HOMEWORK 9 – SOLUTION SUGGESTIONS.

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(a) Normal incidence: $\bar{\bar{R}} = (\bar{\bar{Z}}_s + \eta_o \bar{\bar{I}}_t)^{-1} \cdot (\bar{\bar{Z}}_s - \eta_o \bar{\bar{I}}_t)$

$$\bar{\bar{R}} = \underbrace{\left(\frac{1}{M}\bar{\bar{J}} + \eta_o \bar{\bar{I}}_t\right)^{-1}}_{\dfrac{\eta_o \bar{\bar{I}}_t - \frac{1}{M}\bar{\bar{J}}}{\eta_o^2 + 1/M^2}} \cdot \left(\frac{1}{M}\bar{\bar{J}} - \eta_o \bar{\bar{I}}_t\right) \qquad (\bar{\bar{J}} = \bar{n} \times \bar{\bar{I}}_t)$$

see Eq. (2.188) for 2D inverse

$$\bar{\bar{R}} = \frac{(1-(M\eta_o)^2)\bar{\bar{I}}_t + 2\eta_o M \bar{\bar{J}}}{1+(M\eta_o)^2}$$

$\eta_o M = 3$

$$\bar{\bar{R}} = -\frac{4}{5}\bar{\bar{I}}_t + \frac{3}{5}\bar{\bar{J}}$$

$$\bar{E}^r = \bar{\bar{R}} \cdot \bar{E}^i = \bar{\bar{R}} \cdot (j\bar{u}_x + 2\bar{u}_y)E^i = \frac{-E^i}{5}\left[(6+4j)\bar{u}_x + (8-3j)\bar{u}_y\right]$$

INCIDENT FIELD $\bar{E}^i$: $\bar{p}(\bar{E}^i) = \frac{4}{5}\bar{n}$ (LH elliptical polarization, axis ratio 2, main axis y-directed)

propagates into $-\bar{n}$

REFLECTED FIELD $\bar{E}^r$: $\bar{p}(\bar{E}^r) = \frac{4}{5}\bar{n}$ (RH elliptical polarization, axis ratio 2, main axis points into $3\bar{u}_x + 4\bar{u}_y$)

because propagates into $\bar{n}$

(b) ALL $\hat{E}, \hat{H}, \hat{D}, \hat{B}$ HAVE THE SAME UNITS, $\sqrt{\frac{VAs}{m^3}} = \sqrt{J/m^3}$

(SQ. ROOT OF ENERGY DENSITY)

$$\mathcal{M} = \frac{1}{\cos\vartheta}\begin{pmatrix} 1/\eta_r & \sin\vartheta \\ \sin\vartheta & \eta_r \end{pmatrix}$$

$$\vartheta = 0 \Rightarrow \mathcal{M} = \begin{pmatrix} 1/\eta_r & 0 \\ 0 & \eta_r \end{pmatrix}, \quad \varepsilon_r \mu_r = 1$$

(UNIREFRACTIVE MEDIUM)

FOR $\vartheta \to \pi/2$, ALL PARAMETERS IN $\mathcal{M}$ GROW WITHOUT LIMIT.

$$\hat{D} = \frac{1}{\eta_r \cos\vartheta}\hat{E} + \tan\vartheta\, \hat{H}, \quad \hat{B} = \tan\vartheta\, \hat{E} + \frac{\eta_r}{\cos\vartheta}\hat{H}$$

$\Downarrow$

$$D = \frac{\quad}{\eta_r \cos\vartheta} E + \ldots \tan\vartheta \, H \,, \quad B = \ldots \frac{\quad}{\cos\vartheta}$$

$$\Downarrow$$

$$\eta_r \hat{H} = -\sin\vartheta\, \hat{E} + \cos\vartheta\, \hat{B}$$

$$\eta_r \hat{D} = \frac{1}{\cos\vartheta} \hat{E} + \tan\vartheta\, \eta_r H$$

$$= \frac{1}{\cos\vartheta} \hat{E} - \frac{\sin^2\vartheta}{\cos\vartheta} \hat{E} + \sin\vartheta\, \hat{B} = \cos\vartheta\, \hat{E} + \sin\vartheta\, \hat{B}$$

$$\Rightarrow \quad \begin{pmatrix} \hat{D} \\ \hat{H} \end{pmatrix} = \frac{1}{\eta_r} \underbrace{\begin{pmatrix} \cos\vartheta & \sin\vartheta \\ -\sin\vartheta & \cos\vartheta \end{pmatrix}}_{\text{ROTATION MATRIX}} \begin{pmatrix} \hat{E} \\ \hat{B} \end{pmatrix}$$

IN THIS FORM, THE LIMITS ARE EASY:

$$\vartheta = 0 \quad \Rightarrow \quad \hat{D} = \frac{1}{\eta_r} \hat{E} \,, \quad \hat{H} = \frac{1}{\eta_r} \hat{B}$$

SIMPLE ISOTROPIC

$$\vartheta = \frac{\pi}{2} \quad \Rightarrow \quad \hat{D} = \frac{1}{\eta_r} \hat{B} \quad \& \quad \hat{H} = -\frac{1}{\eta_r} \hat{E}$$

(PEMC!)

(c)

$$\bar{E}_t^r = \bar{\bar{R}}_E \cdot \bar{E}_t^i$$

$$\bar{E}_t^i = \bar{\bar{J}}_t \cdot \eta_0 \bar{H}_t^i$$

$$\bar{E}_t^r = -\bar{\bar{J}}_t \cdot \eta_0 \bar{H}_t^r$$

$$\bar{\bar{J}}_t \cdot \bar{\bar{J}}_t = -\bar{\bar{I}}_t$$

$$-\bar{\bar{J}}_t \cdot \bar{H}_t^r = \bar{\bar{R}}_E \cdot \bar{\bar{J}}_t \cdot \bar{H}_t^i$$

$$\bar{H}_t^r = \underbrace{\bar{\bar{J}}_t \cdot \bar{\bar{R}}_E \cdot \bar{\bar{J}}_t}_{\bar{\bar{R}}_H} \cdot \bar{H}_t^i$$

FOR EXAMPLE, REFLECTION FROM PEC: $\bar{\bar{R}}_E = -\bar{\bar{I}}_t$

$$\Rightarrow \quad \bar{\bar{R}}_H = -\bar{\bar{J}}_t \cdot \bar{\bar{I}}_t \cdot \bar{\bar{J}}_t = +\bar{\bar{I}}_t$$

SH · SURFACE. NORMAL INCIDENCE: $\bar{\bar{R}}_E = \begin{pmatrix} -1 & 0 \\ 0 & +1 \end{pmatrix}$

$\Rightarrow \quad \bar{\bar{R}}_H = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ $\qquad \bar{\bar{J}}_t = \bar{n} \times \bar{\bar{I}} = \bar{\bar{J}}$

$= \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & +1 \end{pmatrix}$ $\qquad = \begin{pmatrix} 0 & -1 \\ +1 & 0 \end{pmatrix}$

hence $\bar{\bar{R}}_H = \bar{\bar{R}}_E$!

NOTE: $\bar{\bar{R}}_E = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$ HOLDS FOR INCIDENCES IN xz & yz planes.

For arbitrary incident wave $\bar{k}^i$, the dyadic is

$$\bar{\bar{R}}_E = \begin{pmatrix} -1 & 0 \\ \alpha & +1 \end{pmatrix} \quad \text{where} \quad \alpha = \frac{2(\bar{u}_x \cdot \bar{k}_t)(\bar{u}_y \cdot \bar{k}_t)}{k_n^2 + (\bar{u}_y \cdot \bar{k}_t)^2}$$

Nevertheless, the same final result holds

$\bar{\bar{R}}_H = \bar{\bar{R}}_E$ even when $\alpha \neq 0$.