Exam: solutions *23 May 2019*

1. For the dyadic $\overline{\overline{B}} = \overline{\overline{I}} + \mathbf{ab}$:
   (a) $\mathrm{tr}\overline{\overline{B}} = 3 + \mathbf{a}\cdot\mathbf{b}$
   (b) $\mathrm{spm}\overline{\overline{B}} = 3 + 2\mathbf{a}\cdot\mathbf{b}$
   (c) $\det\overline{\overline{B}} = 1 + \mathbf{a}\cdot\mathbf{b}$
   (d) $\overline{\overline{B}}^{-1} = \overline{\overline{I}} - \frac{\mathbf{ab}}{1+\mathbf{a}\cdot\mathbf{b}}$
   (e) Straightforward products.

2. Substitute the transformed fields using (4.11) and (4.12) to the constitutive relations:

$$\mathbf{D}_d = \overline{\overline{\varepsilon}}_d\cdot\mathbf{E}_d + \overline{\overline{\xi}}_d\cdot\mathbf{H}_d \quad\Rightarrow\quad \mathbf{B} = -\frac{\alpha}{\beta}\overline{\overline{\varepsilon}}_d\cdot\mathbf{H} - \overline{\overline{\xi}}_d\cdot\mathbf{E} \quad \text{but also} \quad \mathbf{B} = \overline{\overline{\zeta}}\cdot\mathbf{E} + \overline{\overline{\mu}}\cdot\mathbf{H}$$

   and

$$\mathbf{B}_d = \overline{\overline{\zeta}}_d\cdot\mathbf{E}_d + \overline{\overline{\mu}}_d\cdot\mathbf{H}_d \quad\Rightarrow\quad \mathbf{D} = -\overline{\overline{\zeta}}_d\cdot\mathbf{H} - \frac{\beta}{\alpha}\overline{\overline{\mu}}_d\cdot\mathbf{E} \quad \text{but also} \quad \mathbf{D} = \overline{\overline{\varepsilon}}\cdot\mathbf{E} + \overline{\overline{\xi}}\cdot\mathbf{H}$$

   which leaves us with (4.16).

3. The electric field reads

$$\mathbf{E}(\mathbf{r}) = \left[(1+\mathrm{j})\mathbf{u}_x + (3-2\mathrm{j})\mathbf{u}_y\right]\mathrm{e}^{-\mathrm{j}k_0 z} = \mathbf{E}_0\mathrm{e}^{-\mathrm{j}k_0 z}$$

   (a) From (1.35)

$$\mathbf{p}(\mathbf{E}_0) = \frac{\mathbf{E}_0\times\mathbf{E}_0^*}{\mathrm{j}\mathbf{E}_0\cdot\mathbf{E}_0^*} = \frac{2}{3}\mathbf{u}_z$$

   (b) The wave is elliptically polarized ($0 < |\mathbf{p}(\mathbf{E}_0)| < 1$). It is also right-handed because $\mathbf{p}$ points into the propagation direction z.

   How eccentric is the ellipse? Page 11, item 4 says that the axial ratio of the polarization ellipse e is connected to the absolute value of the polarization vector in the following way:

$$\frac{2e}{e^2+1} = |\mathbf{p}(\mathbf{E}_0)| = \frac{2}{3}$$

   which leads to the following major–minor-axis ratio:

$$e = \frac{3+\sqrt{5}}{2} \approx 2.62$$