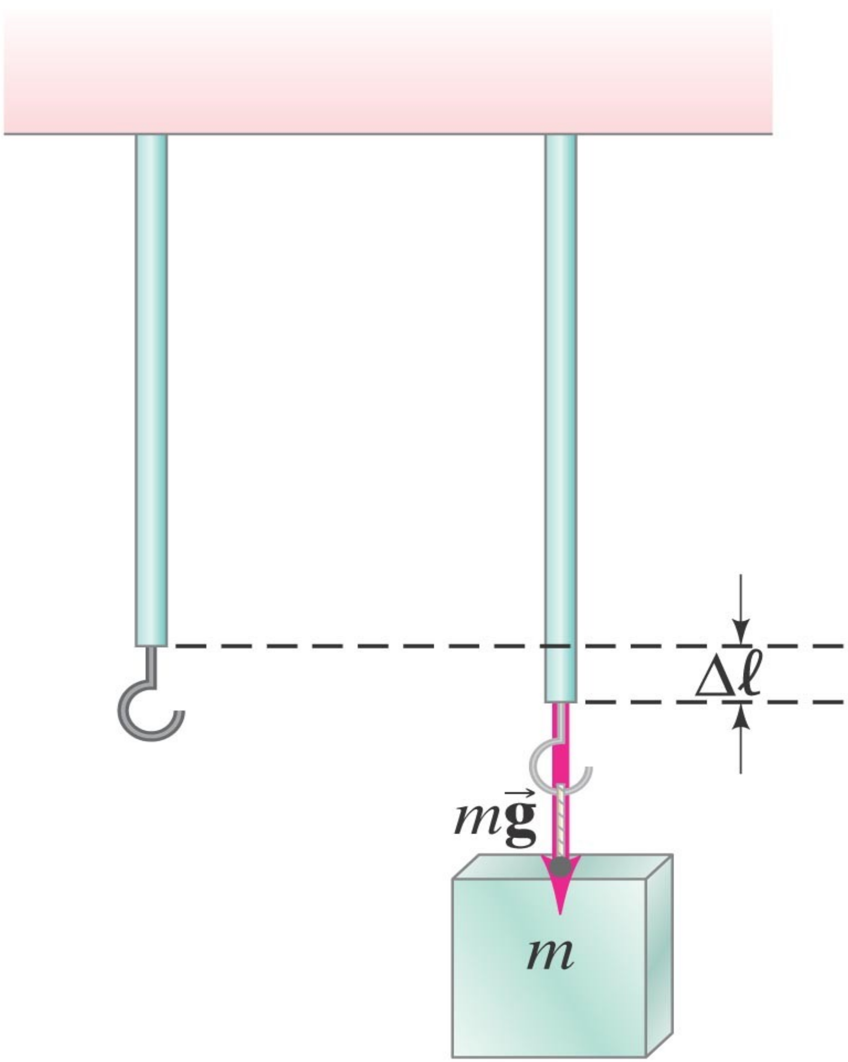
Elastisuus: venymä ja jännitys



Hooken laki:

$$F = k\Delta\ell$$

eli:

$$\Delta\ell = \frac{1}{E}\frac{F}{A}\ell_0$$

jännitys, merk. σ

kimmomoduli eli Youngin moduli

eli:

$$\sigma = E\frac{\Delta\ell}{\ell_0} = E\epsilon$$

(suhteellinen) venymä ε

Elastisuus: venymä ja jännitys

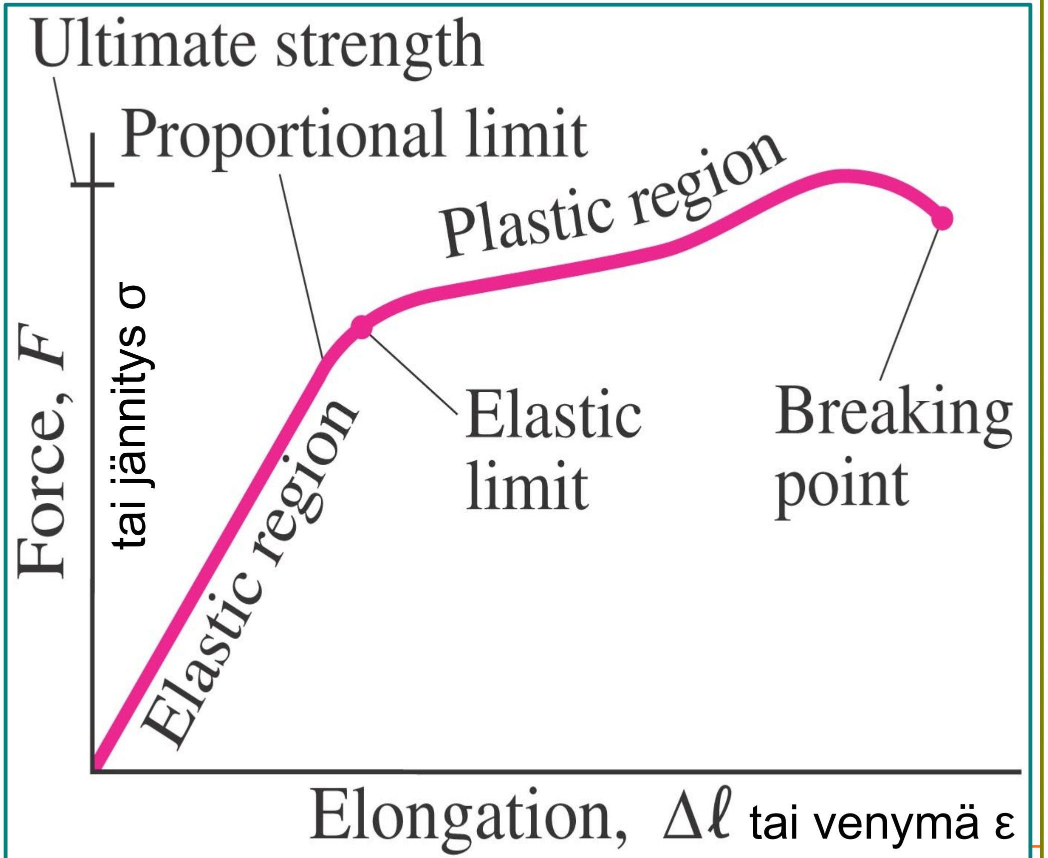
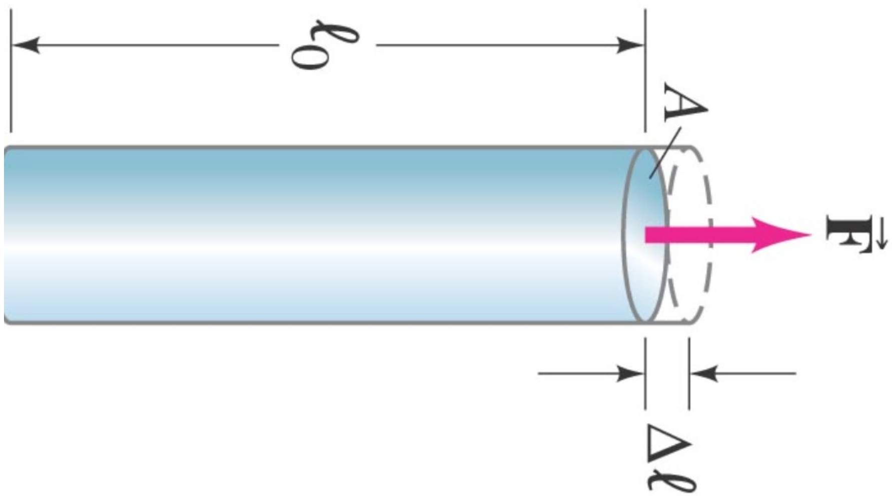


TABLE 12–1 Elastic Moduli

Material	Young's Modulus, E (N/m^2)
Solids	
Iron, cast	100×10^9
Steel	200×10^9
Brass	100×10^9
Aluminum	70×10^9
Concrete	20×10^9
Brick	14×10^9
Marble	50×10^9
Granite	45×10^9
Wood (pine) (parallel to grain)	10×10^9
(perpendicular to grain)	1×10^9
Nylon	5×10^9
Bone (limb)	15×10^9
Liquids	
Water	
Alcohol (ethyl)	
Mercury	
Gases†	
Air, H_2, He, CO_2	

† At normal atmospheric pressure; no variation in temperature during

Elastisuus: Hooken jousi



$$F = \frac{EA}{\ell_0}\Delta\ell$$

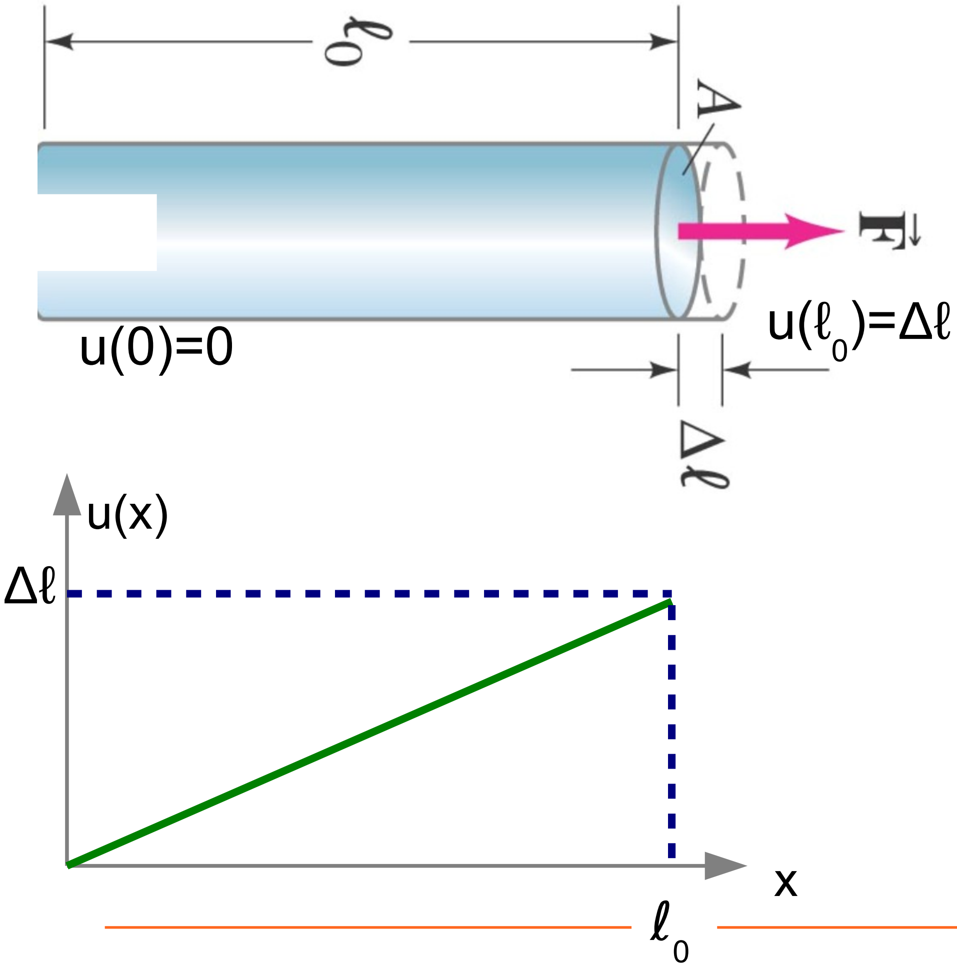
vastaa jousivakiota k

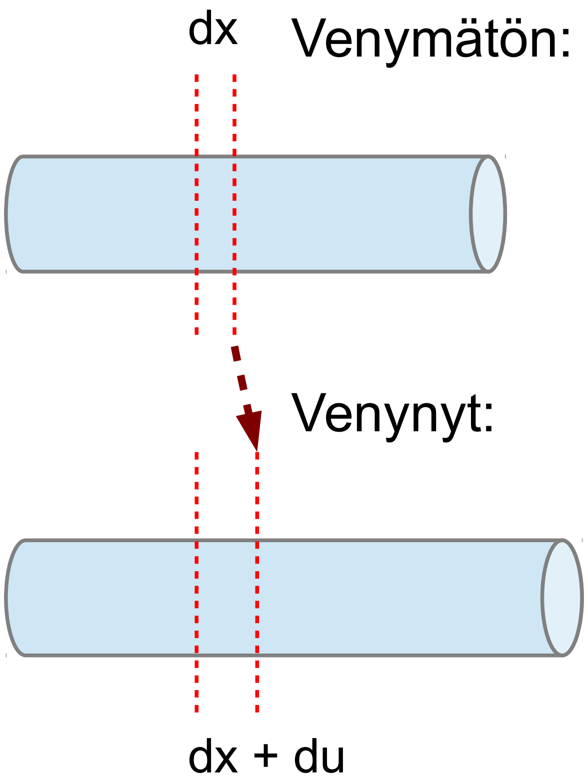
Elastinen sauva toimii siis myös jousena.

- Terässauva: $E = 200 \cdot 10^9$ N/m^2
- $A = 10$ cm^2 $= 0{,}001$ m^2
- $\ell_0 = 1{,}0$ m

$k = 2 \cdot 10^8$ N/m

Elastisuus: Siirtymä





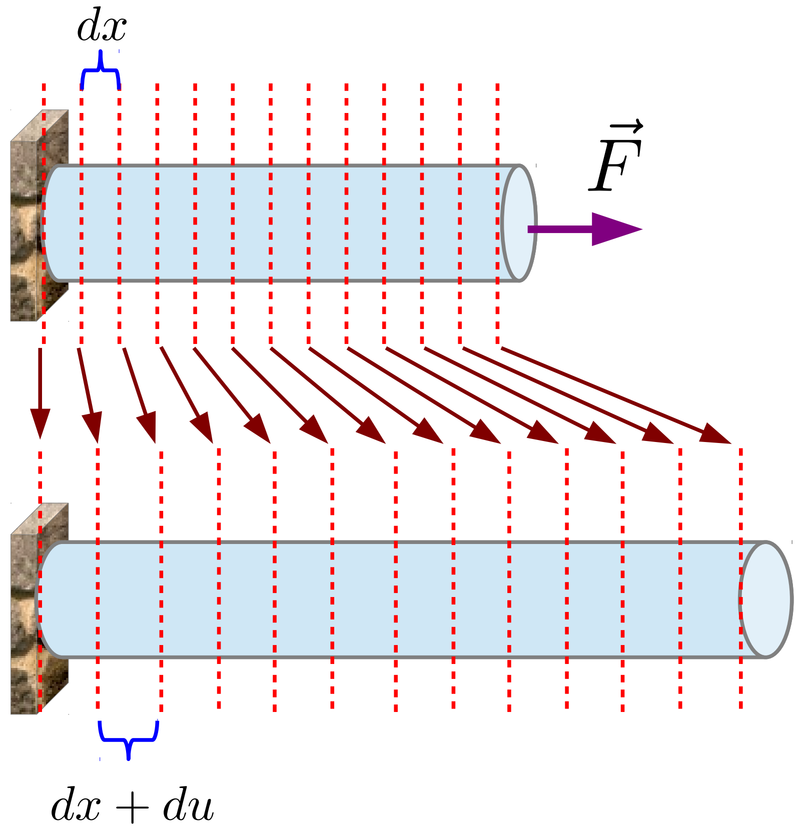
$$\epsilon = \frac{du}{dx} = u'(x) = \frac{1}{E}\sigma$$

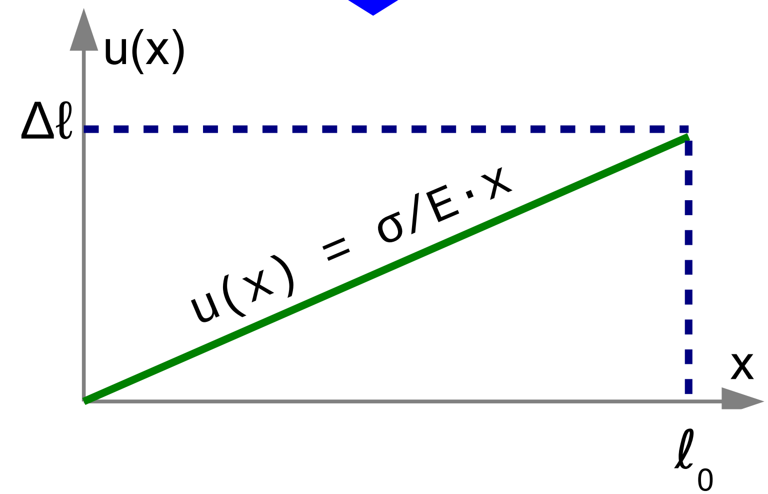
Elastisuus: Venymä ja Siirtymä

$$\epsilon = \frac{du}{dx} = u'(x) = \frac{1}{E}\sigma$$

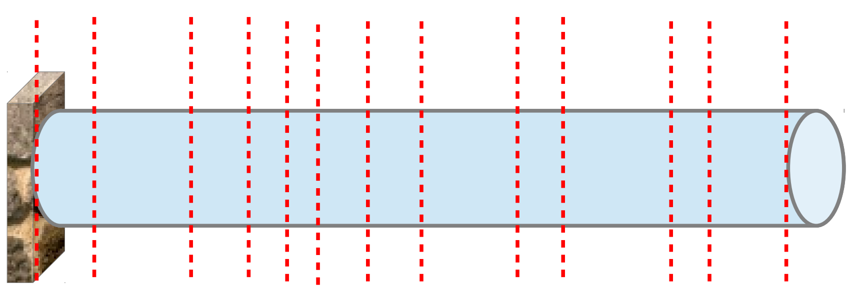
Venymätön:

Venynyt:

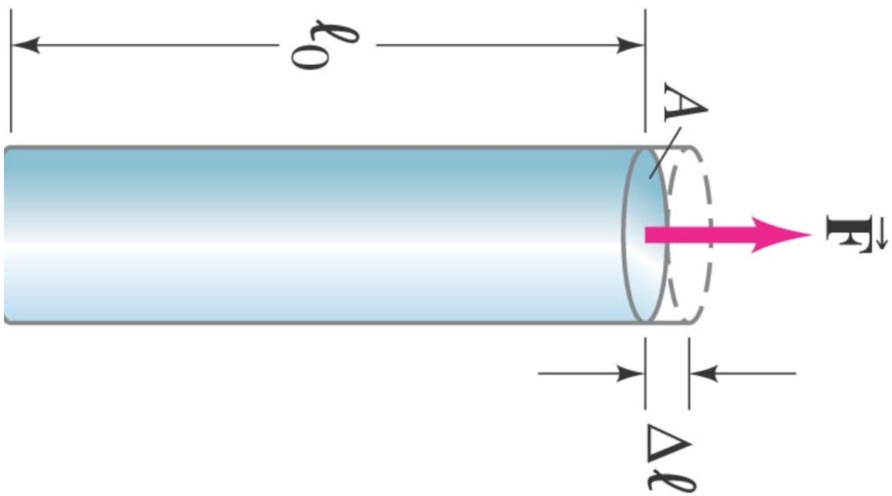




Kun venymä ei ole vakio:

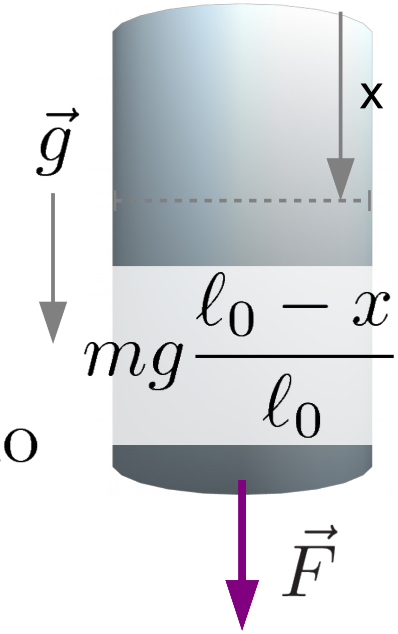


Elastisuus: Siirtymä



$$\sigma = \text{vakio}, \quad u(0) = 0$$

$$u(x) = \int_0^x \frac{\sigma}{E}\,dx = \frac{\sigma}{E}x$$



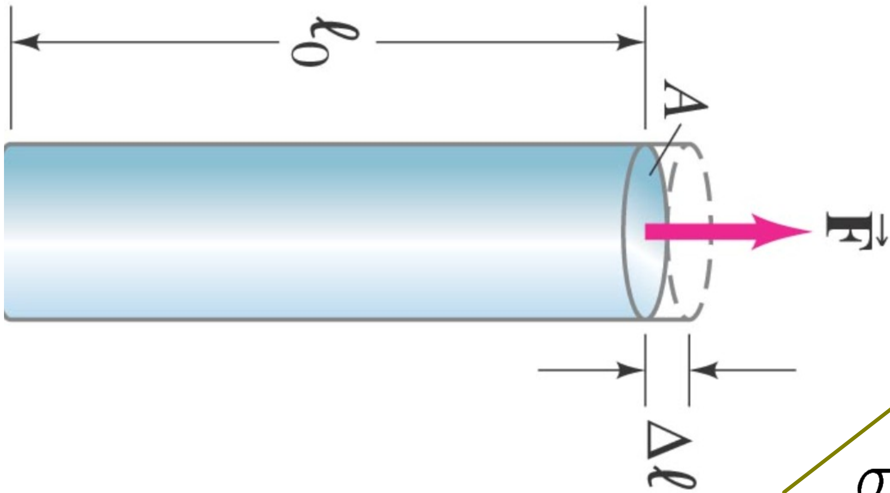
$$\sigma \neq \text{vakio}$$

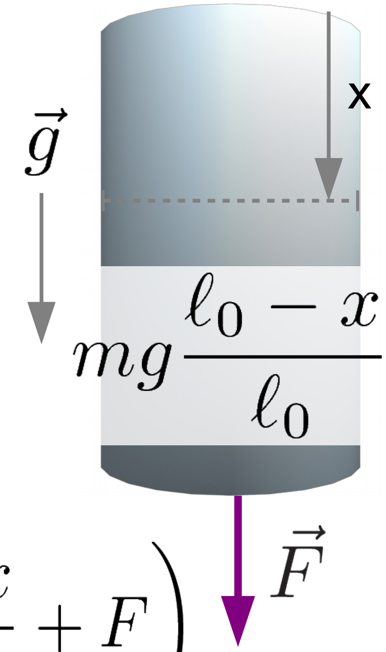
$$\sigma(x) = \frac{1}{A}\left(mg\frac{\ell_0 - x}{\ell_0} + F\right)$$

$$u(x) = \frac{1}{E}\int_0^x \sigma(x)\,dx$$

$$u(x) = \frac{1}{EA}\left(Fx + mg(x - \frac{x^2}{2\ell_0})\right)$$

Elastisuus: Siirtymä ja jännitys



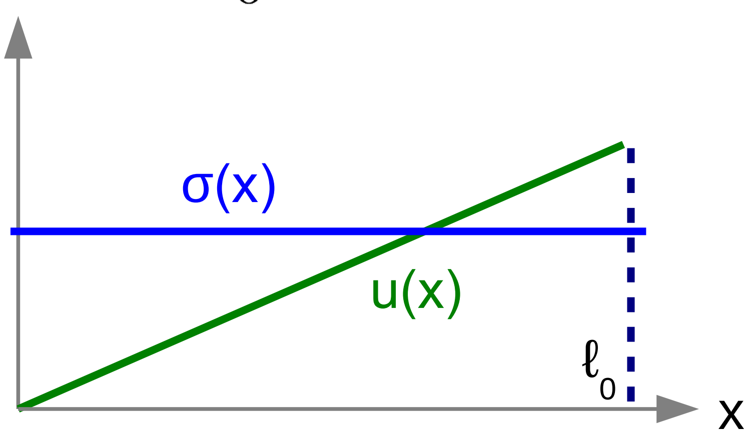


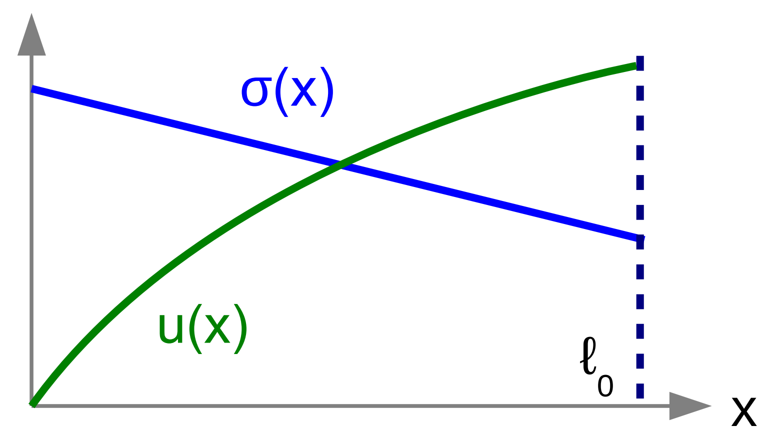
$$\sigma = \text{vakio}, \quad u(0) = 0$$

$$u(x) = \int_0^x \frac{\sigma}{E}\, dx = \frac{\sigma}{E} x$$

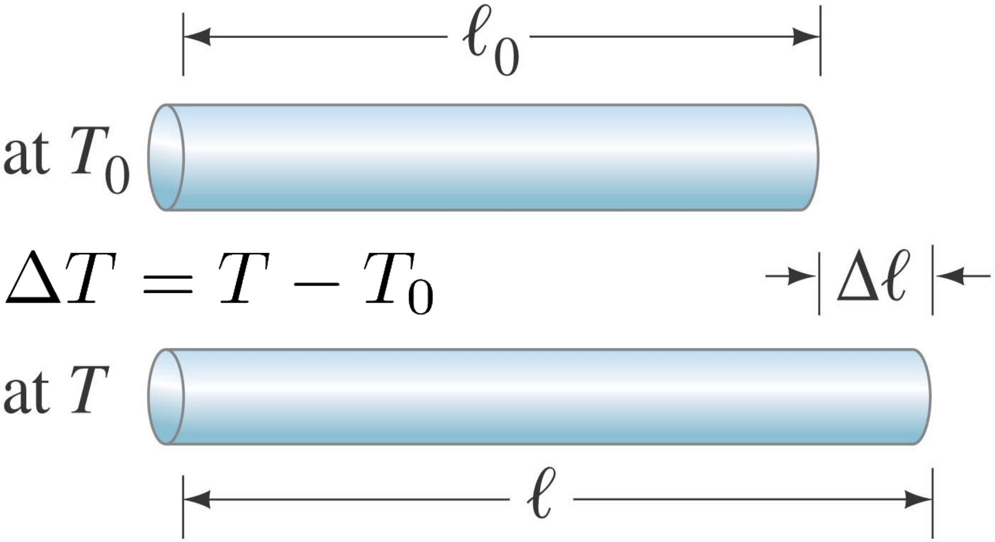
$$\sigma(x) = \frac{1}{A}\left(mg\frac{\ell_0 - x}{\ell_0} + F\right)$$

$$u(x) = \frac{1}{EA}\left(Fx + mg(x - \frac{x^2}{2\ell_0})\right)$$





Elastisuus: Lämpövenymä ja -jännitys



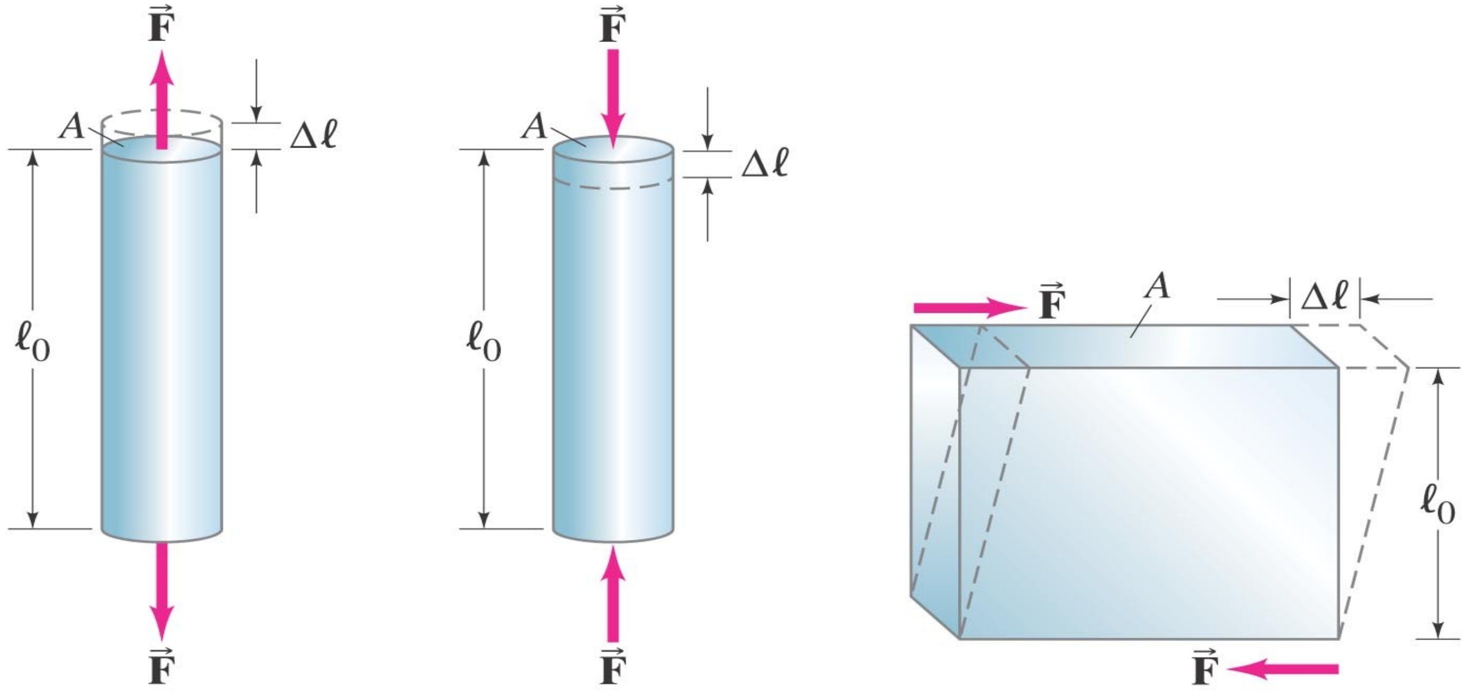
$$\Delta \ell = \alpha \, \ell_0 \Delta T$$

$$\epsilon = \frac{\Delta \ell}{\ell_0} = \frac{1}{E}\sigma + \alpha \Delta T$$

Jos päät kiinnitetty eli ε = 0 ➔ $\sigma = -E \, \alpha \Delta T$

Jos pää vapaa eli σ = 0 ➔ $\epsilon = \alpha \Delta T$

Elastisuus: Venymätyypit



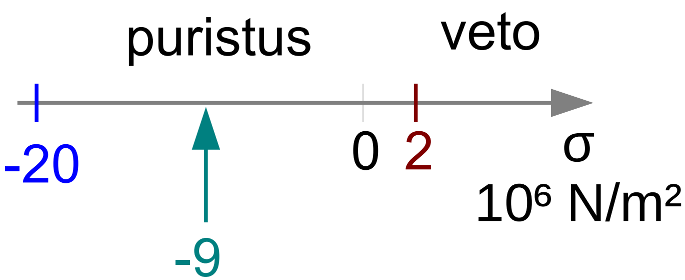
Venytys Puristus Leikkaus

$$\Delta \ell = \frac{1}{E}\frac{F}{A}\ell_0$$

$$\Delta \ell = \frac{1}{G}\frac{F}{A}\ell_0$$

Liukukerroin

Elastisuus: Esijännitetty betoni



$$\begin{cases} \sigma = E\epsilon \\ E = 20 \cdot 10^9 \ \mathrm{N/m^2} \end{cases}$$

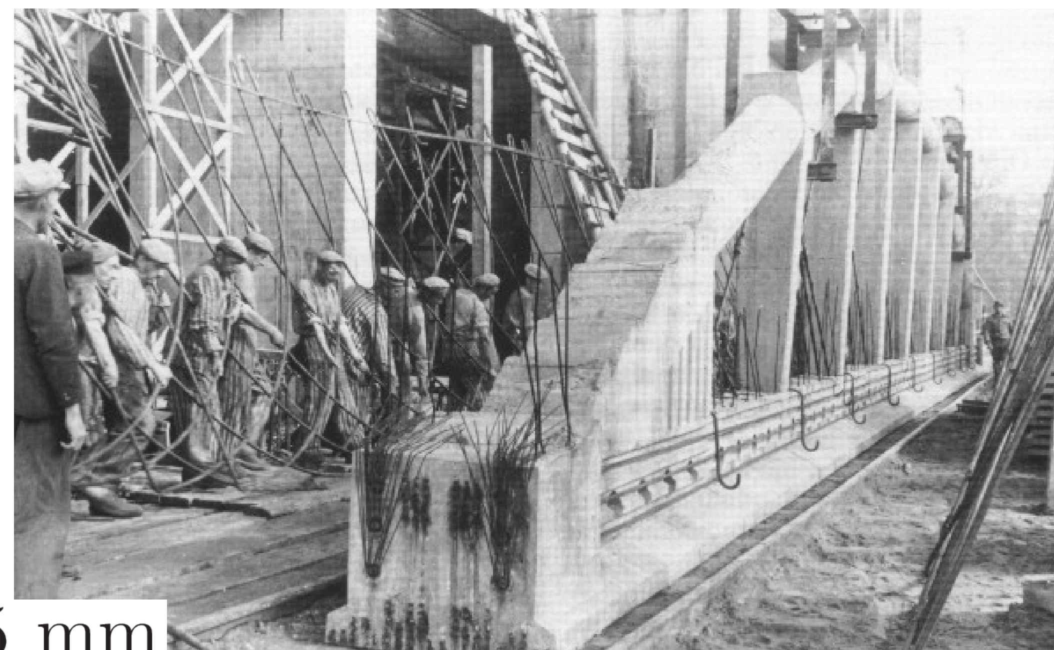
$$\sigma = -9 \cdot 10^6 \ \mathrm{N/m^2}$$

$$\epsilon = -4{,}5 \cdot 10^{-4}$$

$$\ell_0 = 10 \ \mathrm{m} \Rightarrow \Delta\ell = -4{,}5 \ \mathrm{mm}$$

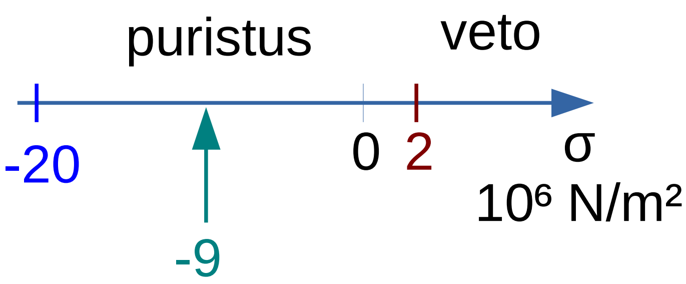
TABLE 12–2 Ultimate Strengths of Materials (force/area)

Material	Tensile Strength (N/m^2)	Compressive Strength (N/m^2)
Iron, cast	170×10^6	550×10^6
Steel	500×10^6	500×10^6
Brass	250×10^6	250×10^6
Aluminum	200×10^6	200×10^6
Concrete	2×10^6	20×10^6



http://www-civ.eng.cam.ac.uk/cjb/4d8/public/history.html

Elastisuus: Esijännitetty betoni jatkuu



Päistään kiinnitetyt betonilaatat: $\varepsilon = 0$

$$\begin{cases} E = 20 \cdot 10^{9}\ \mathrm{N/m^2} \\ \alpha \approx 12 \cdot 10^{-6}\ \dfrac{1}{\mathrm{K}} \end{cases}$$

$$\sigma = -E\,\alpha\Delta T$$

$$\Delta T = -\frac{\sigma}{E\alpha}$$

$$\sigma = 2 \cdot 10^{6}\ \mathrm{N/m^2} \Rightarrow \Delta T = -8{,}33\ \mathrm{K}$$

$$\sigma = \pm 11 \cdot 10^{6}\ \mathrm{N/m^2} \Rightarrow \Delta T = \mp 45{,}8\ \mathrm{K}$$

Elastisuus: Paineen alla

Paineenkin yksikkö on N/m^2 eli voima / pinta-ala

Nesteen tai kaasun aiheuttama paine onkin verrattavissa jännitykseen.

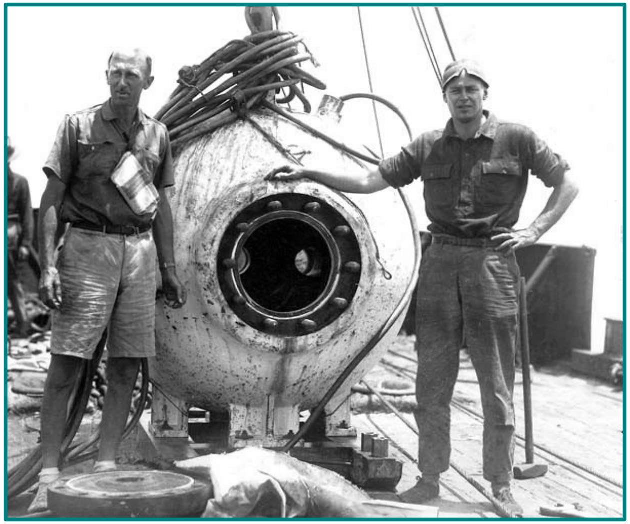
$$\frac{\Delta V}{V} = -\frac{1}{B}\Delta P$$

siis ***tilavuuskimmokerroin*** eli ***puristuskerroin*** on

$$B = -V\frac{dP}{dV}$$

Elastisuus: Paineen alla

Meressä 2000 m:n syvyydessä paine on noin 200 ilmakehän painetta (1 atm = $1{,}0 \cdot 10^5$ N / m²). Kuinka monta prosenttia pallon muotoisen, teräksisen batyskoopin tilavuus muuttuu tässä syvyydessä?

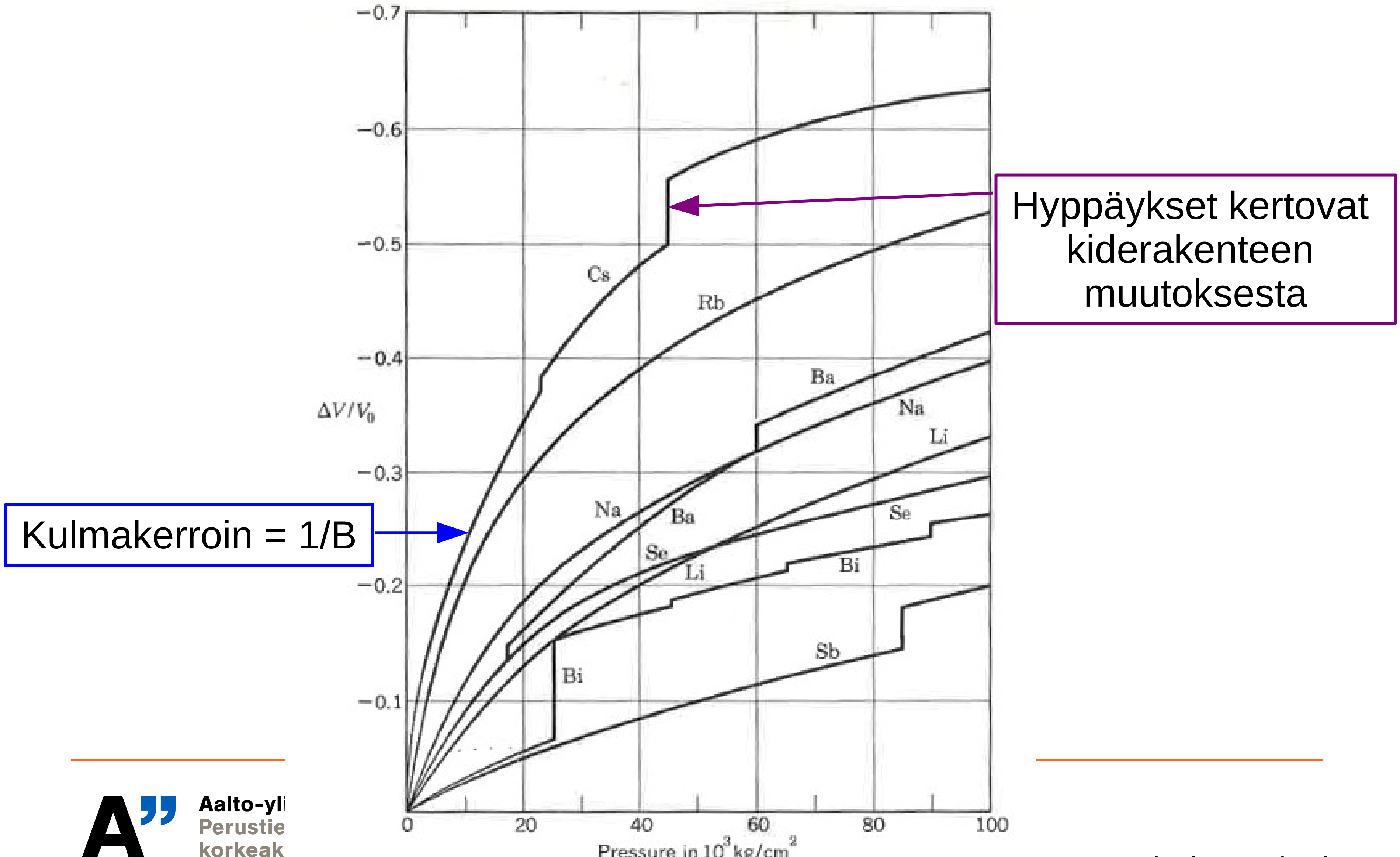


Koska $B = -V\dfrac{dP}{dV}$ niin $\dfrac{\Delta V}{V_0} = -\dfrac{\Delta P}{B}$

ja $\begin{cases} \Delta P = 199 \text{ atm} = 199 \cdot 10^5 \text{ N/m}^2 \\ B = 90 \cdot 10^9 \text{ N/m}^2 \end{cases}$

$$\frac{\Delta V}{V_0} = -2 \cdot 10^{-4} = -2 \cdot 10^{-2} \ \%$$

Elastisuus: Suuren paineen alla



≈ 10⁵ kPa

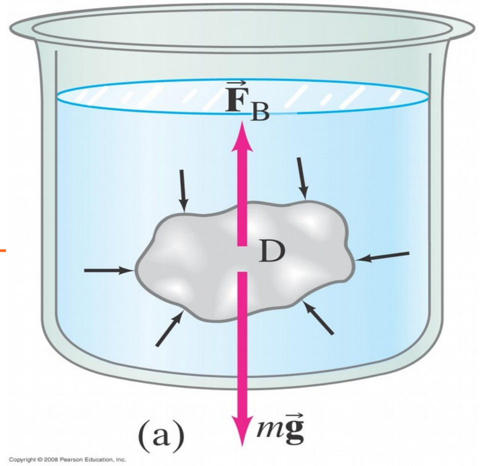
Kuva: C. Kittel: *Introduction to Solid State Physics*

Noste ja Arkhimedeen periaate

Arkhimedes Syrakusalaisen (287 eKr. - 212 eKr.)
teos ”*Kelluvista kappaleista I*”:

Propositio 5: Any solid lighter than a fluid will, if placed in the fluid, be so far immersed that the weight of the solid will be equal to the weight of the fluid displaced.

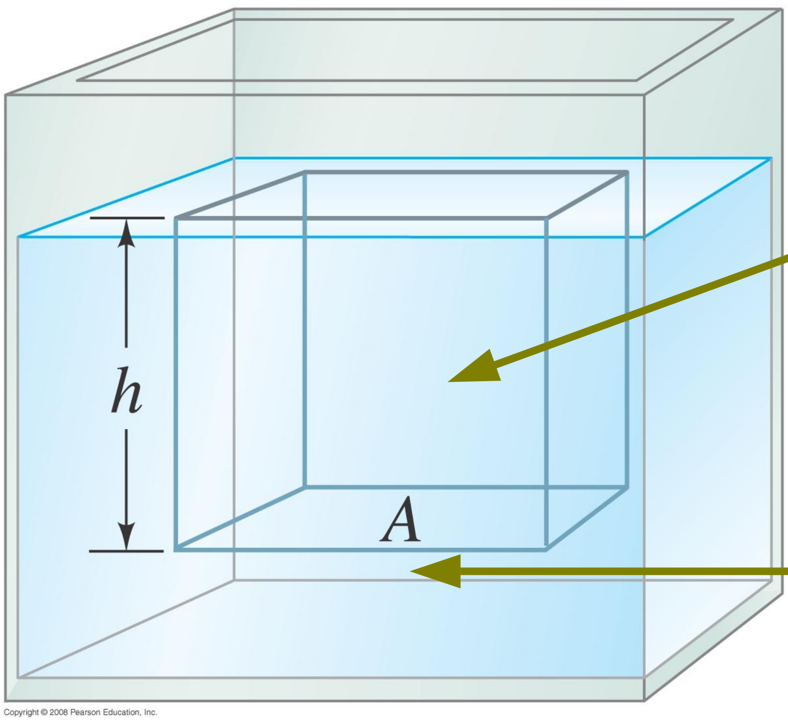
Propositio 7: A solid heavier than a fluid will, if placed in it, descend to the bottom of the fluid, and the solid will, when weighed in the fluid, be lighter than its true weight by the weight of the fluid displaced.



Noste ja Arkhimedeen periaate

Mistä noste syntyy?

1. Hydrostaattinen paine
2. Kappaleen syrjäyttämä neste



$$m = \rho V$$

Paine on puristusjännitystä

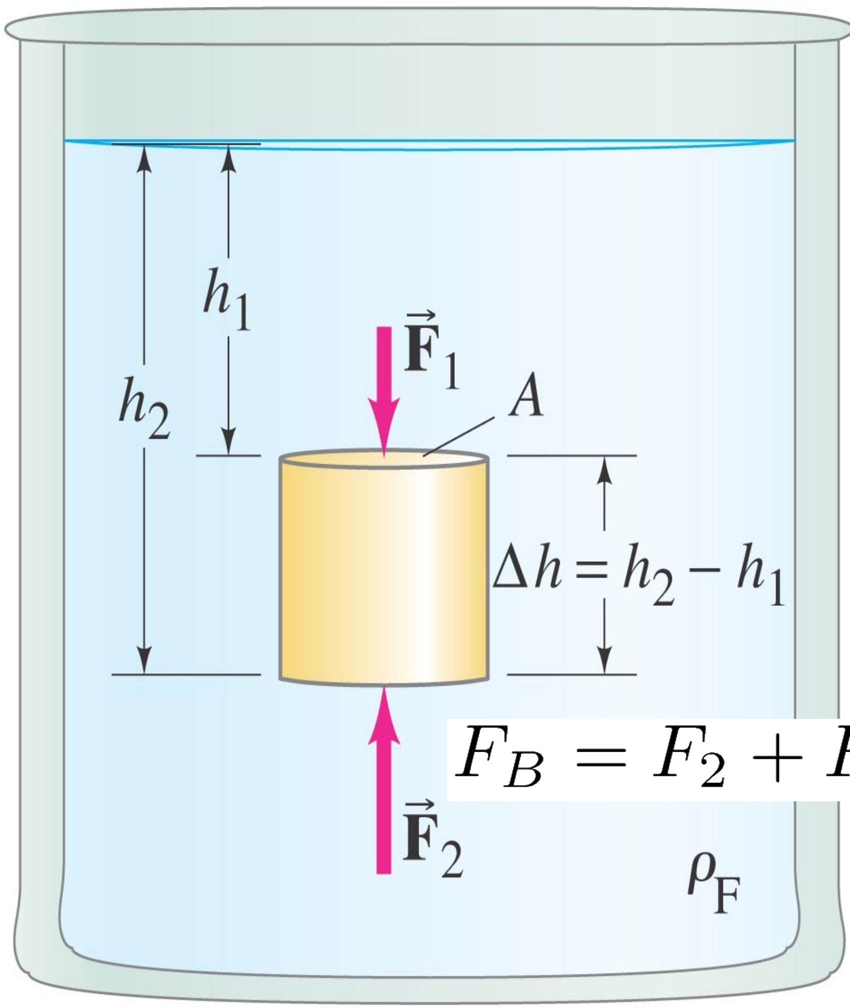
$$P_G = \frac{F}{A} = \frac{\rho A h g}{A} = \rho g h$$

$$P = P_i + P_G = P_i + \rho g h$$

Paine

Mittapaine, G = ”gauge”

Noste ja Arkhimedeen periaate

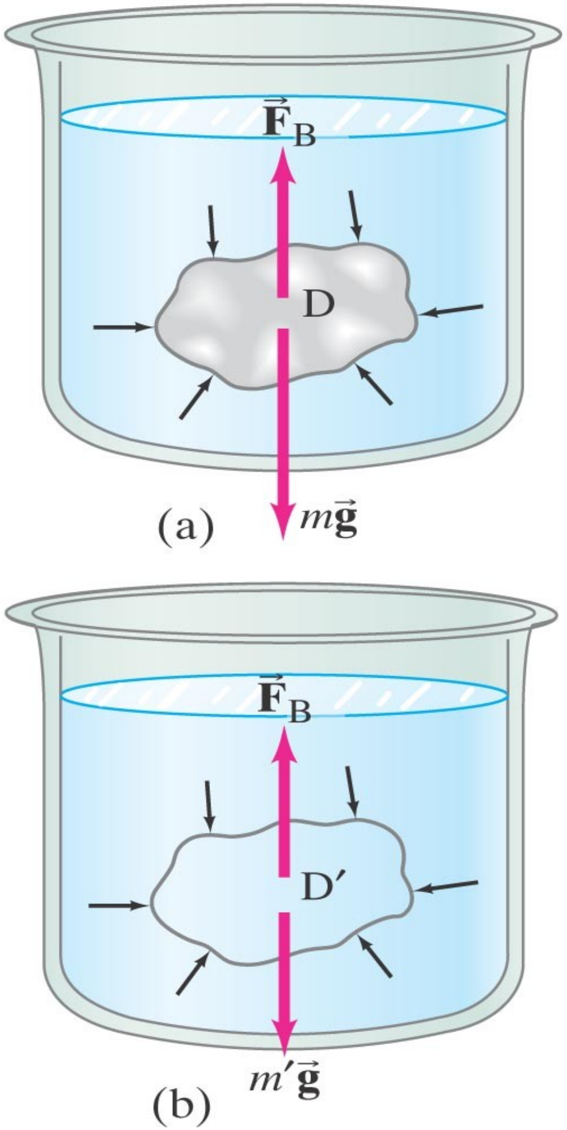


$$\begin{cases} F_1 = -\rho_F g h_1 A \\ F_2 = \rho_F g h_2 A \end{cases}$$

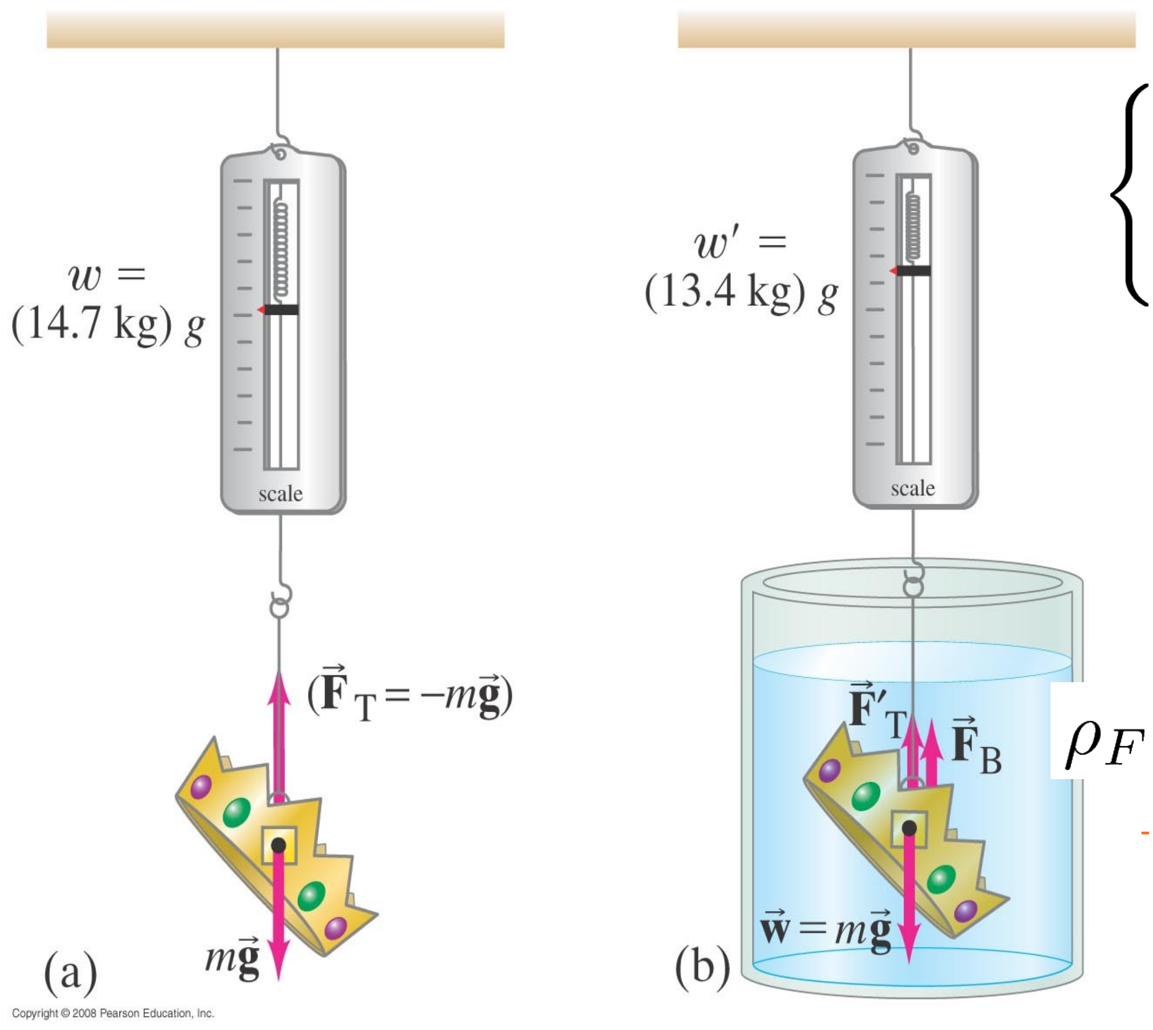
$$F_B = F_2 + F_1 = \rho_F g \Delta h A = \rho_F g V = m_F g$$

Noste ja Arkhimedeen periaate

Koska nesteestä muodostunut vastaava kappale D' pysyisi paikallaan nesteessä, on nosteen oltava D:n syrjäyttämän nesteen painon suuruinen.



Noste, Arkhimedes ja kruunu



$$w' = w - F_B$$

$$\begin{cases} w = \rho_O V g \\ F_B = w - w' = \rho_F V g \end{cases}$$

Jaetaan puolittain

$$\frac{w}{w - w'} = \frac{\rho_O}{\rho_F}$$

Kun

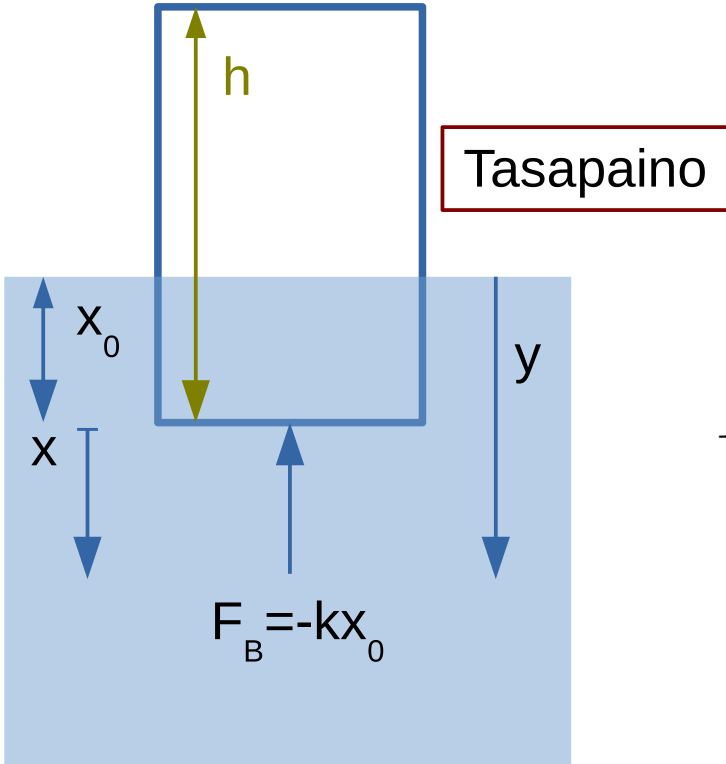
$$\rho_F = \rho_{\mathrm{H_2O}} = 1{,}00\ \mathrm{kg/dm^3}$$

niin

$$\rho_O = 11{,}3\ \mathrm{kg/dm^3}$$

(lienee lyijyä)

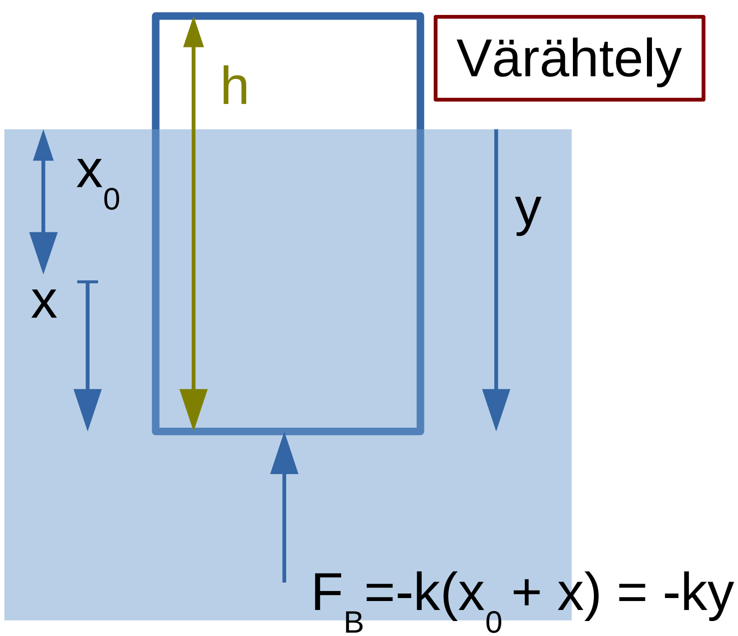
Noste ja jousi



$$F_B = -\rho g V = -\rho g A y = -ky$$

Värähtelee kuten jousi tasapainopisteen

$$x_0 = \frac{mg}{k} = \frac{m}{\rho A}$$ ympärillä.



Kulmataajuus:

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{\rho g A}{\rho_O A h}} = \sqrt{\frac{\rho g}{\rho_O h}}$$