6 (a)

perjantai 5. huhtikuuta 2019 8.45

$$\bar{\bar{X}} = \pm\left(\bar{\bar{I}} - 2\frac{\bar{a}\bar{b}}{\bar{a}\cdot\bar{b}}\right)$$

obviously the $\pm$ cases give the same eigenvectors & opposite eigenvalues (let's look at +)

$$\Rightarrow \bar{\bar{X}} = \bar{\bar{I}} - 2\frac{\bar{a}\bar{b}}{\bar{a}\cdot\bar{b}}$$

Clearly $\bar{a}$ is a right eigenvector: $\bar{\bar{X}}\cdot\bar{a} = -\bar{a}$ $\quad \lambda = -1$

And all vectors $\bar{c}$ orthogonal to $\bar{b}$ (in other words $\bar{c}\cdot\bar{b}=0$) are r-eigenvectors: $\bar{\bar{X}}\cdot\bar{c} = +\bar{c}$ $\quad (\lambda = +1)$

Simplest ones: $\bar{c} = \bar{a}\times\bar{b}$ & $(\bar{a}\times\bar{b})\times\bar{b}$

Left eigenvectors likewise: $\bar{b}\cdot\bar{\bar{X}} = -\bar{b}$

$$(\bar{b}\times\bar{a})\cdot\bar{\bar{X}} = +\bar{b}\times\bar{a}$$

$$[(\bar{b}\times\bar{a})\times\bar{a}]\cdot\bar{\bar{X}} = +(\bar{b}\times\bar{a})\times\bar{a}$$

6 (b)

For a 2D dyadic $\bar{\bar{A}}$ with $\bar{u}\cdot\bar{\bar{A}} = 0$ & $\bar{\bar{A}}\cdot\bar{u} = 0$ ($\bar{u}$ unit vector)

$\bar{\bar{A}} \overset{\times}{\times} \bar{\bar{A}} = \alpha\,\bar{u}\bar{u}$ (why?)

take trace $\Rightarrow$ $\bar{\bar{A}} \overset{\times}{\times} \bar{\bar{A}} : \bar{\bar{I}} = \alpha$ $\Rightarrow$ $\alpha = \mathrm{tr}\,\bar{\bar{A}} \overset{\times}{\times} \bar{\bar{A}} = 2\,\mathrm{spm}\,\bar{\bar{A}}$

(2.74) Planar inverse

$$\bar{\bar{A}}^{-1} = \frac{(\bar{\bar{A}} \overset{\times}{\times} \bar{\bar{A}}^{*(2)})^T}{\bar{\bar{A}}^{(2)} : \bar{\bar{A}}^{*(2)}}$$

NUMERATOR: $(\bar{\bar{A}} \overset{\times}{\times} (\mathrm{spm}\,\bar{\bar{A}}^{*}\,\bar{u}\bar{u}))^T = \mathrm{spm}\,\bar{\bar{A}}^{*}\,\bar{\bar{A}}^T \overset{\times}{\times} \bar{u}\bar{u}$

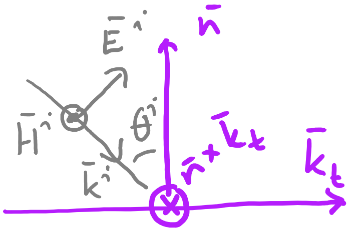
DENOMINATOR: $\mathrm{spm}\,\bar{\bar{A}}\,\bar{u}\bar{u} : \mathrm{spm}\,\bar{\bar{A}}^{*}\,\bar{u}\bar{u} = \mathrm{spm}\,\bar{\bar{A}}\;\mathrm{spm}\,\bar{\bar{A}}^{*}$

$\Downarrow$ (2.187)

6 (c)

REMEMBER: $|\bar{E}^{i}| = \eta_0 |\bar{H}^{i}|$ in a plane wave in free space

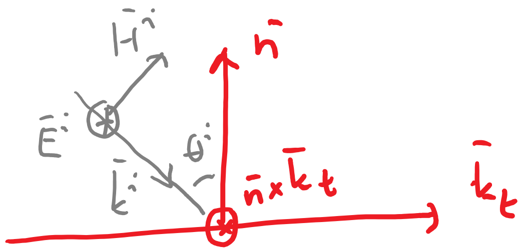
PARALLEL POL.



$$|\bar{E}_t^i| = \cos\theta^i \, \eta_0 |\bar{H}_t^i|$$

$$\cos\theta^i = \frac{k^n}{k_0}$$

PERPENDICULAR POL.



$$\cos\theta^i |\bar{E}_t^i| = \eta_0 |\bar{H}_t^i|$$

$\bar{\bar{J}}_t$ takes care of these, and directions !!!

unit vectors : $\bar{u}_{\parallel} = \dfrac{\bar{k}_t}{\sqrt{\bar{k}_t\cdot\bar{k}_t}}$ & $\bar{u}_{\perp} = \dfrac{\bar{n}\times\bar{k}_t}{\sqrt{\bar{k}_t\cdot\bar{k}_t}}$ $(\bar{n}\times\bar{u}_{\parallel} = \bar{u}_{\perp},\ \bar{n}\times\bar{u}_{\perp} = -\bar{u}_{\parallel})$

$(\bar{k}_t\cdot\bar{u}_{\perp} = 0)$

PARALLEL P.: $\bar{H}^i = \bar{H}_t^i = -\bar{u}_{\perp} H^i$

$$\bar{\bar{J}}_t \cdot \eta_0 \bar{H}_t^i = -\eta_0 H^i \frac{1}{k_0 k_n} \bar{n}\times\left(\bar{k}_t\bar{k}_t + k_n^2 \bar{\bar{I}}_t\right)\cdot\bar{u}_{\perp}$$

$$= -\eta_0 H^i \frac{k_n}{k_0} \bar{n}\times\bar{u}_{\perp} = \bar{u}_{\parallel} \cos\theta^i \, \eta_0 H^i$$

(like $\bar{E}_t^i$ in the picture)

PERPENDICULAR P.:

$\bar{E}^i = \bar{E}_t^i = \bar{u}_{\perp} E^i$

$$\Rightarrow -\bar{\bar{J}}_t\cdot\bar{E}_t^i = -\frac{k_n}{k_0} \bar{n}\times\bar{u}_{\perp} E^i = \bar{u}_{\parallel} \cos\theta^i E^i$$

(like $\eta_0 \bar{H}_t^i$ in the picture)