Aalto University
School of Science

Department of Computer Science

Combinatorics of Efficient Computations

Approximation Algorithms

Lecture 11: Maximum Satisfiability

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2019

Maximum Satisfiability (Max Sat)

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- Note: $\textsc{Satisfiability}$ ($\textsc{Sat}$) is NP-complete where one is to decide whether a given propositional formula (in conjunctive normal form) has a satisfying assignment. E.g., $(x_1 \vee \bar{x}_2 \vee x_3) \wedge (x_2 \vee \bar{x}_3 \vee x_4) \wedge (x_1 \vee \bar{x}_4)$

A simple randomized algorithm

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$$E[W] = E\left[\sum_{j=1}^{m} w_j Y_j\right] = \sum_{j=1}^{m} w_j E[Y_j] = \sum_{j=1}^{m} w_j \Pr[C_j \text{ sat.}]$$

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- Thus, $E[W] \geq \frac{1}{2} \sum_{j=1}^{m} w_j \geq \frac{1}{2} \cdot \text{OPT}$ $\square$

Derandomization by Conditional Expectation

Thm. 2 The previous algorithm can be derandomized, i.e., there is a deterministic $\frac{1}{2}$-approximation algorithm for $\textsc{Max Sat}$.

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- $E[W] = \frac{1}{2} \cdot (E[W|x_1 = 0] + E[W|x_1 = 1])$
- $\rightsquigarrow$ for $x_1 = b_1$ chosen in this way, we have: $E[W|x_1 = b_1] \geq E[W] \geq \frac{1}{2} \cdot \mathrm{OPT}$

Derandomization by Conditional Expectation

- (by induction) we have set $x_1, \ldots, x_i$ to $b_1, \ldots, b_i$ so that

$$E[W | x_1 = b_1, \ldots, x_i = b_i] \geq E[W] \geq \frac{1}{2} \cdot \mathsf{OPT}$$

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- Now (similarly to the base case):

$$\begin{aligned} &E[W|x_1 = b_1, \ldots, x_i = b_i] \\ &\qquad = \tfrac{1}{2}(E[W|x_1 = b_1, \ldots, x_i = b_i, x_{i+1} = 0] \\ &\qquad\qquad + E[W|x_1 = b_1, \ldots, x_i = b_i, x_{i+1} = 1]) \end{aligned}$$

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- $\rightsquigarrow$ set $x_{i+1} = 1$ if and only if

$$\begin{aligned} &E[W|x_1 = b_1, \ldots, x_i = b_i, x_{i+1} = 1] \\ &\geq E[W|x_1 = b_1, \ldots, x_i = b_i, x_{i+1} = 0] \end{aligned}$$

$$\rightsquigarrow E[W|x_1 = b_1, \ldots, x_i = b_i, x_i = b_{i+1}] \geq \ldots \geq \tfrac{1}{2} \cdot \mathsf{OPT}$$

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- Consider a partial assignment $x_1 = b_1, \ldots, x_i = b_i$ and a clause C_j.
- If C_j is already satisfied, then it contributes w_j to $E[W | x_1 = b_1, \ldots, x_i = b_i]$.

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- If C_j is already satisfied, then it contributes w_j to $E[W|x_1 = b_1, \ldots, x_i = b_i]$.
- If C_j is not satisfied, and contains k unassigned variables, then it contributes precisely $w_j(1 - (\frac{1}{2})^k)$ to $E[W|x_1 = b_1, \ldots, x_i = b_i]$.

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- Note: the conditional expectation is simply the sum of the contributions from each clause. $\square$

Derandomization by Conditional Expectation

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Requirement: respective conditional probabilities can be appropriately estimated for each random decision.

The algorithm simply chooses the best option at each step.

Quality of the obtained solution is then at least as high as the expected value.

An ILP

$$
\begin{array}{rll}
\text{maximize} & \displaystyle\sum_{j=1}^{m} w_j z_j & \\
\text{subject to} & \displaystyle\sum_{i \in P_j} y_i + \sum_{i \in N_j} (1 - y_i) \geq z_j, & j = 1, \ldots, m \\
& y_i \in \{0, 1\}, & i = 1, \ldots, n \\
& 0 \leq z_j \leq 1, & j = 1, \ldots, m
\end{array}
$$

where $C_j = \bigvee_{i \in P_j} x_i \vee \bigvee_{i \in N_j} \bar{x}_i$ for each $j = 1, \ldots, m$

Note: $z_j = 1$ when C_j is satisfied, and $z_j = 0$ otherwise.

… and its relaxation

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Randomized Rounding

Thm. 3 Let $(\mathbf{y}^*, \mathbf{z}^*)$ be an optimal solution to the LP-relaxation. Independently setting each variable x_i to 1 (`true`) with probability y_i^* provides a $(1 - \frac{1}{e})$-approximation for $\textsc{Max Sat}$.

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Proof.

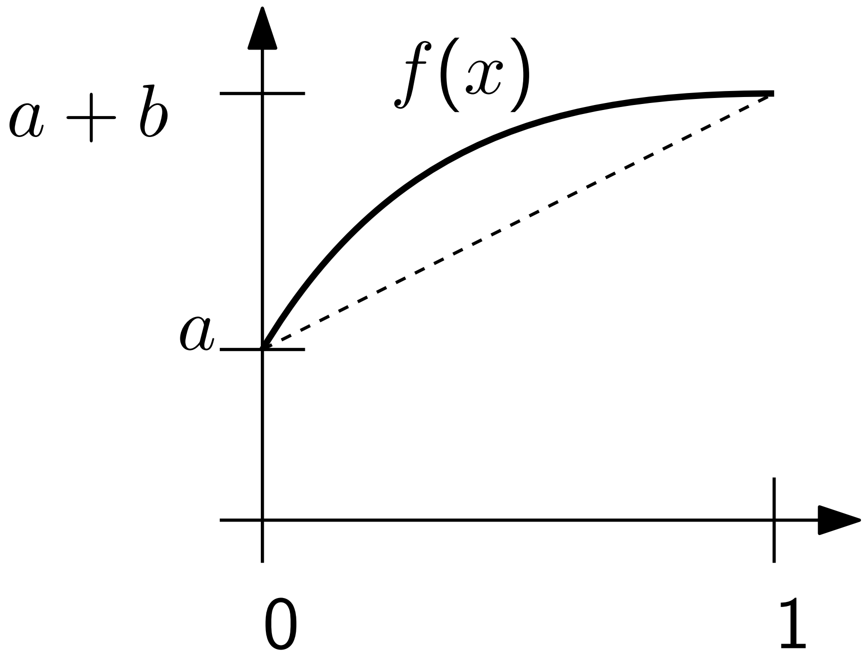
Fact#1: arithmetic-geometric mean inequality (agmi)

For all non-negative numbers $a_1, \ldots, a_k$:

$$\left(\prod_{i=1}^{k} a_i\right)^{1/k} \leq \frac{1}{k}\left(\sum_{i=1}^{k} a_i\right)$$

Randomized Rounding (proof)

Fact#2: Let $f(0) = a$ and $f(1) = a + b$ for a function which is concave on $[0, 1]$ (i.e., $f''(x) \leq 0$ on $[0, 1]$). Then we have $f(x) \geq bx + a$ for $x \in [0, 1]$



Randomized Rounding (proof)

Consider a fixed clause C_j of length l_j. We have:

$$\Pr[C_j \text{ not sat.}] = \prod_{i \in P_j} (1 - y_i^*) \prod_{i \in N_j} y_i^*$$

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$$\overset{\text{(agmi).}}{\leq} \left[\frac{1}{l_j} \left(\sum_{i \in P_j} (1 - y_i^*) + \sum_{i \in N_j} y_i^* \right) \right]^{l_j}$$

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$$= \left[1 - \frac{1}{l_j} \left(\sum_{i \in P_j} y_i^* + \sum_{i \in N_j} (1 - y_i^*) \right) \right]^{l_j}$$

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&= \left[1 - \frac{1}{l_j} \underbrace{\left(\sum_{i \in P_j} y_i^* + \sum_{i \in N_j} (1 - y_i^*) \right)}_{\geq z_j^*} \right]^{l_j} \\
&\overset{\text{LP-Relax.}}{\leq} \left(1 - \frac{z_j^*}{l_j} \right)^{l_j}
\end{aligned}$$

Randomized Rounding (proof)

The function $f(z_j^*) = 1 - \left(1 - \frac{z_j^*}{l_j}\right)^{l_j}$ is concave.
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Thus

$$\Pr[C_j \text{ sat.}] \geq f(z_j^*)$$

$$\geq \left[1 - \left(1 - \frac{1}{l_j}\right)^{l_j}\right] z_j^*$$

$$\text{Note :} \forall k \in \mathbb{Z}^+, \left(1 - \frac{1}{k}\right)^k > \frac{1}{e}$$

$$\geq \left(1 - \frac{1}{e}\right) z_j^*$$

Randomized Rounding (proof)

Therefore,

$$
\begin{aligned}
E[W] &= \sum_{j=1}^{m} \mathsf{Pr}[C_j \text{ sat.}] \cdot w_j \\
&\geq \left(1 - \frac{1}{e}\right) \sum_{j=1}^{m} w_j z_j^* \\
&\geq \left(1 - \frac{1}{e}\right) \mathsf{OPT}
\end{aligned}
$$

$\square$

Thm. 4 The above algorithm can be derandomized by the method of conditional expectation.

Take the better between the two solutions!

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We use another probabilistic argument. With probability $\frac{1}{2}$ choose the solution of Thm. 1 otherwise choose Thm. 3.

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We use another probabilistic argument. With probability $\frac{1}{2}$ choose the solution of Thm. 1 otherwise choose Thm. 3.

The better solution is at least as good as the expectation of the above algorithm.

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The probability that clause C_j is satisfied is at least:

$$P = \frac{1}{2}\left[\overbrace{\left(1-\left(1-\frac{1}{l_j}\right)^{l_j}\right)}^{\text{LP-Rounding}} + \overbrace{\left(1-2^{-l_j}\right)}^{\text{rand. Alg.}}\right] z_j^*$$

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We claim that this is at least $\frac{3}{4} \cdot z_j^*$. (the rest follows similarly to Thm. 1 and Thm. 3 by the linearity of expectation).

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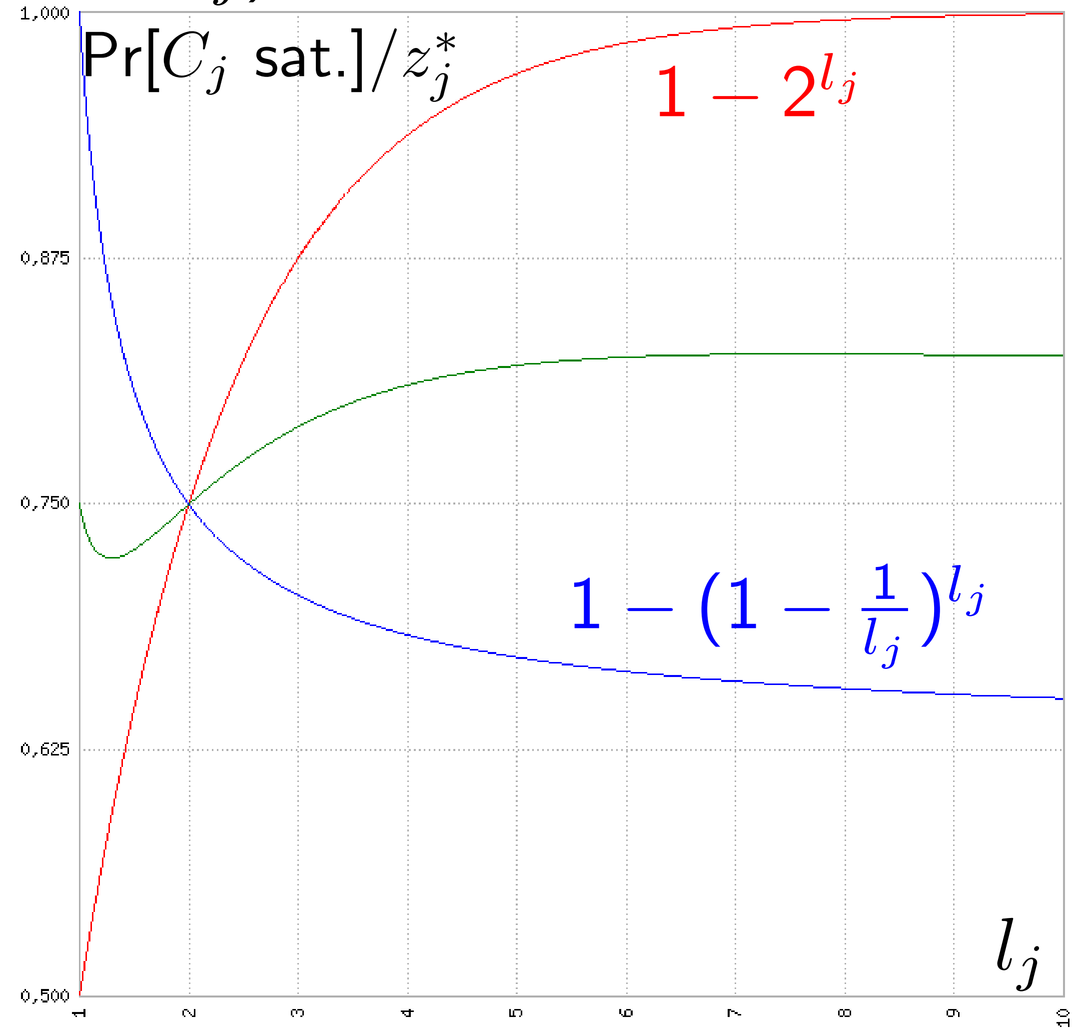
For $l_j \geq 3$, $1 - (1 - \frac{1}{l_j})^{l_j} \geq (1 - \frac{1}{e})$ and $1 - 2^{-l_j} \geq 7/8$. Thus, we have:

$$\frac{P}{z_j^*} \geq \frac{1}{2}\left[\left(1-\frac{1}{e}\right) + \frac{7}{8}\right] \approx 0{,}753 > \frac{3}{4} \qquad \square$$

Visualization and Derandomization

Randomized alg. is better for large values of l_j
Randomized LP-rounding is better for small values of l_j
($\rightsquigarrow$ probability of satisfying clause C_j)



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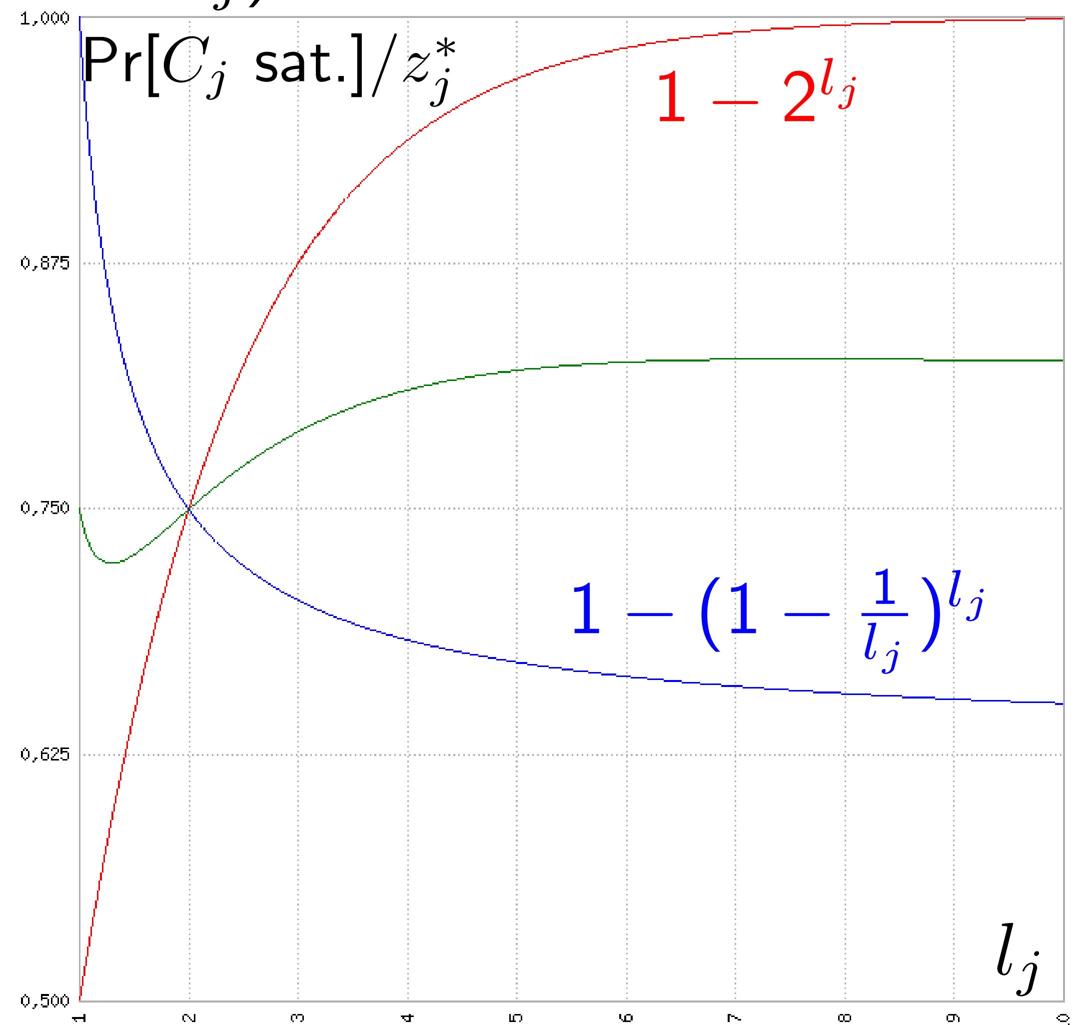

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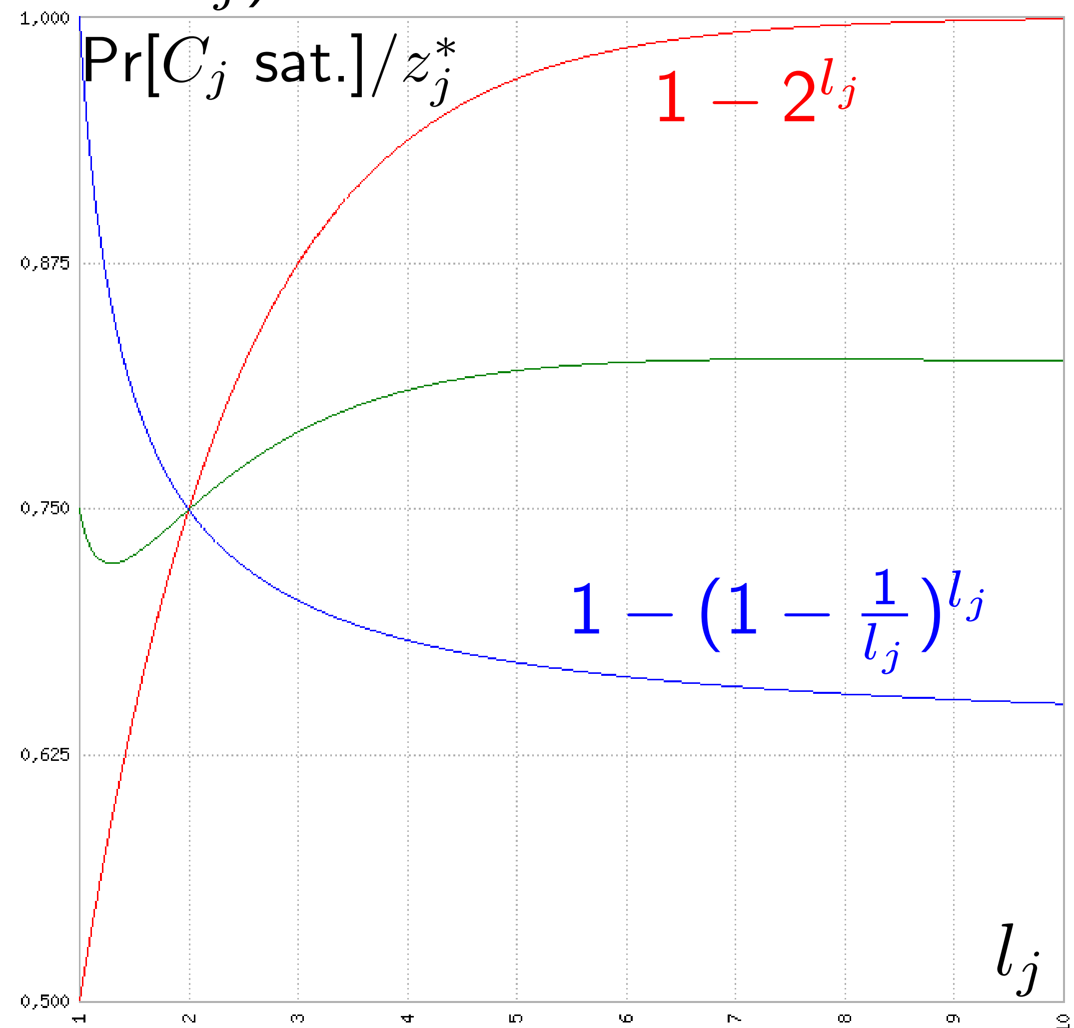

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This algorithm can also be derandomized by conditional expectation.