$$
\begin{aligned}
& \nabla \times \bar{E}=-j \omega \bar{B}-\bar{y}_{m} \\
& \nabla \times \bar{F}=j \omega \bar{D}+\bar{y} \\
& \nabla \cdot \bar{D}=\rho \\
& \nabla \cdot \bar{B}=\rho_{m}
\end{aligned}
$$

SIMPLE DUALITY

$$
\begin{array}{ll}
\rho \rightarrow \rho_{m} & \rho_{m} \rightarrow \rho \\
\bar{D} \rightarrow \bar{B} & \bar{B} \rightarrow \bar{D} \\
\bar{E} \rightarrow-\bar{H} & \bar{H} \rightarrow-E \\
\bar{j}_{m} \rightarrow j & \bar{j} \rightarrow \bar{j}_{m}
\end{array}
$$

Duality:

$$
\begin{aligned}
& \bar{E}_{d}=\alpha \bar{H} \\
& \bar{H}_{\alpha}=\beta \bar{E}
\end{aligned}
$$

$$
\begin{array}{ll}
\nabla \times \bar{E}_{d}=-j \omega \bar{B}_{d}-\bar{\partial}_{m d} \quad \Rightarrow \alpha \nabla \times \bar{H}=-j \omega \bar{B}_{d} \cdot \bar{J}_{m d} \\
& =\alpha(\bar{J}+j \omega \bar{D}) \quad \\
\nabla \times \bar{H}_{d}=j \omega \bar{D}_{d}+\bar{j}_{d} \quad \bar{B}_{d}=-\alpha \bar{D} \\
=\beta \nabla_{\times} \bar{E}=\beta\left(-\bar{J}_{m}-j \omega \bar{B}\right) \quad \bar{D}_{d}=-\beta \bar{B} \quad \bar{J}_{m d}=-\alpha \bar{j} \\
& \bar{j}_{d}=-\beta \bar{J}_{m}
\end{array}
$$

$$
\begin{aligned}
& \nabla \cdot \bar{D}_{d}=\rho_{d} \\
& =-\beta \nabla \cdot \bar{B}=-\beta \rho_{m} \\
& \nabla \cdot \bar{B}_{d}=\rho_{m d} \\
& =-\alpha \nabla \cdot \bar{D}=-\alpha \rho \\
& \bar{n}-\overline{\bar{n}} \cdot \bar{L} \cdot+\overline{\bar{z}} \cdot \bar{H}_{d}
\end{aligned}
$$

$$
\begin{aligned}
& \bar{D}_{\alpha}=\overline{\bar{\varepsilon}}_{d} \cdot \bar{E}_{\alpha}+\bar{\xi}_{d} \cdot \bar{H}_{d} \\
& -\beta \bar{B}=\bar{\varepsilon}_{d} \cdot \alpha \bar{H}+\bar{\xi}_{d} \cdot \beta \bar{E} \\
& \bar{B}=\underbrace{-\bar{\xi}_{d} \cdot \bar{E}}_{\tilde{\bar{\zeta}}}-\underbrace{-\frac{\alpha}{\beta} \bar{\varepsilon}_{d}}_{\dot{\mu}} \cdot \bar{H} \\
& \bar{B}_{d}=\bar{\xi}_{d} \cdot \bar{E}+\bar{\mu}_{d} \cdot \bar{H} \\
& =-\alpha \bar{D}=\alpha \dot{\zeta}_{d} \cdot \bar{H}+\beta \bar{\mu}_{\alpha} \cdot \bar{E}
\end{aligned}
$$

$$
\begin{aligned}
& \dot{\bar{\varepsilon}}_{d}=-\frac{\beta}{\alpha} \overline{\bar{\mu}} \\
& \dot{\xi}_{d}=-\bar{j} \\
& \overline{\bar{\zeta}}_{d}=-\bar{\xi} \\
& \dot{\bar{\mu}}_{d}=-\frac{\alpha}{\beta} \bar{\varepsilon}
\end{aligned}
$$

(1) INVOLUTION

$$
\alpha \bar{H}_{d}=\alpha \beta \bar{E}=\bar{E} \underbrace{\alpha \bar{H}} \quad \alpha \beta=+1 \quad \bar{H}_{d}=\beta \bar{E}
$$

(2) $\left(\varepsilon_{0}, \mu_{0}\right)_{d}=\varepsilon_{0}, \mu_{0}$

$$
\varepsilon_{d}=-\frac{\beta}{\alpha} \mu
$$

$$
\begin{aligned}
& \varepsilon_{0 d}=-\frac{f}{\alpha} \mu_{0}=\varepsilon_{0} \\
& \mu_{0 d}=-\frac{\alpha}{\beta} \varepsilon_{0}=\mu_{0} \\
& \left(\varepsilon_{0 d} \mu_{0 d}=\varepsilon_{0} \mu_{0}\right)
\end{aligned}
$$

$$
\mu_{d}=-\frac{\alpha}{\beta} \varepsilon
$$

$$
\frac{\alpha}{\beta}=\alpha^{2}=-\eta_{0}^{2}
$$

$$
\alpha= \pm j \eta_{0}
$$

$$
\begin{aligned}
\bar{E}_{d} \times \bar{H}_{d}^{*} & =\alpha \bar{H} \times(\beta \bar{E})^{*} \\
& = \pm j \eta_{0} \bar{H} \times\left( \pm \frac{j}{\eta_{0}} \bar{E}^{*}\right) \\
& =-\bar{H} \times \bar{E}^{*} \\
& =E^{*} \times \bar{H}
\end{aligned}
$$

$$
\alpha= \pm j \eta_{0}
$$

$$
\beta=i \frac{j}{\eta_{0}}
$$

(Real part remains) invariant

$$
\beta=\frac{1}{\alpha}=\frac{1}{ \pm j \eta_{0}}
$$

$$
= \pm \frac{1}{j \eta_{0}}=\mp \frac{j}{\eta_{0}}
$$

$$
\begin{aligned}
& \varepsilon_{\varepsilon_{d}}=\varepsilon_{r d} \varepsilon_{0}=-\frac{\beta}{\alpha} \mu=-\left(-\frac{1}{\eta_{0}^{2}}\right) \mu \\
&=\frac{\varepsilon_{0}}{\mu_{a}} \mu_{r} \mu_{\mu-} \\
& \varepsilon_{r d}=\mu_{r}
\end{aligned}
$$

$$
\begin{array}{rlr}
\text { PE }_{d}=P M C & \text { o large } \\
k_{d}=\omega \sqrt{\mu_{d} \varepsilon_{d}}=\omega \sqrt{\mu \varepsilon}=k & \varepsilon=\varepsilon^{\prime}-j 0 / \omega \\
\eta_{d}=\sqrt{\frac{\mu_{d}}{\varepsilon_{d}}}=\sqrt{\frac{\eta_{0}^{2} \varepsilon \eta_{0}^{2}}{\mu}} & \varepsilon_{d}=\frac{\mu}{\eta_{0}^{2}} \\
=\eta_{0}^{2} \frac{1}{\eta} & \mu_{d} & =-\frac{\alpha}{\beta} \varepsilon \\
\eta_{d} \eta=\eta_{0}^{2} & & =\eta_{0}^{2} \varepsilon
\end{array}
$$

$\partial$

$$
\begin{aligned}
& \text { f } \quad \begin{aligned}
& \text { dual: } \bar{j}_{\alpha}=I L \bar{u} \delta(\bar{j}) \\
& \bar{j}_{m}=-\beta \bar{j}_{m}= j_{\eta_{0}} \bar{j}_{m} \\
& \eta_{0} \bar{j}_{a} \quad \frac{j}{\eta_{0}} I_{m} L \bar{u} \delta(\bar{r})
\end{aligned}
\end{aligned}
$$

$$
\begin{aligned}
& \theta \bar{E}_{d}(\bar{\sigma})=j \omega \mu_{0} I_{d} L \frac{e^{-i k_{d} r}}{4 \pi r} \sin \theta \bar{u}_{\theta} \\
& \alpha \bar{H}(r)=j \omega \mu_{0}\left(-\beta \frac{I}{m} L\right) \frac{e^{-j k r}}{4 \pi r} \sin \theta \bar{u}_{\theta} \\
& \bar{H}(r)=-j \omega \mu_{0} \frac{\beta}{\alpha} I_{m} L \frac{e^{-j \omega r}}{4 \pi r} \\
& \sin \theta \bar{u}_{\theta} \\
&=+j \omega \mu_{0} \frac{1}{\eta_{0}^{2}} I_{m L} \frac{e^{-i \omega r}}{4 \pi r} \sin \theta \bar{u}_{\theta}
\end{aligned}
$$

Self. duality

$$
\begin{aligned}
& \bar{E}, \bar{H} \\
& \bar{E}_{ \pm}=\frac{1}{2}\left(\bar{E}_{\mp}{ }_{j} \eta_{0} \bar{H}\right) \\
& \bar{E}_{+d}=\frac{1}{2}\left(\alpha \bar{H}-j \eta_{0} \beta \bar{E}\right) \\
& =\frac{1}{2}\left(-j \eta_{0} \bar{H}-j \eta_{0} \frac{1}{-j \eta_{0}} \bar{E}\right)=\bar{E}_{+} \\
& \bar{E}_{ \pm} \cdot \bar{E}_{ \pm}=0
\end{aligned}
$$

$$
\begin{aligned}
& \text { - + Q } \\
& \cdot+Q \\
& \text { PEC } \\
& \hat{1} \\
& \text { - - Q } \\
& \begin{array}{l}
\because \\
\hline
\end{array} \\
& -+ \\
& \because \pm \\
& \text { oj } \\
& t j \\
& +j \\
& \underset{p \in C}{ } \\
& \vec{\theta} \left\lvert\, \begin{array}{lll} 
& \vdots & \vdots \\
\theta & \vec{\theta} & 0
\end{array}\right. \\
& \text { Top view } \\
& \text { io }
\end{aligned}
$$

CABINET

$\frac{\text { APERTURE }}{j \not} 1$

$+$


Self. complementary antennas


$$
\begin{aligned}
z_{d} z & =\left(\frac{\eta_{0}}{2}\right)^{2} \\
z_{d} & =z \Rightarrow 189 \Omega
\end{aligned}
$$

BOWTTE ANTENNA

