7(a)

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$$\chi(\omega) = \int_{-\infty}^{\infty} H(t)\,(\beta+\gamma t)\,e^{-t/T}\,e^{-j\omega t}\,dt = \int_0^{\infty} (\beta+\gamma t)\,e^{-(j\omega+\frac{1}{T})t}\,dt$$

$$= \Big/_0^{\infty} \beta\,\frac{e^{-(\,)t}}{-(j\omega+\frac{1}{T})} + \Big/_0^{\infty} \gamma\,\frac{(t+\frac{1}{(\,)})\,e^{-(\,)t}}{-(j\omega+\frac{1}{T})}$$

$$= \frac{\beta}{j\omega+\frac{1}{T}} + \frac{\gamma}{(j\omega+1/T)^2} = \frac{\beta T}{1+j\omega T} + \frac{\gamma T^2}{(1+j\omega T)^2}$$

$$\chi(0) = \beta T\left(1+\frac{\gamma T}{\beta}\right) = \varepsilon(0)-\varepsilon_0 \quad \& \quad \chi(\infty) = 0 = \varepsilon(\infty)-\varepsilon_0$$

7(b)

Medium is passive $\Leftrightarrow \operatorname{Im}\{\varepsilon(\omega)\} \le 0 \Leftrightarrow \operatorname{Im}\{\chi(\omega)\} \le 0$

Debye ($\gamma=0$) is passive, because then $\operatorname{Im}\chi(\omega) = \underbrace{-\frac{\omega T}{1+(\omega T)^2}}_{\text{always} \le 0\ (\omega \ge 0)}$

$\gamma \ne 0$

$$\Rightarrow \operatorname{Im}\chi(\omega) = \operatorname{Im}\left\{\beta T\,\frac{1+\frac{\gamma T}{\beta}+j\omega T}{(1+j\omega T)^2}\right\}$$

$$= -\frac{\beta T\,\omega T}{(1+\omega^2T^2)^2}\left(1+2\frac{\gamma T}{\beta}+\omega^2T^2\right)$$

$\leftarrow$ always ≤ 0 for $\frac{\gamma T}{\beta} \ge 0$

If $\frac{\gamma T}{\beta}$ is negative enough $\to \operatorname{Im}\{\chi(\omega)\}$ changes sign.

$$\frac{\partial}{\partial\omega}\operatorname{Im}\{\chi(\omega)\}\Big|_{\omega=0} = -T^2\beta\,\underbrace{\left(1+\frac{2\gamma T}{\beta}\right)}_{\text{changes sign at } \gamma = -\frac{1}{2}\beta/T}$$

Requirement for passivity: $\boxed{\gamma \ge -\frac{\beta}{2T}}$

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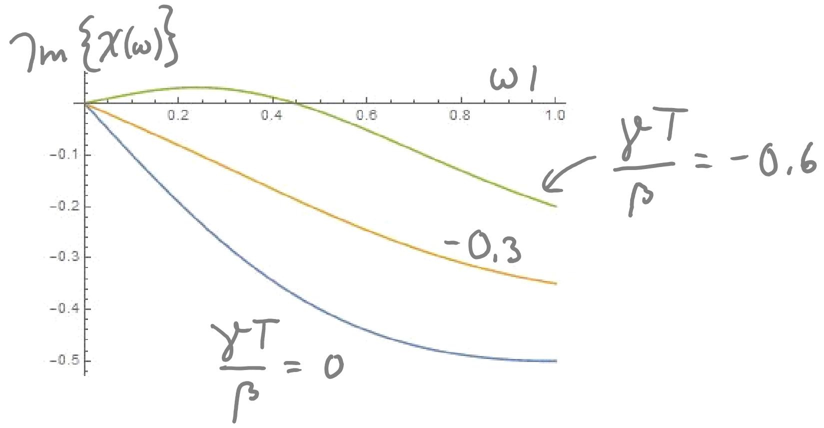

Here can be seen when the $a = -\frac{1}{2}$ limit is violated!

7(c)

The reflection dyadic becomes $\bar{\bar{R}} = -\bar{u}_x\bar{u}_x + \bar{u}_y\bar{u}_y$

(i) $\bar{E}^i = E^i\bar{u}_x \Rightarrow \bar{E}^r = -E^i\bar{u}_x = -\bar{E}^i$

(ii) $\bar{E}^i = E^i\bar{u}_y \Rightarrow \bar{E}^r = +E^i\bar{u}_y = +\bar{E}^i$

(iii) $\bar{E}^i = \dfrac{\bar{u}_x + \bar{u}_y}{\sqrt{2}}E^i \Rightarrow \bar{E}^r = \dfrac{-\bar{u}_x + \bar{u}_y}{\sqrt{2}}E^i \Rightarrow \bar{E}^r \cdot \bar{E}^i = 0$

(iv) $\bar{E}^i = \dfrac{\bar{u}_x + j\bar{u}_y}{\sqrt{2}}E^i \Rightarrow \bar{E}^r = \dfrac{-\bar{u}_x + j\bar{u}_y}{\sqrt{2}}E^i$

(v)

	PEC reflection	SHS reflection
(i)	$R = -1$	$R = -1$
(ii)	$R = -1$	$R = +1$ (like PMC)

(iii) $R = -1$

totally cross-polarized

(iv)

RH ↓ ↑ LH

PEC

(handedness changes in reflection)

RH ↓ ↑ RH

SHS

(handedness remains the same!)