Homework 10

(a) $\bar{D} = \varepsilon \bar{E} + \xi \bar{H} \Rightarrow \bar{D}_T^* = \varepsilon \bar{E}_T^* + \xi \bar{H}_T^* \Rightarrow \begin{cases} \bar{D}_T = \varepsilon^* \bar{E}_T + \xi^* \bar{H}_T \\ \bar{D}_T = \varepsilon_T \bar{E}_T + \xi_T \bar{H}_T \end{cases} \Rightarrow$

$$\begin{cases} \varepsilon_T(\omega) = \varepsilon^*(\omega) \\ \xi_T(\omega) = \xi^*(\omega) \end{cases} \quad (1)$$

From Ismo V. Lindell's book, remember that $F_T(\omega) = F^*(\omega)$.

$\bar{D} = \varepsilon \bar{E} + \xi \bar{H} \Rightarrow -\bar{D}_P = -\varepsilon \bar{E}_P + \xi \bar{H}_P \Rightarrow \begin{cases} \bar{D}_P = \varepsilon \bar{E}_P - \xi \bar{H}_P \\ \bar{D}_P = \varepsilon_P \bar{E}_P + \xi_P \bar{H}_P \end{cases} \Rightarrow$

$$\begin{cases} \varepsilon_P = \varepsilon \\ \xi_P = -\xi \end{cases} \quad (2)$$

$\xrightarrow[(2)]{(1)} \varepsilon_{PT}(\omega) = \varepsilon^*(\omega)$ and $\xi_{PT}(\omega) = -\xi^*(\omega)$

In the same way, we can conclude that $\mu_{PT}(\omega) = \mu^*(\omega)$.

$\xi_{PT}(\omega) = -\xi^*(\omega) \Rightarrow \chi_{PT}(\omega) - j\kappa_{PT}(\omega) = -\chi^*(\omega) - j\kappa^*(\omega) \Rightarrow \begin{cases} \chi_{PT}(\omega) = -\chi^*(\omega) \\ \kappa_{PT}(\omega) = \kappa^*(\omega) \end{cases}$

(b) $\begin{cases} E_d = AE + \eta_0 BH \\ \eta_0 H_d = CE + \eta_0 DH \end{cases} \xrightarrow[\text{confusion}]{\text{for avoiding}} \begin{cases} E_d = \alpha E + \eta_0 \beta H \\ \eta_0 H_d = \gamma E + \eta_0 Z H \end{cases}$

$\nabla \times E_d = \alpha \nabla \times E + \eta_0 \beta \nabla \times H = -j\omega \alpha B - \alpha J_m + \eta_0 \beta J + \eta_0 \beta j\omega D \Rightarrow$

(2)

$$\nabla\times E_d = -j\omega\left(\alpha B - \eta_0\beta D\right) - \left(\alpha J_m - \eta_0\beta J\right) \Rightarrow$$

$$\boxed{B_d = \alpha B - \eta_0\beta D} \quad \boxed{J_{m_d} = \alpha J_m - \eta_0\beta J}$$

$$\nabla\times H_d = \frac{\gamma}{\eta_0}\nabla\times E + Z\,\nabla\times H = \frac{\gamma}{\eta_0}\left[-j\omega B - J_m\right] + Z\left[J + j\omega D\right] \Rightarrow$$

$$\nabla\times H_d = \left[ZJ - \frac{\gamma}{\eta_0}J_m\right] + j\omega\left[ZD - \frac{\gamma}{\eta_0}B\right] \Rightarrow$$

$$\boxed{D_d = -\frac{\gamma}{\eta_0}B + ZD} \quad \boxed{J_d = -\frac{\gamma}{\eta_0}J_m + ZJ}$$

(c) $$\left(\frac{d^2}{dz^2} + k^2\right)G(z) = -\delta(z)$$

$$z \neq 0 \longrightarrow G''(z) + k^2 G(z) = 0 \Rightarrow G(z) = Ae^{-jk|z|}$$

$$s^2 + k^2 = 0 \Rightarrow s = \pm jk$$

$$\left(\frac{d^2}{dz^2} + k^2\right)G(z) = -\delta(z) \Rightarrow \int_{-\alpha}^{+\alpha}\left(\frac{d^2}{dz^2} + k^2\right)G(z)\,dz = -\int_{-\alpha}^{+\alpha}\delta(z)\,dz = -1$$

$$\int_{-\alpha}^{+\alpha}\left(\frac{d^2}{dz^2} + k^2\right)G(z)\,dz = \left.\frac{dG(z)}{dz}\right|_{-\alpha}^{+\alpha} + k^2\int_{-\alpha}^{+\alpha}G(z)\,dz$$

Let us make $\alpha \longrightarrow 0$ we can write $$\lim_{\alpha\to 0}\left.\frac{dG(z)}{dz}\right|_{-\alpha}^{+\alpha} = -j2kA$$

$$\lim_{\alpha \to 0} k^2 \int_{-\alpha}^{+\alpha} G(z)\,dz = 0$$

$$\Rightarrow \left.\frac{dG(z)}{dz}\right|_{-\alpha}^{+\alpha} + k^2 \int_{-\alpha}^{+\alpha} G(z)\,dz \xlongequal{\alpha \to 0} -j2kA$$

Now: $-j2kA = -\int_{-\alpha}^{+\alpha} \delta(z)\,dz = -1 \Rightarrow A = \dfrac{1}{j2k}$

$$\Rightarrow \boxed{G(z) = \frac{e^{-jk|z|}}{j2k}}$$