HOMEWORK 8 – SOLUTION SUGGESTIONS

8(a)

(i) $\dot{\bar{\bar{Z}}}_s = -j\eta_0 \sinh u \, \bar{\bar{I}}_t + j\eta_0 \cosh u \, \bar{\bar{L}}$

(ii) u is real $\Rightarrow$ amplitudes imaginary $\Rightarrow$ $\dot{\bar{\bar{Z}}}_s$ lossless

(iii) $\dot{\bar{\bar{Z}}}_s$ is reciprocal because $\dot{\bar{\bar{Z}}}_s^T = \dot{\bar{\bar{Z}}}_s$ (symmetric)

(b) $\dot{\bar{\bar{R}}} = (\dot{\bar{\bar{Z}}}_s + \eta_0 \bar{\bar{I}}_t)^{-1} \cdot (\dot{\bar{\bar{Z}}}_s - \eta_0 \bar{\bar{I}}_t) = \dots = -\underbrace{\dfrac{\cosh u}{\sinh u + j}}_{-|A| e^{j\psi},\ |A| = 1} \bar{\bar{L}}$

(note that $\bar{\bar{L}} \cdot \bar{\bar{L}} = \bar{\bar{I}}_t$ & $\cosh^2 u - \sinh^2 u = 1$)

Compact form: $\dot{\bar{\bar{R}}} = -e^{j\psi} \bar{\bar{L}}$

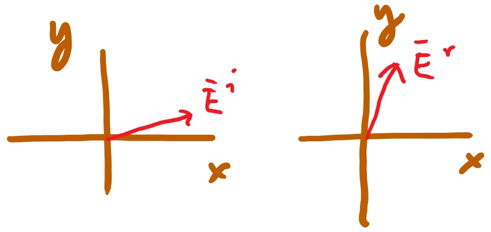
where $\tan\psi = \dfrac{-1}{\sinh u}$

(eigenvalues of $\dot{\bar{\bar{R}}}$ are $\pm e^{j\psi}$ $\leftarrow$ absolute value $+1$ (lossless))

(i) LP $\qquad$ $\bar{\bar{L}}$ changes x- and y-components

$\bar{E}^i = E^i (\bar{u}_x \cos\alpha + \bar{u}_y \sin\alpha)$

$\Rightarrow \bar{E}^r = -e^{j\psi} E^i (\bar{u}_x \sin\alpha + \bar{u}_y \cos\alpha)$



(ii) RHCP $(e^{+jk_0 z})$

$\bar{E}^i = E^i \dfrac{\bar{u}_x + j\bar{u}_y}{\sqrt{2}}$ $\qquad \Rightarrow \bar{E}^r = -e^{j\psi} E^i j \underbrace{\dfrac{\bar{u}_x - j\bar{u}_y}{\sqrt{2}}}_{\text{RHCP } (e^{-jk_0 z})}$

(iii) LHCP $\qquad \bar{E}^i = E^i \dfrac{\bar{u}_x - j\bar{u}_y}{\sqrt{2}}$

$\Rightarrow \bar{E}^r = +e^{j\psi} E^i j \underbrace{\dfrac{\bar{u}_x + j\bar{u}_y}{\sqrt{2}}}_{\text{LHCP}}$

(iv) In all cases, all power reflected for PEC & Fu Lin

PEC: LP remains, phase shift 180°

CP: handedness changes, phase shift 180°

Fu Lin: LP remains for $\bar{u}_x \pm \bar{u}_y$ -polarized wave otherwise rotated

CP: handedness remains the same RH → RH, LH → RH

phase shifts

$\psi + \pi$ LP

$\psi + \pi/2$ RHCP

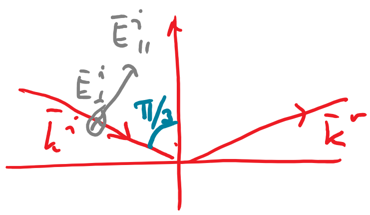
$\psi - \pi/2$ LHCP

(c) $\theta_1 = \frac{\pi}{3}$ SNELL: $\sin\theta_1 = \sqrt{\varepsilon_r}\,\sin\theta_2$ $(\varepsilon_r = 2)$

$$\cos\theta_1 = \frac{1}{2}\,, \quad \cos\theta_2 = \frac{\sqrt{5}}{2\sqrt{2}}$$

$$R_{\parallel} = \frac{\eta\cos\theta_2 - \eta_0\cos\theta_1}{\eta\cos\theta_2 + \eta_0\cos\theta_1} = \frac{\cos\theta_2 - \sqrt{2}\cos\theta_1}{\cos\theta_2 + \sqrt{2}\cos\theta_1} = 9 - 4\sqrt{5} \simeq +0{,}056$$

$$R_{\perp} = \frac{\eta\cos\theta_1 - \eta_0\cos\theta_2}{\eta\cos\theta_1 + \eta_0\cos\theta_2} = \frac{\sqrt{5} - 3}{2} \simeq -0{,}382$$



$$\bar{k}^i = k_0\left(\bar{u}_x \sin\frac{\pi}{3} - \bar{u}_z\cos\frac{\pi}{3}\right)$$

$$\bar{E}^i \sim \underbrace{\bar{u}_x\cos\frac{\pi}{3} + \bar{u}_z\sin\frac{\pi}{3}}_{\bar{E}^i_{\parallel}} + \underbrace{j\,\bar{u}_y}_{\bar{E}^i_{\perp}} \qquad (|\bar{E}^i_{\parallel}| = |\bar{E}^i_{\perp}|)$$

$$\bar{p}(\bar{E}^i) = \bar{u}_x\sin\frac{\pi}{3} - \bar{u}_z\cos\frac{\pi}{3}$$

$(|\bar{p}| = 1 \Rightarrow \text{CP})$

$(\bar{p} \parallel \bar{k}^i \Rightarrow \text{RH})$

$$\bar{k}^r = k_0 \left(\bar{u}_x \sin\frac{\pi}{3} + \bar{u}_z \cos\frac{\pi}{3} \right) = k_0 \left(\bar{u}_x \frac{\sqrt{3}}{2} + \bar{u}_z \frac{1}{2} \right)$$

$$\bar{E}^r \sim \left(\bar{u}_x \cos\frac{\pi}{3} - \bar{u}_z \sin\frac{\pi}{3} \right) R_{\parallel} + j \bar{u}_y R_{\perp}$$

$$\bar{p}(\bar{E}^r) = \bar{u}_x \frac{\sqrt{3}}{7} + \bar{u}_z \frac{1}{7}$$

$$\left(|\bar{E}^r_{\perp}| = \left| \frac{R_{\perp}}{R_{\parallel}} \right| |\bar{E}^r_{\parallel}| \right)$$

$\bar{E}^r$ elliptically polarized, axis ratio $\frac{7+3\sqrt{5}}{2} \simeq 6{,}85$

$\bar{p} \parallel \bar{E}^r \Rightarrow$ RH ellipt. pol.

$$\left| \frac{R_{\perp}}{R_{\parallel}} \right| = e$$

ALSO: $|\bar{p}(\bar{E}^r)| = \frac{2e}{e^2+1} \simeq 0{,}286$