

Phys-E0414: Advanced quantum mechanics

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• TAs: Fredrik Brange & Pedro Portugal; 235

12 online lectures (2×45 min), Tuesdays 10.15-12.00

12 online exercise classes ; Thursdays 12.15-14.00

1 exam (3 hours); December 9, 13.00-16.00

• Lectures & exercises take place online via zoom.

You can enter the online classroom via MyCourses.

If prompted, the password is "einstein".

• Exercise classes: 1 exercise sheet w. ~ 3 problems.

The last exercise is also a homework exercise.

• Grading: 1-5; Exam: 60%; Homework: 40%

• Course material: Zettili: "Quantum Mechanics: Concepts and Applications" + hand-written lecture notes on MyCourses

• Required background: - Some quantum mechanics
- Some linear algebra
- Some complex analysis

Course content:

Week 1: Intro to course, history of QM

Week 2: Math. of QM, two-level systems,
time-evolution, time-independent
Schrödinger equation

Week 3: Two-level systems (continued)
Bloch sphere, measurements,
time-dependent Schrödinger equation

Week 4: Quantum harmonic oscillator,
coherent states, displacement operator

Week 5: Orbital angular momentum,
Stern-Gerlach experiment, spin

Week 6: Perturbation theory & examples
(WKB method)

Fall break

- Week 7: Composite systems, addition of two spins, Pauli principle, exchange coupling
- Week 8: Entanglement; creation & detection
Bell and CHSH inequality
- Week 9: Qubits, quantum computing,
Deutsch's algorithm, Grover's algorithm
- Week 10: Quantum communication and
quantum teleportation
- Week 11: Open quantum systems,
density matrices, decoherence
- Week 12: Summary of course

History of quantum mechanics (some of the major steps)

1900: Max Planck; theory of black body radiation

1905: Albert Einstein; explanation of the photoelectric effect (+ brownian motion & relativity theory)

1913: Niels Bohr; quantum model of atoms

1918: Max Planck, Nobel Prize (NP)

1921: Albert Einstein, NP

1922: Niels Bohr, NP

1922: Stern-Gerlach experiment, spin of the electron

1924: Louis de Broglie; wave-particle duality of electrons

1925: Wolfgang Pauli; the exclusion principle

1925: Werner Heisenberg; matrix formulation of quantum mechanics

1926: Erwin Schrödinger; wave mechanics & Schrödinger equation

1927: Werner Heisenberg; uncertainty principle

1927: Max Born; probabilistic interpretation of QM

1929: de Broglie, NP

(1931): (Kurt Gödel: incompleteness theorem)

1932: Werner Heisenberg, NP

1933: Schrödinger, NP

1933: Otto Stern; measures the magnetic moment of the proton

1935: Einstein, Podolsky, and Rosen: EPR paradox

(1936: Alan Turing: Describes the Turing Machine of computation)

1940: Pauli; proves the spin-statistics theorem

1943: Otto Stern, NP

1944: Pauli, NP.

→ The QM that we know today was more-or-less developed by WWII.

1950



Nuclear Physics

Quantum Optics

Condensed matter physics

High-energy physics

→ Lasers, semi-conductor technology, electronics, computers, ...

1964: Bell inequality

1969: CSW — " —

1980



Deutsch algorithm (1985)

Shor's algorithm (1994)

Grover's algorithm (1996)

1982: Test of Bell inequality (Aspect)

2015 →

Quantum engineering and technology

IBM, Google, Microsoft, D-Wave, ...

~ 4-15 qubits

~ simple quantum algorithms have been implemented



~ 2025?

Quantum Cryptography

Quantum Communication

Quantum Simulation

Quantum computation

Quantum Sensing

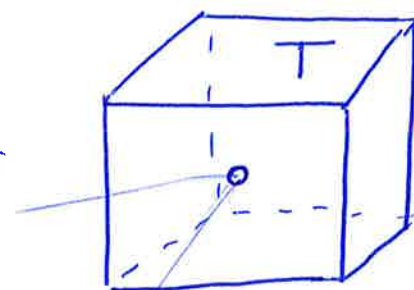


Quantum future!

Planck's radiation law:

- Wien's energy distribution (1859):

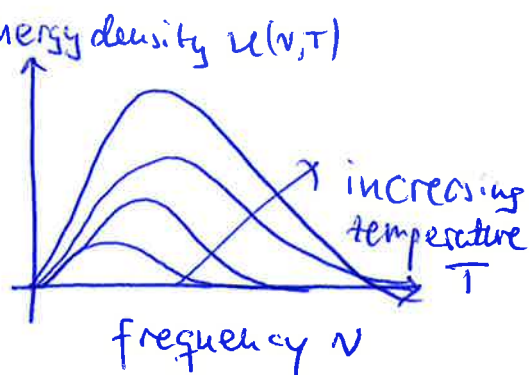
$$u(\nu, T) = A \nu^3 e^{-B\nu/T}$$



blackbody radiation

- Fits well at large frequencies.

- Rayleigh-Jeans law (1900):



Electromagnetic radiation

inside the cavity are standing waves that can be described as harmonic oscillators

Number of modes between ν and $\nu + d\nu$:
(Density)

$$N(\nu) = \frac{8\pi}{c^3} \nu^2 \quad \left[\leftarrow \text{calculate in statistical mechanics} \right]$$

Energy density:

$$u(\nu, T) = N(\nu) \langle E \rangle = \frac{8\pi}{c^3} \nu^2 \langle E \rangle$$

Average energy:

$$\langle E \rangle = \int_0^\infty dE E p(E) = \int_0^\infty dE E \frac{e^{-\beta E}}{\int_0^\infty dE e^{-\beta E}}$$

with $\beta = 1/k_B T$ and k_B is Boltzmann's constant (determined by Planck!)

Notice that

$$\langle E \rangle = -\partial_{\beta} \ln Z,$$

where $Z = \int_0^{\infty} dE e^{-\beta E}$ is the partition function:

$$\langle E \rangle = -\partial_{\beta} \ln Z = \frac{-\partial_{\beta} Z}{Z} = \frac{\int_0^{\infty} dE E e^{-\beta E}}{\int_0^{\infty} dE e^{-\beta E}}$$

$$\text{Now, } Z = \int_0^{\infty} dE e^{-\beta E} = \frac{1}{-\beta} [e^{-\beta E}]_0^{\infty} = \frac{1}{\beta} = k_B T$$

$$\text{and } \langle E \rangle = -\partial_{\beta} \ln \frac{1}{\beta} = \partial_{\beta} \ln \beta = \frac{1}{\beta} = k_B T //$$

$$\Rightarrow u(\nu, T) = \frac{8\pi}{c^3} \nu^2 k_B T //$$

Fits at low frequencies, but diverges at high frequencies $\Rightarrow$ the ultraviolet catastrophe

- Planck's distribution:

Planck postulated that the energy of each oscillator is quantized such that

$$E_n = n h \nu, \quad n = 0, 1, 2, \dots$$

are the only allowed energies. Here, h is known as Planck's constant.

Planck's postulate changes the average energy:

$$\begin{aligned}\langle E \rangle &= \sum_{n=0}^{\infty} E_n p(E_n) = \sum_{n=0}^{\infty} E_n \frac{e^{-\beta E_n}}{\sum_{n=0}^{\infty} e^{-\beta E_n}} \\ &= -\partial_{\beta} \ln Z \quad \text{with } Z = \sum_{n=0}^{\infty} e^{-\beta E_n}\end{aligned}$$

We find that

$$Z = \sum_{n=0}^{\infty} (e^{-\beta h \nu})^n = \frac{1}{1 - e^{-\beta h \nu}}$$

thus,

$$\begin{aligned}\langle E \rangle &= -\partial_{\beta} \ln Z = \partial_{\beta} \ln(1 - e^{-\beta h \nu}) \\ &= \frac{h \nu e^{-\beta h \nu}}{1 - e^{-\beta h \nu}} = \frac{h \nu}{e^{\beta h \nu} - 1}\end{aligned}$$

$$\Rightarrow \boxed{u(\nu, T) = \frac{8\pi}{c^3} \nu^2 \frac{h \nu}{e^{\beta h \nu} - 1}}$$

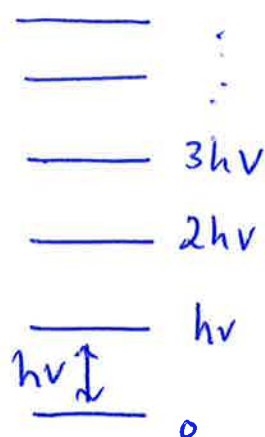
Planck's radiation law works!

But why is energy quantized?

→ quantum mechanics

How do we recover classical physics?

Quantum world



$h \rightarrow 0$

Classical world



$$h = 6.626 \times 10^{-34} \text{ J s (Planck's constant)}$$

$$k_B = 1.3807 \times 10^{-23} \text{ J/K (Boltzmann's constant)}$$

$$S = k_B \ln \Omega$$

Other natural constants: $G, e, \epsilon_0, \dots$

Comparison of scales:

$$\frac{L}{d_{\text{molecule}}} \ll 1 \quad \frac{L}{d_{\text{atom}}} \gg 1$$

We recover the classical world, when the energy quantization is small

compared to the thermal energy $k_B T$, i.e.

$$\epsilon \equiv \frac{h\nu}{k_B T} = \beta h\nu \ll 1 \quad (\text{equivalent to } h \rightarrow 0!)$$

From Planck's law, we get

$$u(\nu, T) = \frac{8\pi\nu^2}{c^3} \frac{h\nu}{e^{\beta h\nu} - 1}$$

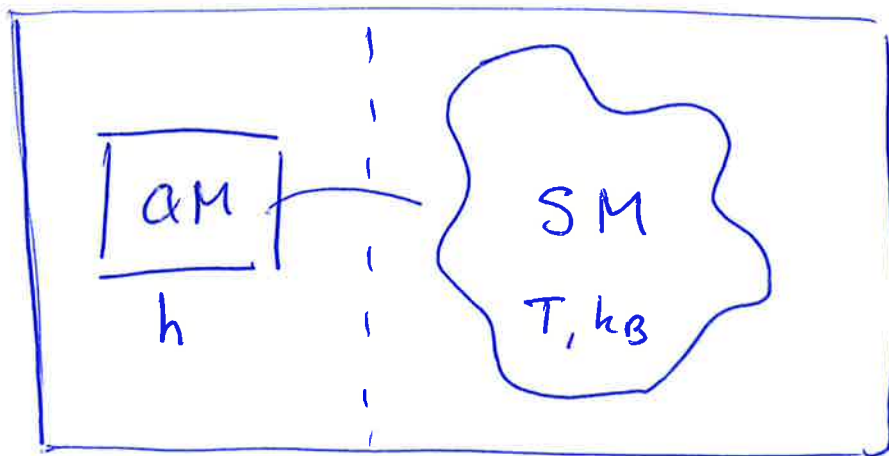
$$\stackrel{\beta h\nu \ll 1}{\approx} \frac{8\pi\nu^2}{c^3} \frac{h\nu}{1 + \beta h\nu - 1}$$

$$= \frac{8\pi\nu^2}{c^3} \frac{1}{\beta} = \frac{8\pi\nu^2}{c^3} k_B T$$

→ Rayleigh-Jeans law

and h has disappeared!

→ classical limit



In quantum mechanics, we use $h \rightarrow 0$,

and we need statistical mechanics to take $\beta h\nu \ll 1$.