GREEN FUNCTION

$$L(\nabla) f(\bar{r}) = g(\bar{r}) \qquad G(\bar{r}-\bar{r}')$$

$$f(\bar{r}) = -\int G(\bar{r},\bar{r}')\, g(\bar{r}')\, dV'$$

$$L(\nabla) f(\bar{r}) = \int -L(\nabla)\, G(\bar{r}-\bar{r}')\, g(\bar{r}')\, dV' = g(\bar{r})$$

$$\int \delta(\bar{r}-\bar{r}')\, g(\bar{r}')\, dV' = g(\bar{r})$$

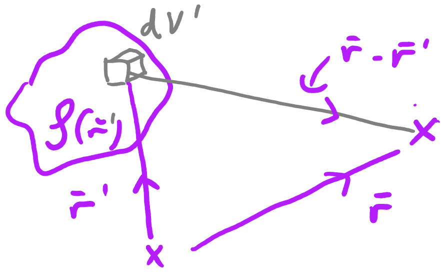
$$L(\nabla)\, G(\bar{r}-\bar{r}') = -\delta(\bar{r}-\bar{r}')$$

G.F. = field of a point source !

ELECTROSTATICS

$$\nabla^2 \phi = -\frac{\rho}{\varepsilon}$$

Q . —r— x $\quad \phi = \dfrac{Q}{4\pi\varepsilon r}$



$$\phi(\bar r) = \int \frac{\rho(\bar r')\, dv'}{4\pi\varepsilon |\bar r - \bar r'|}$$

$$= \int G(\bar r - \bar r')\, \rho(\bar r')\, dv'$$

MAGNETOSTATICS

$$\nabla \times \bar H = \bar J \qquad \Rightarrow \quad \nabla \times (\nabla \times \bar A) = \mu \bar J$$

$$\nabla \cdot \bar B = 0 \quad \Rightarrow \quad \bar B = \nabla \times \bar A$$

$$\nabla(\underbrace{\nabla \cdot \bar A}_{0}) - \nabla^2 \bar A = \mu \bar J$$

$$\nabla^2 \bar A = -\mu \bar J$$

$$\bar A(\bar r) = \int \frac{\mu \bar J(\bar r')}{4\pi |\bar r - \bar r'|}\, dv'$$

DYNAMICS.

(ISOTROPIC)

$$\nabla \times \bar{E} = -j\omega \bar{B} = -j\omega\mu \bar{H}$$

$$\nabla \times (\nabla \times \bar{E}) = -j\omega\mu \nabla \times \bar{H} = -j\omega\mu (\bar{J} + j\omega\varepsilon \bar{E})$$

$$-\nabla \times (\nabla \times \bar{E}) + \underbrace{\omega^2 \mu\varepsilon}_{k^2} \bar{E} = +j\omega\mu \bar{J}$$

$$\left[\nabla\nabla \overset{\times}{\times} \bar{\bar{I}} + k^2 \bar{\bar{I}}\right) \cdot \bar{E} = j\omega\mu \bar{J}$$

INVERSION ?

$$\bar{E}(\bar{r}) = -j\omega\mu \int \bar{\bar{G}}(\bar{r}-\bar{r}') \cdot \bar{J}(\bar{r}')\, dv'$$

GREEN DYADIC !

$$\left(\bar{\bar{I}} + \frac{\nabla\nabla}{k^2}\right) G(\bar{r}-\bar{r}')$$

$$\frac{e^{-jk|\bar{r}-\bar{r}'|}}{4\pi|\bar{r}-\bar{r}'|}$$

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$$E = \bar{E}_1 + \bar{E}_2$$

$$\bar{E}_1(\bar{r}) = -j\omega\mu \int \frac{e^{-jk|\bar{r}-\bar{r}'|}}{4\pi|\bar{r}-\bar{r}'|} \bar{J}(\bar{r}')\, dV'$$

$$\bar{E}(\bar{r}) = -\nabla\phi - j\omega\bar{A}$$

$$\nabla\times\bar{E} + j\omega \underset{\uparrow \atop \nabla\times\bar{A}}{\bar{B}} = 0 \quad \Rightarrow \quad \nabla\times(\underbrace{\bar{E}+j\omega\bar{A}}_{-\nabla\phi}) = 0$$

$$\nabla^2\phi + k^2\phi = -\rho/\varepsilon$$

$$\nabla^2\bar{A} + k^2\bar{A} = -\mu\bar{J}$$

$$\swarrow$$

$$\nabla^2\phi + k^2\phi = 0$$

$$\frac{1}{r}(r\phi)'' + k^2\phi = 0$$

$$(r\phi)'' + k^2(r\phi) = 0$$

$$\nabla^2\phi = 0$$

$$\frac{1}{r}(\underbrace{r\phi}_{Ar+B})'' = 0$$

$$\phi = A + \frac{B}{r}$$

$$r\phi = e^{\pm jkr}$$

$$G(\bar{r}-\bar{r}') = \frac{e^{-jk|\bar{r}-\bar{r}'|}}{4\pi|\bar{r}-\bar{r}'|}$$

$$\bar{E}_1(\bar{r}) = -j\omega\mu \int \frac{e^{-jk|\bar{r}-\bar{r}'|}}{4\pi|\bar{r}-\bar{r}'|} \bar{J}(\bar{r}') \, dv'$$ (VECTOR POTENTIAL)

$$\bar{E} = -\nabla\phi - j\omega\bar{A}$$

$$\int \frac{\mu\bar{J} \, e^{-jk|\bar{r}-\bar{r}'|}}{4\pi|\bar{r}-\bar{r}'|} \, dv'$$

$$\bar{E}_2(\bar{r}) = ?$$

$$\phi(\bar{r}) = \int \frac{\rho(\bar{r}') \, e^{-jk|\bar{r}-\bar{r}'|}}{4\pi\varepsilon|\bar{r}-\bar{r}'|} \, dv'$$

CONTINUITY $\quad \nabla \cdot \left(\nabla\times\bar{H} = \bar{J} + j\omega\bar{D}\right) \Rightarrow \nabla\cdot\bar{J} = -j\omega \underbrace{\nabla\cdot\bar{D}}_{\rho}$

$$\rho = j\frac{\nabla\cdot\bar{J}}{\omega}$$

$$\phi(\bar{r}) = \int \frac{j \, \nabla\cdot\bar{J}(\bar{r}') \, e^{-jk|\bar{r}-\bar{r}'|}}{\omega \, 4\pi\varepsilon \, |\bar{r}-\bar{r}'|} \, dv'$$

$$= j\omega\mu \int \frac{\nabla\cdot\bar{J} \, e^{-jk|\bar{r}-\bar{r}'|}}{\underbrace{\omega^2\mu\varepsilon}_{k^2} \, 4\pi|\bar{r}-\bar{r}'|} \, dv'$$

$$\bar{E}_2(\bar{r}) = -j\omega\mu \int \frac{\nabla\nabla}{k^2} \frac{e^{-jk|\bar{r}-\bar{r}'|}}{4\pi|\bar{r}-\bar{r}'|} \cdot \bar{J} \, dv'$$

$$\bar{E}(\bar{r}) = -j\omega\mu \int \bar{\bar{G}}(\bar{r}-\bar{r}') \cdot \bar{J}(\bar{r}')\, dV'$$

$$\bar{H}(\bar{r}) = \int \nabla G(\bar{r}-\bar{r}') \times \bar{J}(\bar{r}')\, dV'$$

$$\frac{\nabla \times \bar{E}}{-j\omega\mu} = \bar{H}$$

$$\bar{\bar{G}}(\bar{r}-\bar{r}') = \left(\bar{\bar{I}} + \frac{\nabla\nabla}{k^2}\right) G(\bar{r}-\bar{r}')$$

$$\bar{E}_d = j\eta_0 \bar{H}$$

$$\bar{H}_d = \frac{\bar{E}}{j\eta_0}$$

$$\bar{J}_d = -\frac{1}{j\eta_0} \bar{J}_m$$

$$\mu_d = \eta_0^2 \varepsilon$$

$$\bar{E}_d = -j\omega\mu_d \int \bar{\bar{G}}(\bar{r}-\bar{r}') \cdot \bar{J}_d(\bar{r}')\, dV'$$

$$j\eta_0 \bar{H} = -j\omega\, \eta_0^2 \varepsilon \int \bar{\bar{G}}(\bar{r}-\bar{r}') \cdot \frac{-\bar{J}_m}{j\eta_0}\, dV'$$

$$\bar{H}(\bar{r}) = -j\omega\varepsilon \int \bar{\bar{G}}(\bar{r}-\bar{r}') \cdot \bar{J}_m\, dV'$$

$$\bar{E}(\bar{r}) = -\int \nabla G(\bar{r}-\bar{r}') \times \bar{J}_m\, dV'$$

PLANE WAVE, SOURCE-FREE MEDIUM

$\bar{\bar{\varepsilon}}, \bar{\bar{\mu}}, \bar{\bar{\xi}}, \bar{\bar{\zeta}}$

$$\nabla e^{-j\bar{k}\cdot\bar{r}} = -j\bar{k}\, e^{-j\bar{k}\cdot\bar{r}}$$

$$\nabla\times\bar{E} = -j\bar{k}\times\bar{E} = -j\omega\bar{B} = -j\omega\bar{\bar{\mu}}\cdot\bar{H} - j\omega\bar{\bar{\zeta}}\cdot\bar{E}$$

$$\bar{\bar{\mu}}\cdot\bar{H} = \frac{\bar{k}}{\omega}\times\bar{E} - \bar{\bar{\zeta}}\cdot\bar{E}$$

$$\bar{H} = \bar{\bar{\mu}}^{-1}\cdot\left(\frac{\bar{k}\times\bar{\bar{I}}}{\omega} - \bar{\bar{\zeta}}\right)\cdot\bar{E}$$

$$\nabla\times\bar{H} = -j\bar{k}\times\bar{H} = j\omega\bar{\bar{\varepsilon}}\cdot\bar{E} + j\omega\bar{\bar{\xi}}\cdot\bar{H}$$

$$\bar{\bar{\varepsilon}}\cdot\bar{E} = -\frac{\bar{k}\times\bar{\bar{I}}}{\omega}\cdot\bar{H} - \bar{\bar{\xi}}\cdot\bar{H} = -\left(\bar{\bar{\xi}} + \frac{\bar{k}\times\bar{\bar{I}}}{\omega}\right)\cdot\bar{H}$$

$$\bar{\bar{\varepsilon}}\cdot\bar{E} = -\left(\bar{\bar{\xi}} + \frac{\bar{k}\times\bar{\bar{I}}}{\omega}\right)\cdot\bar{\bar{\mu}}^{-1}\cdot\left(\frac{\bar{k}\times\bar{\bar{I}}}{\omega} - \bar{\bar{\zeta}}\right)\cdot\bar{E}$$

$$\left[\bar{\bar{\varepsilon}} + \left(\bar{\bar{\xi}} + \frac{\bar{k}\times\bar{\bar{I}}}{\omega}\right)\cdot\bar{\bar{\mu}}^{-1}\cdot\left(\frac{\bar{k}\times\bar{\bar{I}}}{\omega} - \bar{\bar{\zeta}}\right)\right]\cdot\bar{E} = 0$$

$$\bar{\bar{D}}_E\left(\frac{\bar{k}}{\omega}\right)\cdot\bar{E} = 0$$

BI-ISOTROPIC

$$\varepsilon = \varepsilon_r \varepsilon_0 \qquad \mu = \mu_r \mu_0$$

$$\bar{\bar\varepsilon} = \varepsilon \bar{\bar I} \qquad \bar{\bar\mu} = \mu \bar{\bar I}$$

$$\bar{\bar\xi} = \xi \bar{\bar I} \qquad \bar{\bar\zeta} = \zeta \bar{\bar I}$$

$$\xi = (\chi - j\kappa)\sqrt{\mu_0\varepsilon_0} \qquad \zeta = (\chi + j\kappa)\sqrt{\mu_0\varepsilon_0}$$

$$\bar{\bar D}_E = \varepsilon \bar{\bar I} + \left(\xi \bar{\bar I} + \frac{\bar k \times \bar{\bar I}}{\omega}\right)\cdot \frac{\bar{\bar I}}{\mu}\cdot\left(\frac{\bar k \times \bar{\bar I}}{\omega} - \zeta \bar{\bar I}\right)$$

$$= \frac{1}{\omega^2\mu}\left(\omega^2\mu\varepsilon \bar{\bar I} + \left(\omega\xi\bar{\bar I} + \bar k \times \bar{\bar I}\right)\cdot\left(\bar k \times \bar{\bar I} - \omega\zeta\bar{\bar I}\right)\right)$$

$$= \frac{1}{\omega^2\mu}\left(k_0^2\varepsilon_r\mu_r\bar{\bar I} - \omega^2 \underbrace{\xi\zeta}_{\mu_0\varepsilon_0(\chi^2+\kappa^2)}\bar{\bar I} + \omega\underbrace{(\xi-\zeta)}_{-2j\kappa\sqrt{\mu_0\varepsilon_0}}\bar k \times \bar{\bar I} + \underbrace{\bar k \times (\bar k \times \bar{\bar I})}_{-\bar k \bar k \times \bar{\bar I}}\right)$$

$$= \frac{k_0^2}{\omega^2\mu}\underbrace{\left[\bar{\bar I}\left(\mu_r\varepsilon_r - \chi^2 - \kappa^2\right) - 2j\kappa\frac{\bar k \times \bar{\bar I}}{k_0} + \frac{\bar k \times (\bar k \times \bar{\bar I})}{k_0^2}\right]}_{\bar{\bar B}}$$

$$n^2 = \mu_r\varepsilon_r$$

$$k_0^2\bar{\bar B} = \left(\bar k \times \bar{\bar I} - jk_+\bar{\bar I}\right)\cdot\left(\bar k \times \bar{\bar I} + jk_-\bar{\bar I}\right)$$

$$= \bar k \times (\bar k \times \bar{\bar I}) - j\,\bar k \times \bar{\bar I}\,(k_+ - k_-) + k_+k_-\bar{\bar I}$$

$$k_+ = k_0\left(\sqrt{n^2-\chi^2} + \kappa\right), \qquad k_- = k_0\left(\sqrt{n^2-\chi^2} - \kappa\right)$$

SOLUTION : $\det\left(\bar{k}\times\bar{\bar{I}} - jk_+\bar{\bar{I}}\right) = 0$

$$\left(\bar{k}\times\bar{\bar{I}} - jk_+\bar{\bar{I}}\right) \,{}^{\times}_{\times}\, \left(\bar{k}\times\bar{\bar{I}} - jk_+\bar{\bar{I}}\right)$$

$$= \underbrace{2\bar{k}\bar{k} - 2jk_+\bar{k}\times\bar{\bar{I}} - 2k_+^2\bar{\bar{I}}}_{:\left(\bar{k}\times\bar{\bar{I}} - jk_+\bar{\bar{I}}\right)}$$

$$\bar{k}\times\bar{\bar{I}} \,{}^{\times}_{\times}\, \bar{k}\times\bar{\bar{I}} = 2\bar{k}\bar{k}$$

$$\bar{k}\times\bar{\bar{I}} \,{}^{\times}_{\times}\, \bar{\bar{I}} = \bar{k}\times\bar{\bar{I}}$$

$$\bar{\bar{I}} \,{}^{\times}_{\times}\, \bar{\bar{I}} = 2\bar{\bar{I}}$$

$$\bar{k}\times\bar{\bar{I}} : \bar{k}\times\bar{\bar{I}} = 2\bar{k}\cdot\bar{k}$$

$$= -2jk_+\bar{k}\cdot\bar{k} - 4jk_+\bar{k}\cdot\bar{k} + j6k_+^3$$

$$\det = -jk_+\left(\bar{k}\cdot\bar{k} - k_+^2\right)$$

$$= 0 \quad \Rightarrow \quad \bar{k}\cdot\bar{k} = k_+^2$$

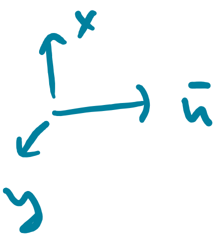
$$\bar{k} = \pm\bar{u}k_+$$

POLARIZATION ?

$$\left(\underset{\bar{u}k_+}{\underset{\uparrow}{\bar{k}}}\times\bar{\bar{I}} - jk_+\bar{\bar{I}}\right)\cdot\bar{E}_0 = 0$$

$$\Downarrow$$

$$\bar{u}\times\bar{E}_0 = j\bar{E}_0$$



$$\bar{u}\times\left(\bar{u}_x - j\bar{u}_y\right) = \bar{u}_y + j\bar{u}_x$$

$$= j\left(\bar{u}_x - j\bar{u}_y\right)$$

RHCP !