fiber

Once  $\beta$  is determined from Equation (11), three of the coefficients (A, B, C, and D) can be determined in terms of the fourth by solving the individual boundary-value equations of the matrix. Let's relate coefficients A and B to each other:

Using the continuity of  $E_\phi$  and  $H_\phi$  we get, respectively:

$$B = \frac{i v \beta}{\omega \mu a} \left[ \frac{1}{\kappa^2} + \frac{1}{\gamma^2} \right] \left[ \frac{J_v'(\kappa a)}{\kappa J_v(\kappa a)} + \frac{K_v'(\gamma a)}{\gamma K_v(\gamma a)} \right]^{-1} A \quad (12a)$$

*(using the electric field continuity)*

$$B = \frac{i \omega \kappa a}{\beta v} \left[ \frac{n_{core}^2}{\kappa} \frac{J_v(\kappa a)}{J_v(\kappa a)} + \frac{n_{clad}^2}{\gamma} \frac{K_v(\gamma a)}{K_v(\gamma a)} \right] \left[ \frac{1}{\kappa^2} + \frac{1}{\gamma^2} \right]^{-1} A \quad (12b)$$

*(using the magnetic field continuity)*

The quantity B/A is of particular interest because it is a measure of the relative amount of  $E_z$  and  $H_z$  in a mode (i.e.  $B/A = H_z/E_z$ ). Note that the ratio is purely imaginary, and  $E_z$  and  $H_z$  are out of phase by  $\pi/2$ .

### Field Distribution in the Step-index Fiber

For nontrivial solutions (i.e. nonzero amplitudes) the four equations will simultaneously equal zero if and only if the determinant of the matrix equals zero. We get the "characteristic equation" for the step-index fiber:

$$\frac{\beta^2 v^2}{a^2} \left[ \frac{1}{\gamma^2} + \frac{1}{\kappa^2} \right]^2 = \left[ \frac{J_v'(\kappa a)}{\kappa J_v(\kappa a)} + \frac{K_v'(\gamma a)}{\gamma K_v(\gamma a)} \right] \left[ \frac{k_0^2 n_{core}^2 J_v'(\kappa a)}{\kappa J_v(\kappa a)} + \frac{k_0^2 n_{clad}^2 K_v'(\gamma a)}{\gamma K_v(\gamma a)} \right] \quad (11)$$

This equation requires numerical or graphical solution. There is only one unknown:  $\beta$ . The terms  $\kappa$  and  $\gamma$  are functions of  $\beta$  and the local index. Due to the oscillatory nature of  $J_v(\kappa a)$ , there can be several values of  $\beta$  for a given structure.

There are two dimensional degrees of freedom in a cylindrical waveguides, so the solutions to the wave equation are labelled with two indices,  $v$  and  $m$ . The  $m$  value is called the **radial mode number** and it represents the number of radial nodes in the field distribution. The integer  $v$  is called the **angular mode number**, and it represents the number of angular nodes existing in the field distribution.

### Transverse Electric and Magnetic Mode in the Step-index Fiber

Since  $v$  represents the angular dependence of the solution, the field solutions will be rotationally invariant when  $v = 0$ . The **characteristic equation** (11) simplifies to:

$$\left[ \frac{J_v'(\kappa a)}{\kappa J_v(\kappa a)} + \frac{K_v'(\gamma a)}{\gamma K_v(\gamma a)} \right] \left[ \frac{k_0^2 n_{core}^2 J_v'(\kappa a)}{\kappa J_v(\kappa a)} + \frac{k_0^2 n_{clad}^2 K_v'(\gamma a)}{\gamma K_v(\gamma a)} \right] = 0 \quad (13)$$

### Transverse Electric and Magnetic Mode in the Step-index Fiber

Let's compare Equation 13 to Equations 12a and 12b: The two terms in Equation 13 appear individually in Equations 12a and 12b, where the amplitude A was related to the amplitude B. If the first term in Equation 13 is set to zero, then A must be zero to keep the magnitude of B finite. If A = 0, then  $E_z = 0$ , and the electric field is transverse. Such modes are called **TE modes**.

Similarly, if the second term in Equation 13 is set to zero, the magnitude of B must be zero and the magnetic field is transverse ( $H_z = 0$ ). These modes are called **TM modes**.

Thus, if  $v = 0$ , the allowed modes are TE or TM.

### Transverse Electric Mode in the Step-index Fiber

Consider the TE mode: the first term in equation 13 should be set equal to zero.

$$\left[ \frac{J'_v(\kappa a)}{\kappa J_v(\kappa a)} + \frac{K'_v(\gamma a)}{\gamma K_v(\gamma a)} \right] \left[ \frac{k_0^2 n_{core}^2 J'_v(\kappa a)}{\kappa J_v(\kappa a)} + \frac{k_0^2 n_{clad}^2 K'_v(\gamma a)}{\gamma K_v(\gamma a)} \right] = 0 \quad (13)$$

Using Bessel function relations:

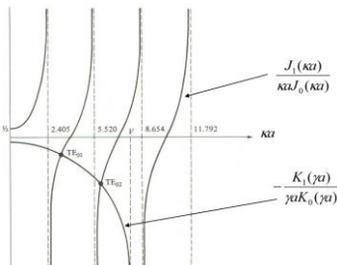
$$\begin{aligned} J'_v(x) &= J_{v-1}(x) - \frac{v}{x} J_v(x) & K'_v(x) &= -K_{v+1}(x) - \frac{v}{x} K_v(x) \\ J'_v(x) &= -J_{v+1}(x) + \frac{v}{x} J_v(x) & K'_v(x) &= K_{v-1}(x) + \frac{v}{x} K_v(x) \end{aligned} \quad \&$$

The eigenvalue equation for TE modes becomes:

$$\frac{J_1(\kappa a)}{\kappa J_0(\kappa a)} = -\frac{K_1(\gamma a)}{\gamma K_0(\gamma a)}$$

### Graphical Determination of the Propagation constants of TE modes ( $v = 0$ ):

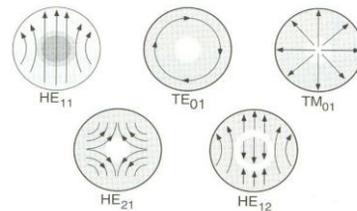
The eigenvalue equation for TE modes:  $\frac{J_1(\kappa a)}{\kappa J_0(\kappa a)} = -\frac{K_1(\gamma a)}{\gamma K_0(\gamma a)}$



$$\kappa^2 = k_0^2 n_{core}^2 - \beta^2$$

### Summary of Mode Shapes

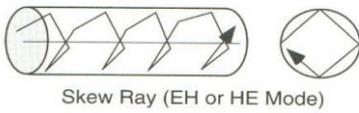
Below we schematically show the electric field lines in a step index fiber for a few modes:



**The Hybrid Modes ( $\nu \neq 0$ ):**

When  $\nu \neq 0$ , the characteristic equation is more complicated to solve. The values of  $\beta$  correspond to modes that have finite components of both  $E_z$  and  $H_z$ . These modes are called hybrid modes, EH or HE depending on the relative magnitude of the longitudinal E and H components:

- if  $A=0$  the mode is called TE mode
- if  $B=0$  the mode is called TM mode
- if  $A > B$  the mode is called EH ( $E_z$  dominates  $H_z$ )
- if  $B > A$  the mode is called HE ( $H_z$  dominates  $E_z$ )

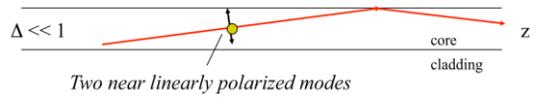


**Weak-guidance Approximation**

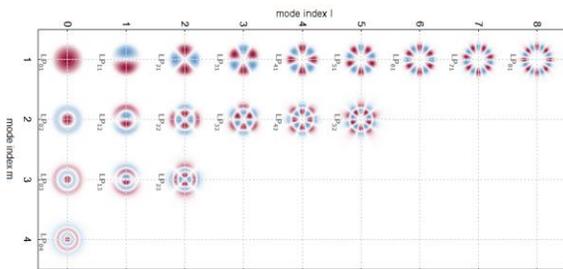
The analysis may be simplified when considering telecommunication grade optical fibers. These fibers have the relative index difference  $\Delta \ll 1$  ( $\Delta = (n_{core} - n_{clad}) / n_{core}$  typically less than 1%).

So, the propagation is preferentially along the fiber axis ( $\theta \approx 90^\circ$ ). In other word, the field is therefore predominantly transverse.

Modes are approximated by linearly polarized components. (both  $E_z$  and  $H_z$  are nearly zero)



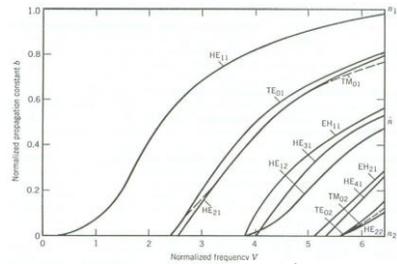
**Linearly Polarized Modes**



The mode subscripts  $l$  and  $m$  describe the electric field intensity profile. There are  $2l$  field maxima around the fiber core circumference and  $m$  field maxima along the fiber core radial direction.

rp-photonics

**Normalized Propagation Constant as a Function of Normalized Frequency**



Normalized propagation constant:  $b = \frac{\beta/k_0 - n_{clad}}{n_{core} - n_{clad}}$ ,  $\beta/k_0 = n_{eff}$

Normalized frequency (V Number):  $V = \frac{2\pi a}{\lambda} \sqrt{n_1^2 - n_2^2} = \frac{2\pi a}{\lambda} NA$

**Number of Guided Modes**

The total number of guided modes M for a step-index fiber is approximately related to the V number (for  $V \gg 1$ ) as follows:

$$M \approx V^2/2$$

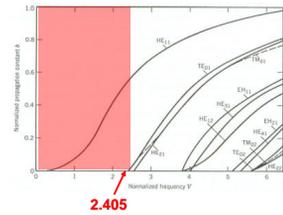
e.g., for the multimode step-index “water fiber” with a core diameter of 5 mm operating at a wavelength of 0.532 μm. If the water refractive index is 1.33, estimate (a) the normalized frequency for the “fiber”; (b) the number of guided modes.

(a)  $V = (2\pi/\lambda) a n_1 (2\Delta)^{1/2} = 2.76 \times 10^4$

(b)  $M \approx V^2/2 = 3.83 \times 10^8$



**Normalized Propagation Constant as a Function of Normalized Frequency**



A fiber becomes single-mode when its V number < 2.405 (the first root of the J0 Bessel function).

In a single-mode fiber only the HE<sub>11</sub> mode can propagate. This mode is often called the fundamental mode of the fiber, or LP<sub>01</sub> mode (weakly guiding approximation).

**Single Mode Fiber & Single Mode Condition**

A cutoff wavelength defines the boundary between multi-mode and single-mode operation of a fiber. The fiber is single-mode with wavelengths longer than the cutoff wavelength:

$$V = \frac{2\pi a}{\lambda} \sqrt{n_1^2 - n_2^2} = \frac{2\pi a}{\lambda} NA, \quad \text{Single mode condition: } V \text{ number} < 2.405$$

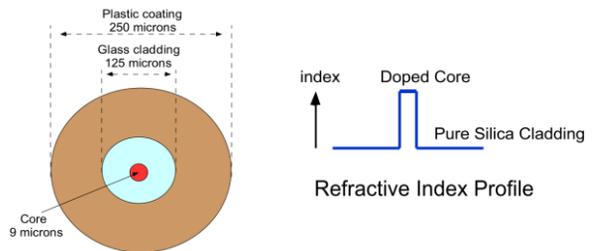
$$\rightarrow \lambda_c = \frac{2\pi a}{2.405} \sqrt{n_{core}^2 - n_{clad}^2} \approx \frac{2\pi a}{2.405} n_{core} \sqrt{2\Delta}$$

e.g., Determine the cutoff wavelength for a step-index fiber to exhibit single-mode operation when the core refractive index is 1.46 and the core radius is 4.5 μm, with the relative index difference of 0.25 %.

$$\lambda_c = (2\pi a n_1 / 2.405) (2\Delta)^{1/2} = 1214 \text{ nm}$$

Hence, the fiber is single-mode for  $\lambda > 1214 \text{ nm}$ .

**Typical Single Mode Fiber**



Typical parameters of a single-mode telecommunication fiber: Core radius  $a \sim 4 \mu\text{m}$

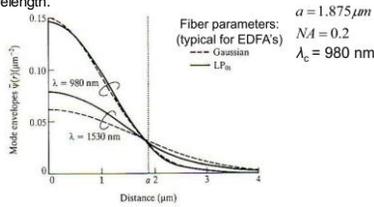
$$\Delta = (n_{core} - n_{clad}) / n_{core} \sim 0.005 \quad (n_{core} \sim 1.45)$$

### Gaussian Approximation

The transverse field of the  $HE_{11}$  mode is described by the  $J_0$  Bessel function in the core region. However, since Bessel functions are cumbersome to use in practice, the mode field distribution is often approximated by a Gaussian shape:

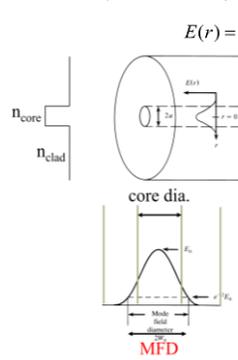
$$E(r) = E_0 \exp\left[-\left(\frac{r}{w}\right)^2\right]$$

Comparison between an exact Bessel mode intensity distribution (LP01) and a Gaussian mode approximation, for the wavelengths  $\lambda = 980$  nm and  $\lambda = 1530$  nm. At wavelengths well above cut-off, e.g. at 1530 nm, significant error is seen. This is important to take into account in designing Er-doped fiber amplifiers pumped at 980 nm wavelength.



### Mode-field Diameter

The Mode Field Diameter (MFD) (spot size) is determined by the points where the power is down by  $e^{-2}$  (or amplitude by  $e^{-1}$ )



“Corning SMF-28” single-mode fiber has MFD:

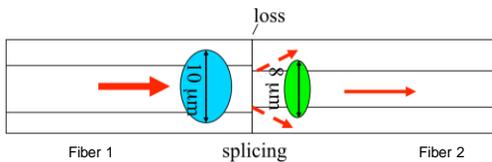
9.2  $\mu\text{m}$  at 1310 nm  
10.4  $\mu\text{m}$  at 1550 nm

core diameter: 8.2  $\mu\text{m}$

MFD > core diameter

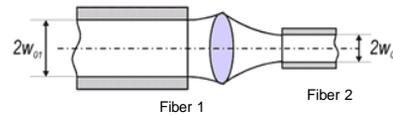
### Mode-field Diameter Mismatch

Mismatches in mode-field diameter can increase fiber splice loss



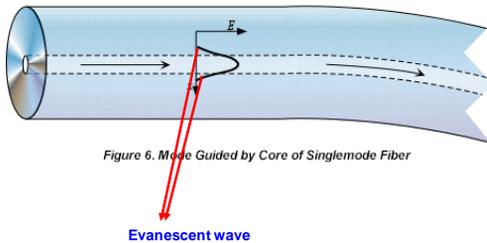
### Mode Coupling in Waveguides

Mode coupling is very crucial to reduce the coupling loss for waveguide alignment



Mode field adaptation by an optical lens or lens system

## Evanescent wave



## Evanescent wave: Useful or Useless

- Evanescent waves are electromagnetic waves that penetrate tens of nm through a surface and propagate along the surface.
- They may be used to sample volume “outside” a waveguide for sensing
- They may be used to locally excite a sample on the other side of an interface for imaging

## Transmission properties of Optical Fibers

Attenuation (loss)	}	<ul style="list-style-type: none"> <li>• Silica-based fibers have ~0.2dB/km (i.e., ~95% launched power remains after 1 km fiber transmission)</li> </ul>
Bandwidth		<ul style="list-style-type: none"> <li>• Bandwidth determines the number of bits of information transmitted in a given time period (mainly depends on dispersion).</li> </ul>

## Attenuation in Fiber

Signal attenuation is usually expressed in the logarithmic unit of the decibel, which has the advantage that the operations of **multiplication** (and division) reduce to **addition** (and subtraction).



$$\text{Loss (dB)} = -10 \log_{10} (P_o / P_i) = 10 \log_{10} (P_i / P_o)$$

**Please calculate the loss of 50% in Decibel!**

The attenuation per unit length (dB/kM):  $\frac{\text{Loss}}{L} = \frac{10 \log_{10} (P_i / P_o)}{L}$

\*In electronics, Loss =  $20 \log_{10} (V_o / V_i)$

## dBm

• dBm is a specific unit of power in decibels when the reference power is 1 mW:

$$\text{dBm} = 10 \log_{10} (\text{Power}/1\text{mW})$$

e.g., 1 mW = 0 dBm ; 100  $\mu$ W = -10 dBm

$$\Rightarrow \text{Loss (dB)} = \text{input power (dBm)} - \text{output power (dBm)}$$

e.g., Input power = 1 mW (0 dBm), output power = 100  $\mu$ W (-10 dBm)

$$\Rightarrow \text{loss} = -10 \log_{10} (100 \mu\text{W}/1 \text{ mW}) = 10 \text{ dB}$$

$$\text{OR loss} = 0 \text{ dBm} - (-10 \text{ dBm}) = 10 \text{ dB}$$

## Attenuation in Fiber

1. Material absorption
2. Scattering loss
3. Nonlinear loss
4. Bending loss
5. Mode coupling loss  
(Splice and connection)

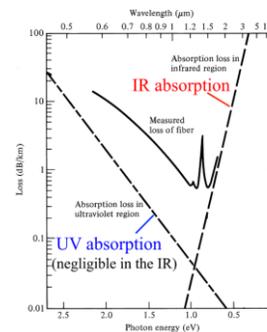
### Attenuation in Fiber Material absorption

#### □ Intrinsic material absorption

Due to materials used to make fibers (e.g., fused silica and doped fused silica). The material absorbs at certain wavelengths corresponding to the electronic and vibrational resonances of the molecules. For SiO<sub>2</sub> molecules the electronic resonances occur in the ultraviolet region ( $\lambda < 0.4 \mu\text{m}$ ), and the lattice resonances occur in the infrared region ( $\lambda > 7 \mu\text{m}$ ). Due to the amorphous nature of fused silica, these resonances are in the form of absorption bands whose tails extend to the wavelength region of our interest.

### Attenuation in Fiber Material absorption

#### □ Intrinsic material absorption



### Attenuation in Fiber

#### Material absorption

☐ Extrinsic material absorption

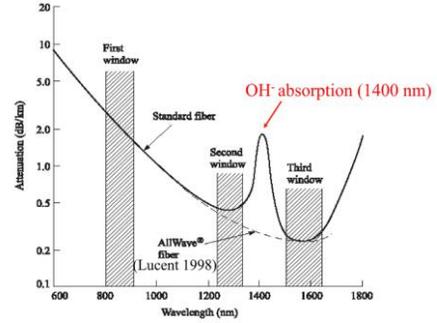
Due to impurities. E.g., transition-metal impurities (Fe, Cu, Ni, Mn, and Cr) absorbs strongly in the wavelength range 0.6-1.6 μm). Their amount needs to be kept below 0.1 ppb in order not to increase the losses more than 0.1 dB/km.

- The main source of extrinsic absorption in modern fibers is the OH-absorption. A vibrational resonance of the OH ion occurs near 2.73 μm and its harmonics and combination tones with silica produce absorption peaks at 1.39-, 1.24-, and 0.95-μm wavelengths. In modern fibers, OH ion concentration is reduced to < 10<sup>-8</sup> lowering the peak absorption at 1.39 μm to less than 1dB/km.

### Attenuation in Fiber

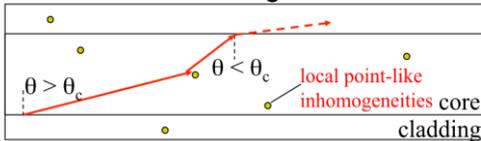
#### Material absorption

☐ Extrinsic material absorption



### Attenuation in Fiber

#### Scattering loss



☐ Rayleigh scattering: results from random inhomogeneities that are small in size compared with the wavelength.

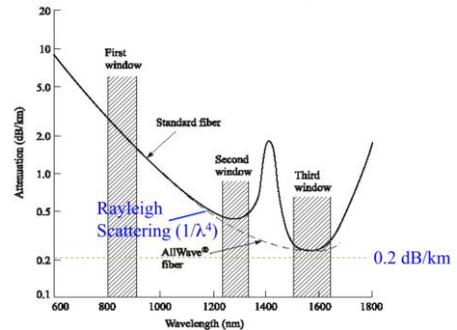
$$\alpha_R = \frac{8\pi^2}{3\lambda^4} (n^2 - 1)\beta k_B T$$

\*These inhomogeneities exist in the form of refractive index fluctuations which are frozen into the amorphous glass fiber upon fiber pulling. Such fluctuations always exist and cannot be avoided !

### Attenuation in Fiber

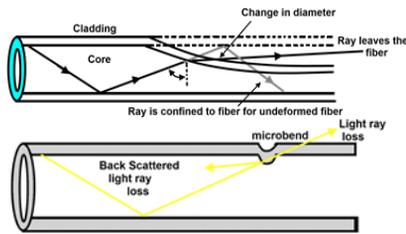
#### Scattering loss

#### Rayleigh scattering



## Attenuation in Fiber Scattering loss

- Waveguide scattering: results from imperfections in the waveguide structure of a fiber, e.g., nonuniformity in the size and shape of the core, perturbations in the core-cladding boundary, and defects in the core or cladding.

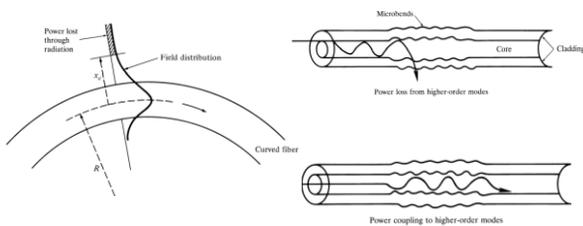


## Attenuation in Fiber Nonlinear loss

- Light is confined over long distances in an optical fiber, nonlinear optical effects can become important even at a relatively moderate optical power.
- Nonlinear optical processes such as stimulated Brillouin scattering and stimulated Raman scattering can cause significant attenuation in the power of an optical signal.
- Other nonlinear processes can induce mode mixing or frequency shift, all contributing to the loss of a particular guided mode at a particular frequency.
- Nonlinear effects are intensity dependent, and thus they can become very important at high optical powers.

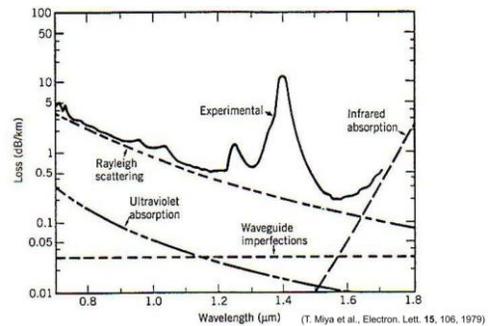
## Attenuation in Fiber

### Bending loss & Mode-coupling to Higher-order Modes



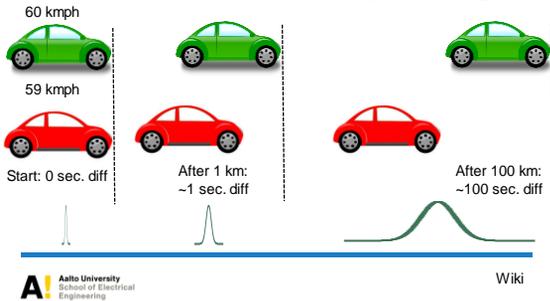
## Attenuation in Fiber

### Loss spectrum of a single-mode fiber produced in 1979



## Dispersion

- In optics, dispersion is the phenomenon in which the phase velocity (and group velocity) of a wave depends on its frequency (wavelength).



## Dispersion Limit

- For a given data rate, the bit period is given by  $T$  where  $T=1/\text{bit rate}$ .
- Clearly if the pulse broadens, light in a "1" will leak into adjacent bit periods.
- If too much light leaks, it is possible that the receiver will mistake the light that has leaked from a "1" into an adjacent "0" as meaning that the "0" should be a "1".
- An error will therefore occur (as the propagation distance and the data rate increase).



Dispersion is the limiting factor in determining the amount of information that can be sent over an optical link.

## Dispersion in Waveguides

- Material dispersion:** Different wavelengths travel at different velocities due to the wavelength dependence of the index of refraction.
- Modal dispersion:** Different waveguide modes propagate at different velocities. This is not an issue in modern systems that use single-mode fibers
- Waveguide dispersion:** Different wavelengths travel at different velocities due to the wavelength dependence of the propagation constant  $\beta$  (caused by the waveguide structure, i.e. index profile).
- Polarization mode dispersion:** Fiber birefringence causes different polarizations to propagate at different speeds.

## Material Dispersion

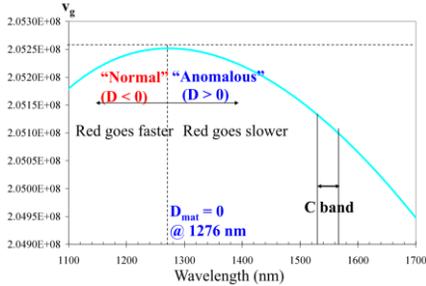
- The refractive index of material (e.g., Silica) is wavelength dependent. Different wavelength light has different refractive index, which is called material dispersion.



We cannot modify material dispersion as it only depends on the material itself.

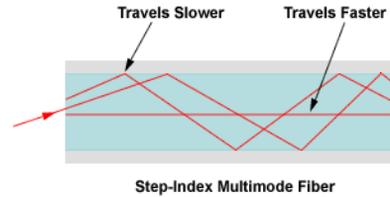
### Material Dispersion

Variation of  $V_g$  with wavelength for fused silica



Material dispersion  $D_{mat} = 0$  at  $\lambda \sim 1276$  nm for fused silica. This  $\lambda$  is referred to as the **zero-dispersion wavelength**  $\lambda_{zd}$ .

### Modal Dispersion



### Modal Dispersion

- Modal dispersion is caused by the variation in propagation constant between different modes; it is also called inter-mode dispersion.
- Modal dispersion appears only when more than one mode is excited in a multimode fiber. It exists even when chromatic dispersion disappears.
- Multimode dispersion does not depend on the source linewidth (even a single wavelength can be simultaneously carried by multiple modes in a waveguide).
- Multimode dispersion would not occur if the waveguide allows only one mode to propagate - the advantage of single-mode waveguides!



### Modal Dispersion

Estimate modal dispersion pulse broadening using phase velocity

- A zero-order mode traveling near the waveguide axis needs time:

$$t_0 = L/v_{m=0} \approx Ln_1/c \quad (v_{m=0} \approx c/n_1)$$

- The highest-order mode traveling near the critical angle needs time:

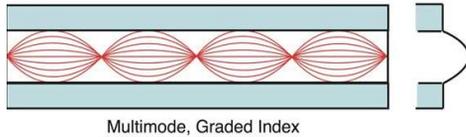
$$t_m = L/v_m \approx Ln_2/c \quad (v_m \approx c/n_2)$$

=> The pulse broadening due to modal dispersion:

$$\Delta T \approx t_0 - t_m \approx (L/c) (n_1 - n_2) \approx (L/2cn_1) NA^2 \quad (n_1 \sim n_2)$$

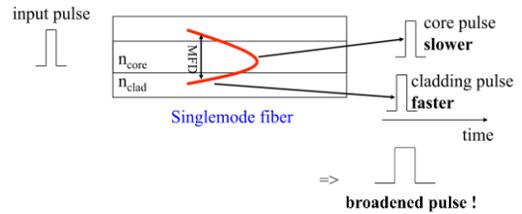
### Modal Dispersion

**Graded-Index multimode fibers can solve the problem of modal dispersion.**



### Waveguide Dispersion

Waveguide dispersion is chromatic dispersion which arises from waveguide effects: the dispersive phase shifts for a wave in a waveguide differ from those which the wave would experience in a homogeneous medium.



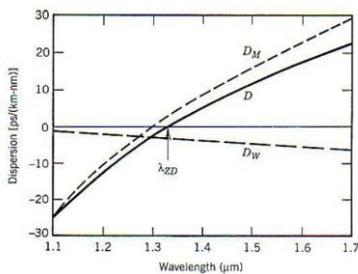
The dependence of Waveguide dispersion on  $\lambda$  may be controlled by altering the core radius, the NA, or the V number.

### Dispersion

Combining the material and waveguide dispersion in **single-mode fibers** we get the dispersion parameter D:

$$D = -\frac{\lambda}{c} \frac{d^2 n_{core}}{d\lambda^2} - \frac{n_{core} \Delta}{c \lambda} V \frac{d^2(Vb)}{dV^2}$$

$(D_{Material})$                        $(D_{Waveguide})$



### Dispersion in Low-bit-rate systems

- Broadening of the light pulse due to chromatic dispersion:

$$\Delta T = D L \Delta \lambda$$

Consider the maximum pulse broadening equals to the bit time period 1/B, then the **dispersion-limited distance**:

$$L_D = 1 / (D B \Delta \lambda)$$

e.g. For  $D = 17 \text{ ps}/(\text{km} \cdot \text{nm})$ ,  $B = 2.5 \text{ Gb/s}$  and  $\Delta \lambda = 0.03 \text{ nm}$ .

$$L_D = 784 \text{ km}$$

### Time Vs Frequency

An optical pulse has a finite spread in wavelength (or frequency) even if light sources with very narrow line-width are used.

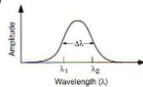
In time domain:

Consider a Gaussian pulse and "1" is desired:  $E(t) = E_0 e^{-t^2/\tau^2}$

The full width at half-maximum is:  $t_{FWHM} = \tau\sqrt{2\ln 2}$

In frequency domain: 
$$E(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} E(t)e^{j\omega t} dt$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} E_0 e^{-t^2/\tau^2} e^{j\omega t} dt = \frac{E_0}{2\pi} \sqrt{\pi} e^{-\frac{\omega^2 \tau^2}{4}}$$



The full-width half-maximum frequency is  $\Delta\omega_{FWHM} = \frac{\sqrt{8\ln 2}}{\tau}$

10 Gbit/s transmission:  $t_{FWHM} \approx 0.05\text{ns} \rightarrow \Delta f = \frac{1}{2\pi} \Delta\omega_{FWHM} \approx 8.8\text{GHz}$

### Dispersion in high-bit-rate systems

Note that spectral width  $\Delta\lambda$  is proportional to the modulation of the lightwave:  $\Delta\omega_{FWHM} = \frac{\sqrt{8\ln 2}}{\tau} \Rightarrow \Delta\lambda \propto B$

$$L_D = 1 / (D B \Delta\lambda) \Rightarrow L_D \propto 1 / B^2$$

Chromatic dispersion scales with  $B^2$

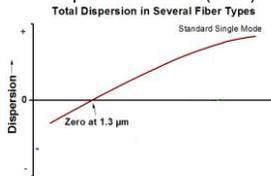
Bit-rate	Dispersion-limited distance ( $L_D$ )
2.5 Gb/s	784 km
10 Gb/s	~50 km
40 Gb/s	~3 km

\*  $D = 17\text{ ps}/(\text{km}\cdot\text{nm})$

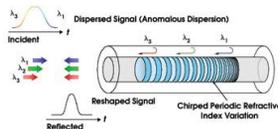
This is why chromatic dispersion compensation must be employed for systems operating at 10 Gbit/s (now at 40 Gbit/s and beyond)

### Dispersion Compensation

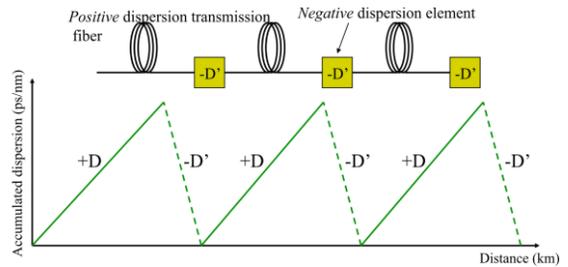
Dispersion compensation fiber (DCF)



Chirped Fiber Bragg Grating Dispersion Compensator



### Dispersion Compensation Scheme



In a dispersion-managed system, positive dispersion transmission fiber alternates with negative dispersion compensation elements, such that the total dispersion is zero end-to-end.

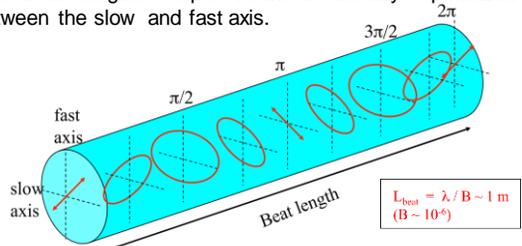
### Fiber Birefringence

- In ideal fibers with perfect rotational symmetry, the two modes are degenerate with equal propagation constants ( $\beta_x = \beta_y$ ), and any polarization state injected into the fiber will propagate unchanged.
- In actual fibers there are imperfections, such as asymmetrical lateral stresses, non-circular cores and variations in refractive-index profiles. These imperfections break the circular symmetry of the ideal fiber and lift the degeneracy of the two modes.
- The modes propagate with different phase velocities, and the difference between their effective refractive indices is called the fiber birefringence:

$$B = |n_y - n_x|$$

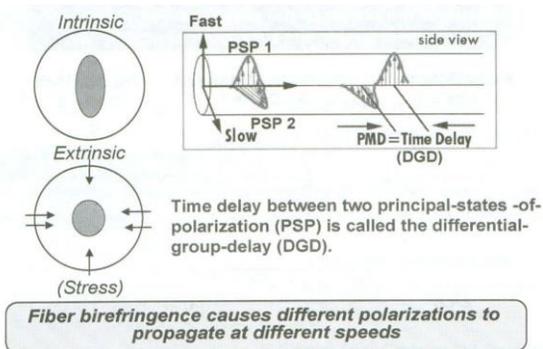
### Fiber Birefringence

State-of-polarization in a constant bi-refringent fiber over one beat length. Input beam is linearly polarized between the slow and fast axis.



\*In optical pulses, the polarization state will also be different for different spectral components of the pulse.

### Polarization Mode Dispersion (PMD)



### Polarization-maintaining Fiber

- The fiber birefringence is enhanced in single-mode polarization-maintaining fibers, which are designed to maintain the polarization of the launched wave.
- Polarization is preserved because the two normal modes have significantly different propagation characteristics. This keeps them from exchanging energy as they propagate through the fiber.
- Polarization-maintaining fibers are constructed by designing asymmetries into the fiber



## Optical Fiber Fabrication

Fabrication of silica fibers by a two-step process

1. Preform fabrication
  - Modified Chemical Vapor Deposition (MCVD) Method
  - Outside Vapor Deposition (OVD) Method
  - Vapor-phase Axial (VAD) Method
2. Drawing into an optical fiber

## How Fiber Is Made?



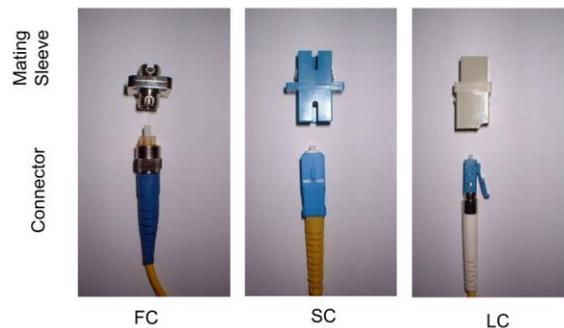
Youtube

## Fiber Devices

- Fiber Connectors / Fusion Splicing
- Isolators
- Optical Circulators
- Wavelength division multiplexer
- Wavelength selective devices (filters)
- Fused Fiber Couplers
- Optical Modulators
- Polarization controllers

.....

## Fiber Connectors



## Fiber Connectors

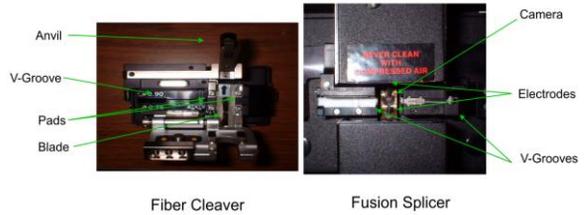
- Sometimes connectors are packaged together with multiple fibers.
- Insertion losses are generally <0.2 dB typical (<0.5 dB max)

Connectors can have a flat polish or they may have an 8° angle polish which reduces back reflections.



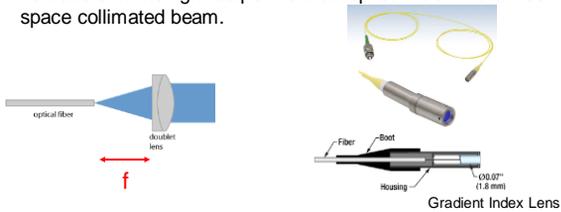
## Fiber Fusion Splicing

Fiber joints (splices) are required during installation and service should cables be damaged and fibers become broken. They are also used extensively in putting devices (or subsystems), such as EDFAs, together. These subsystems consist of several individual components with fiber "pigtailed" that are spliced together. Today the splices are made by fusing the two fiber ends together. The fusion splices are made using an electric discharge to melt the fiber ends, and with the help of surface tension forces the fibers are self-aligned with each other. Several automated, simple splicers are commercially available. The splice loss is typically below 0.1 dB



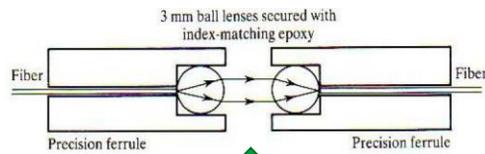
## Fiber Collimators

To transform the light output from an optical fiber into a free-space collimated beam.



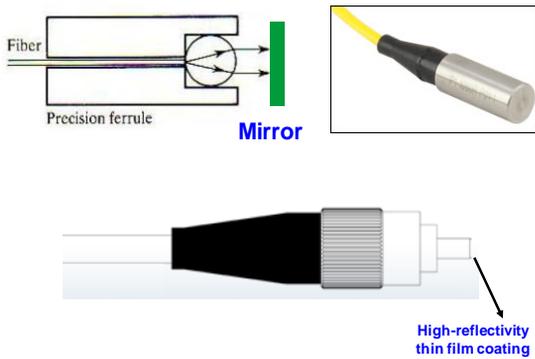
Effective focal length (mm)	Input MFD (μm)	Output Waist Dia. (mm)	Full Angle Beam Diverg. (mrad)	Clear Aperture (mm)	NA	Operation wavelength (nm)	Fiber Type	Return Loss (dB)	AR Coating Range (nm)	Fiber connector
18.75	10.4	3.5	0.57	5.5	0.15	1550	SMF-28	55	1050-1620	FC/APC

## Micro-optic Fiber Components

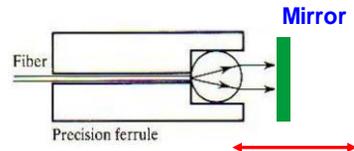


Various passive/active free-space devices

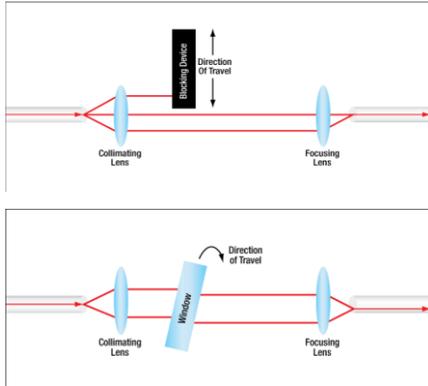
### Fiber Reflector



### Optical Fiber Delay Lines



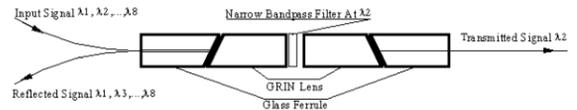
### Variable Fiber Optical Attenuators



### Wavelength selective devices

#### Thin film bandpass filter

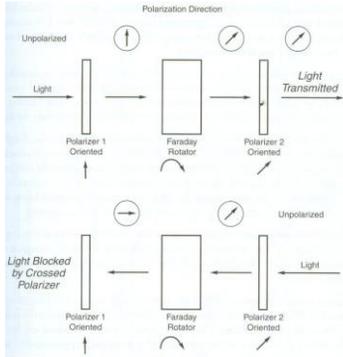
A typical configuration of WDM MUX made by narrow bandpass filters is shown below. In the basic unit, the left side is a dual-fiber collimator, and right side is a single fiber collimator. A narrow bandpass filter is placed between them. The narrow bandpass filters for Dense WDM are dielectric multi-cavity Fabry-Perot filters. Light signals coming from one fiber of the dual-fiber collimator are collimated and impinge on the narrow bandpass filter. One channel ( $\lambda_2$ ) passes through to the output fiber and filters out. The remaining channels are reflected back by the narrow band pass filter to another fiber of the dual-fiber collimator.



Structure of a WDM unit by narrow bandpass filter.

## Isolators

An optical component which allows the transmission of light in only one direction



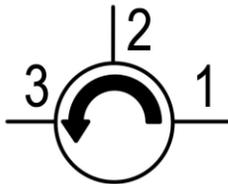
## Isolators



Parameter		Premium	Standard
Insertion loss ( $\lambda_c$ , 23 °C)	Typical	0.19 dB	0.21 dB
Insertion loss ( $\lambda_c$ , 15 nm, 23 °C)	Maximum	0.24 dB	0.27 dB
Insertion loss ( $\lambda_c$ , 15 nm, -5 to 75 °C)	Maximum	0.30 dB	0.35 dB
Polarization dependent loss ( $\lambda_c$ , 15 nm, -5 to 75 °C)	Maximum	0.04 dB	0.06 dB
Return loss (input/output), 23 °C		60 dB / 55 dB	55 dB / 50 dB
Isolation ( $\lambda_c$ , 15 nm, -5 to 75 °C)	Minimum	22 dB	20 dB
Polarization mode dispersion	Maximum	0.02 ps	0.03 ps
Operating temperature			-5 to 75 °C
Storage temperature			-40 to 85 °C
Input optical power			2 W
Tensile load	Maximum		5 N
Package dimensions (D x L)			5.5±0.1 x 30±2 mm
Device marking		JDSU logo, JDSU model name and device serial number	

## Optical Circulators

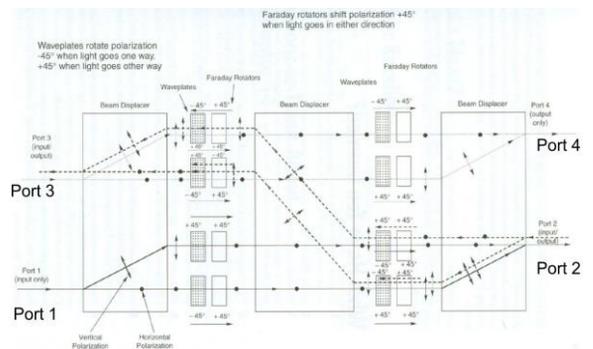
A special fiber-optic component that can be used to separate optical signals that travel in opposite directions in an optical fiber, analogous to the operation of an electronic circulator.



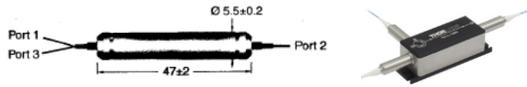
Directs light from port 1→2, 2→3

## Optical Circulators

Directs light from port 1→2, 2→3, 3→4

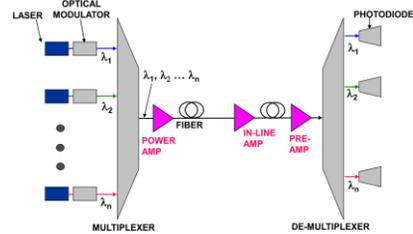


## Optical Circulators



Wavelength range	Insertion loss	Isolation	Return loss	Max optical power
1530-1570nm	0.9-1.3 dB	32 dB	55dB	5W

## Wavelength division multiplexing



Different channel spacings are used for different systems. In general, fewer channels with larger spacings are easier to implement. Fewer wavelengths, however means less aggregate bandwidth capacity for a given bandwidth per channel. To increase aggregate capacity, either the individual channel speed or the number of multiplexed channels must be increased.

## Types of WDM

The simplest implementation required to meet current system demands and potential future upgrades is used.

**CWDM** (Coarse WDM)- Channel spacing of 20 nm.

**WDM** – Channel spacing of 400, 200 or 100 GHz (3.2, 1.6 or 0.8 nm)

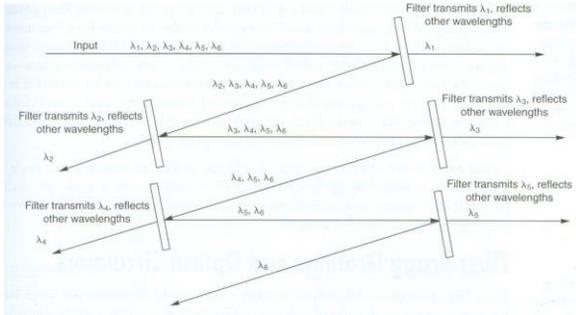
**DWDM** (Dense WDM )- Channel spacing of 100, 50 or 25 GHz (0.8, 0.4 or 0.2 nm)

## Types of WDM

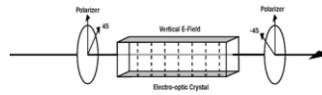


Operation wavelength	Insertion loss	Isolation	Bandwidth	Max power
980/1550nm	<0.55dB	>20dB	20nm	300mW
1550/1560	<0.4dB	>16dB	5nm	300mW

### Wavelength MUX/DEMUX Based on Interference Filters



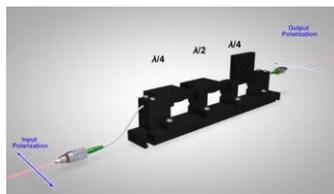
### Optical Modulators



Operation wavelength	Optical on/off extinction ratio	Bit Rate Frequency	Insertion Loss	RF drive voltage
1525nm	20 dB	9.953 Gb/s	4 dB	5.5V

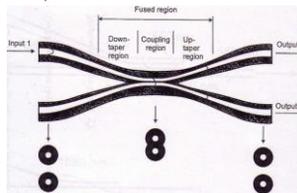
### Polarization Controllers

Polarization Controllers use stress-induced birefringence produced by wrapping the fiber around two or three spools to create independent wave plates that will alter the polarization of the transmitted light in a single mode fiber. The fast axis of the fiber is in the plane of the spool, allowing an arbitrary input polarization state to be adjusted by rotating the paddles.

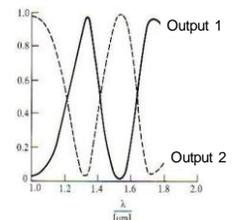


### Fused Fiber Couplers

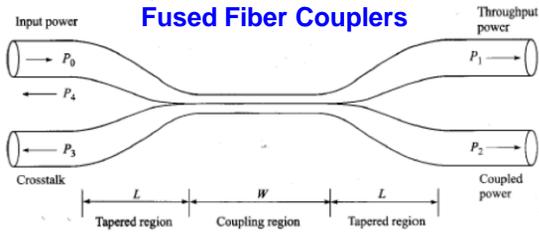
Fused fiber couplers are fabricated by twisting two fibers together, fusing them at high temperature and drawing them out. The cores become narrower in the drawn out region and the modes can couple from one fiber to the other. During fabrication light is launched in the input and output is monitored "in situ" to achieve the desired performance. These type of guided-wave couplers are often called proximity couplers or directional couplers. Fused fiber couplers are highly reliable and their excess losses are very low (< 0.1 dB).



2x2 Fused Fiber Coupler



Splitting ratio as a function of wavelength

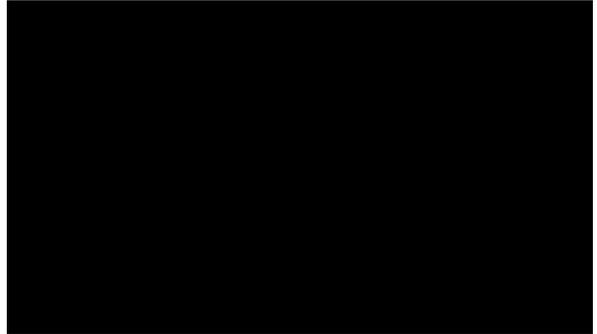


$$\text{Splitting Ratio} = \frac{P_2}{P_1 + P_2}$$

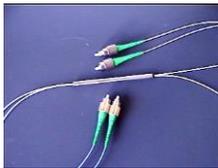
$$\text{Insertion Loss} = 10 \text{Log} \left( \frac{P_i}{P_j} \right)$$

$$\text{Excess Loss} = 10 \text{Log} \left( \frac{P_o}{P_1 + P_2} \right)$$

### How to make 3dB Fused Fiber Couplers



### Fused Fiber Couplers



Parameter (50/50 coupling ratio)	Grade 1	Grade 2
Center wavelength	1310, 1480, 1550, 1590 nm	1310, 1480, 1550, 1590 nm
Insertion loss (without connectors)	Maximum 3.4 dB	3.6 dB
Excess loss (without connectors)	Typical 0.1 dB	0.3 dB
Uniformity	Maximum 0.6 dB	1.0 dB
Polarization dependent loss	Maximum 0.1 dB	0.15 dB
Optical return loss	Minimum 50 dB	50 dB
Directivity	Minimum 55 dB	55 dB
Temperature coefficient	Typical 0.002 dB/°C	0.002 dB/°C
Package dimensions		
S package (D x L)	3.0 x 54 mm	3.0 x 54 mm
L package (D x L)	3.6 x 70 mm	3.6 x 70 mm
H package (L x W x H)	85 x 17.8 x 7.5 mm	85 x 17.8 x 7.5 mm
Operating temperature <sup>1</sup>	-40 to 85 °C	-40 to 85 °C
Storage temperature <sup>1</sup>	-50 to 85 °C	-50 to 85 °C

### Summary about the next week work

- home assignments for Group 1/2
  1. Prepare the home assignment (3 questions) and announce it on the mycourses website before **10AM 30<sup>th</sup>, April**
  2. Collect all answers from your peers before **10AM 7<sup>th</sup>, May**
  3. Evaluate the home assignments of your peers and announce it on the mycourses website before **10AM 9<sup>th</sup>, May**, and give me the results ( the evaluation + answer sheets).
  4. Exercise lecture on **9<sup>th</sup>, May, 12:30-14:00**
  
- Submit your poster PPT (DL: 14<sup>th</sup> May).