Hertz dipole
$$P = \frac{1}{2}\oint \bar{E}_\theta \times \bar{H}_\varphi^* \cdot d\bar{s} = \frac{\omega^4 \mu\sqrt{\mu\varepsilon}}{12\pi}|p|^2$$

DUALITY: magnetic dipole $\quad (j\omega p = IL)$

$$\frac{1}{j\eta}E_\varphi = j\frac{k\frac{j}{\eta}I_m l}{4\pi r}\sin\theta\, e^{-jkr} \Rightarrow E_\varphi = -j\frac{kI_m l}{4\pi r}\sin\theta\, e^{-jkr}$$

$$P = \cdots = \frac{\omega^4\varepsilon\sqrt{\mu\varepsilon}}{12\pi}|I_m l|^2 \qquad (j\omega m = I_m L)$$

Transformation
$$\begin{pmatrix}\bar{E}\\ \eta_0\bar{H}\end{pmatrix}_d = \begin{pmatrix}\bar{E}\\ \bar{E}+\eta_0\bar{H}\end{pmatrix}$$

$$\nabla\times\bar{E}_d = -j\omega\bar{B}_d \Rightarrow \nabla\times\bar{E} = -j\omega\bar{B} = -j\omega\bar{B}_d$$

$$\nabla\times\eta_0\bar{H}_d = j\omega\eta_0\bar{D}_d$$

$$\nabla\times\bar{E} + \eta_0\nabla\times\bar{H} = -j\omega\bar{B} + \eta_0 j\omega\bar{D} \quad \Rightarrow \eta_0\bar{D}_d = \eta_0\bar{D} - \bar{B}$$

$$\begin{pmatrix}\eta_0\bar{D}\\ \bar{B}\end{pmatrix}_d = \begin{pmatrix}\eta_0\bar{D}-\bar{B}\\ \bar{B}\end{pmatrix}$$

$$\bar{D}_d = \varepsilon_d\cdot\bar{E}_d + \xi_d\cdot\bar{H}_d$$

$$\bar{D} - \frac{1}{\eta_0}\bar{B} = \varepsilon_d\cdot\bar{E} + \xi_d\cdot\left(\frac{\bar{E}}{\eta_0} + \bar{H}\right)$$

$$\varepsilon\cdot\bar{E} - \frac{\mu}{\eta_0}\bar{H} = \varepsilon_d\cdot\bar{E} + \frac{1}{\eta_0}\xi_d\cdot\bar{E} + \xi_d\cdot\bar{H}$$

$$\xi_d = -\frac{\mu}{\eta_0} \qquad \varepsilon_d = \varepsilon - \frac{\xi_d}{\eta_0} = \varepsilon + \frac{\mu}{\eta_0^2}$$

$$\bar{B}_d = \zeta_d\cdot\bar{E}_d + \mu_d\cdot\bar{H}_d = \zeta_d\cdot\bar{E} + \mu_d\cdot\left(\bar{H} + \frac{\bar{E}}{\eta_0}\right) = \bar{B} = \mu\bar{H}$$

$= \mu \bar{H}$

$$\mu = \mu_d \qquad \zeta_d = -\frac{\mu_d}{\eta_0} = -\frac{\mu}{\eta_0}$$

SELF-DUAL FIELDS (WAVE FIELDS)

$$\bar{E}_+(\bar{r}) = \frac{1}{2}\left(\bar{E}(\bar{r}) - j\eta_0 \bar{H}(\bar{r})\right) \qquad \bar{H}_+ = ?$$

$$\nabla \times \bar{E}_+ = -j\omega\mu_0 \bar{H}_+$$

$$\frac{1}{2}\nabla \times \left(\bar{E} - j\eta_0 \bar{H}\right) = \frac{1}{2}\left(-j\omega\mu_0 \bar{H} - j\eta_0 j\omega\varepsilon_0 \bar{E}\right)$$

$$= -j\omega\mu_0 \underbrace{\frac{1}{2}\left(\bar{H} + j\frac{\eta_0\varepsilon_0}{\mu_0}\bar{E}\right)}$$

$$\bar{H}_+ = \frac{j}{\eta_0}\,\frac{1}{2}\left(\bar{E} - j\eta_0\bar{H}\right) \qquad \bar{H}_+ = \frac{1}{2}\left(\bar{H} + \frac{j}{\eta_0}\bar{E}\right)$$

$$\bar{H}_+ = \frac{j}{\eta_0}\bar{E}_+ \qquad \bar{E}_+ = -j\eta_0 \bar{H}_+$$

PLANE WAVE: $\bar{E}(\bar{r}) = \bar{E}_0 e^{-j\bar{k}\cdot\bar{r}} = \bar{E}_0 e^{-jk_0\bar{u}\cdot\bar{r}}$

$$\nabla \times \bar{E}(\bar{r}) = -jk_0\bar{u} \times \bar{E}(\bar{r}) = -j\omega\mu_0 \bar{H}(\bar{r}) \qquad \eta_0 \bar{H}(\bar{r}) = \bar{u} \times \bar{E}(\bar{r})$$

$$\bar{E}_+(\bar{r}) = \frac{1}{2}\left(\bar{E}(\bar{r}) - j\eta_0 \bar{H}(\bar{r})\right)$$

$$= \frac{1}{2}\left(\bar{E}(\bar{r}) - j\bar{u} \times \bar{E}(\bar{r})\right)$$

$$\bar{p}(\bar{E}_+) = ? \qquad \bar{p}(\bar{E}^+) = \frac{\bar{E}_+ \times \bar{E}_+^*}{j\bar{E}_+ \cdot \bar{E}_+^*}$$

$$\bar{E}_+ \times \bar{E}_+^* = \frac{1}{4}\left(\bar{E} - j\bar{u}\times\bar{E}\right) \times \left(\bar{E}^* + j\bar{u}\times\bar{E}^*\right)$$

$$\cancel{\bar{E}_+ \times \bar{E}_+^* = \frac{1}{4}(\bar{E} - j\,\bar{u}\times\bar{E}) \times (\bar{E}^* + j\,\bar{u}\times\bar{E}^*)}$$

$$\cancel{= \frac{1}{4}\Big(\bar{E}\times\bar{E}^* + j\,\underbrace{\bar{E}\times(\bar{u}\times\bar{E}^*)}_{\bar{u}\,|\bar{E}|^2} - j\,\underbrace{(\bar{u}\times\bar{E})\times\bar{E}^*}_{-\bar{u}\,|\bar{E}|^2} + (\bar{u}\times\bar{E})\times(\bar{u}\times\bar{E}^*)\Big)}$$

$$\cancel{\bar{u}\,(\bar{u}\times\bar{E})\cdot\bar{E}^* - \bar{E}^*\,\bar{u}\cdot(\bar{u}\times\bar{E})}$$

$$\bar{E}_+ = -j\,\bar{u}\times\bar{E}_+$$

$$\bar{E}_+ = \frac{1}{2}(\bar{E} - j\,\bar{u}\times\bar{E})$$

$$\bar{u}\times\bar{E}_+ = \frac{1}{2}\Big(\bar{u}\times\bar{E} - j\,\underbrace{\bar{u}\times(\bar{u}\times\bar{E})}_{-\bar{E}}\Big) = j\,\bar{E}_+$$

$$\bar{E}_+\times\bar{E}_+^* = \bar{E}_+\times(-j\,\bar{u}\times\bar{E}_+)^* = j\,\bar{E}_+\times(\bar{u}\times\bar{E}_+^*) = j\bar{u}\,|\bar{E}_+|^2$$

$$\bar{p}(\bar{E}_+) = \frac{\bar{E}_+\times\bar{E}_+^*}{j\,|\bar{E}_+|^2} = \frac{j\bar{u}\,|\bar{E}_+|^2}{j\,|\bar{E}_+|^2} = +\bar{u}$$

$$\bar{E}_+ = \frac{1}{2}(\bar{E} - j\,\bar{u}\times\bar{E})$$

$$4\,\bar{E}_+\cdot\bar{E}_+ = (\bar{E} - j\,\bar{u}\times\bar{E})\cdot(\bar{E} - j\,\bar{u}\times\bar{E})$$

$$= \bar{E}\cdot\bar{E} - j\,\underbrace{\bar{E}\cdot(\bar{u}\times\bar{E})}_{0} - j\,\underbrace{(\bar{u}\times\bar{E})\cdot\bar{E}}_{0} - \underbrace{(\bar{u}\times\bar{E})\cdot(\bar{u}\times\bar{E})}_{\bar{E}\cdot(\bar{E}\,\underbrace{\bar{u}\cdot\bar{u}}_{1} - \bar{u}\,\underbrace{\bar{E}\cdot\bar{u}}_{=0})}$$

$$= \bar{E}\cdot\bar{E} - \bar{E}\cdot\bar{E} = 0$$

$$= \bar{E}\cdot\bar{E} - \bar{E}\cdot\bar{E} = \overset{0}{0}$$

$$\bar{E}\cdot\left(\bar{E}\,\underbrace{\bar{u}\cdot\bar{u}}_{1} - \bar{u}\,\underbrace{\bar{E}\cdot\bar{u}}_{=0}\right)$$