



**Aalto University**  
**School of Electrical**  
**Engineering**

# **Lecture 6: DC-DC Conversion**

## **ELEC-E8405 Electric Drives (5 ECTS)**

Marko Hinkkanen

Autumn 2020

# Learning Outcomes

After this lecture and exercises you will be able to:

- ▶ Explain basic operating principles of switched DC-DC conversion
- ▶ Draw the equivalent circuit of a 4-quadrant DC-DC converter
- ▶ Implement a simple pulse-width modulator in the Simulink software

# Outline

**Introduction**

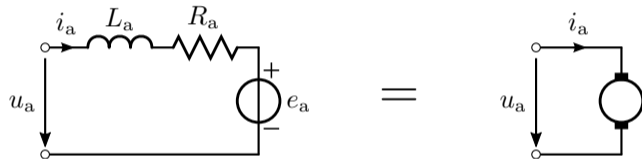
Buck Converter

4-Quadrant DC-DC Converter

Unipolar Pulse-Width Modulation

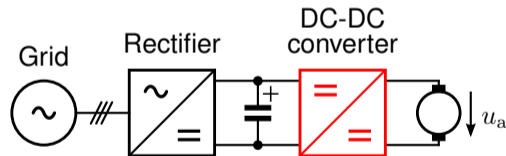
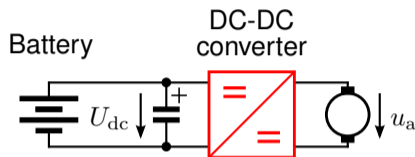
Synchronous Sampling

# Symbol Used for the DC Motor

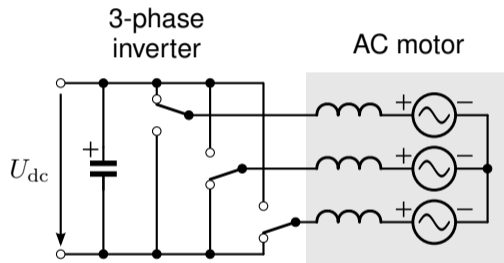
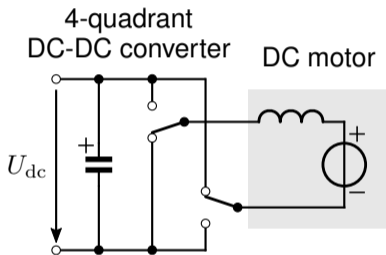


# Introduction

- ▶ DC source voltage  $U_{dc}$  is typically a battery or a diode bridge
- ▶ Armature voltage  $u_a$  has to be adjusted in order to be able to control the speed and torque
- ▶ Topologies and control of DC-DC converters are very similar to those of three-phase inverters

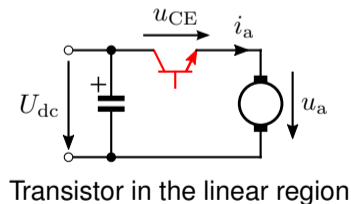
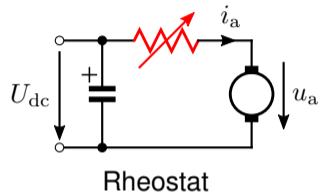


# DC-DC Converters Are Similar to 3-Phase Inverters



# Why Not Linear Voltage Regulation?

- ▶ In principle,  $u_a$  could be adjusted using a rheostat or a transistor in the linear region
- ▶ What would be the efficiency if  $U_{dc} = 100\text{ V}$  and  $u_a = 50\text{ V}$ ? What would be the losses in the transistor if  $i_a = 10\text{ A}$ ?
- ▶ Why **linear voltage regulation does not work** in practice (except in very low-power drives)?



# Outline

Introduction

**Buck Converter**

4-Quadrant DC-DC Converter

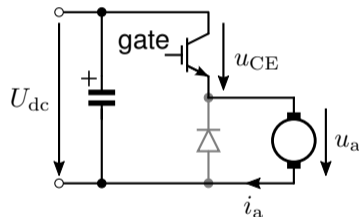
Unipolar Pulse-Width Modulation

Synchronous Sampling



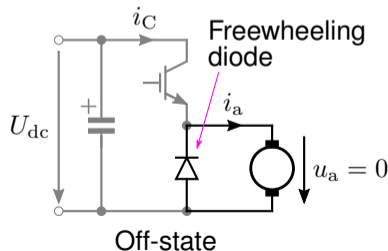
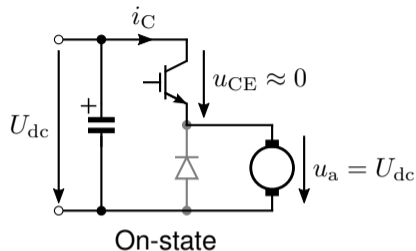
# Switched-Mode DC-DC Conversion

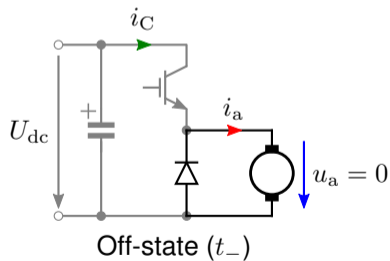
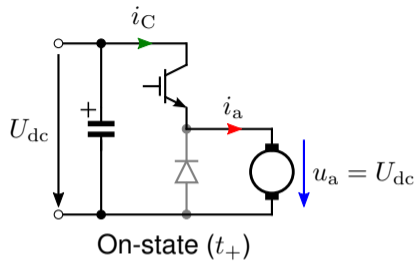
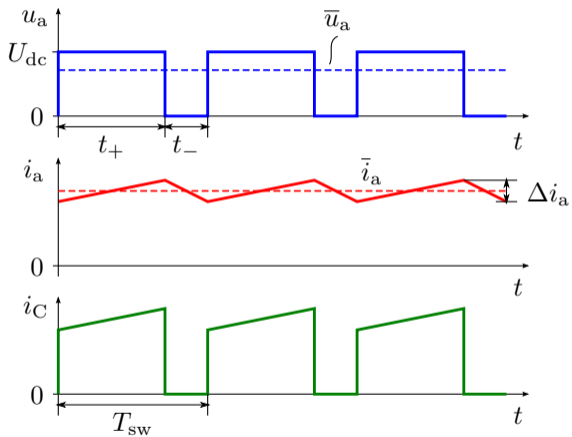
- ▶ To avoid high losses, the transistor is **switched periodically on and off**
- ▶ Typical power semiconductors: IGBT, MOSFET, thyristor, GTO
- ▶ We will assume ideal switches
- ▶ Typical switching frequencies  $f_{sw} = 1 \dots 50$  kHz
- ▶ Figure shows a buck converter (step-down converter)

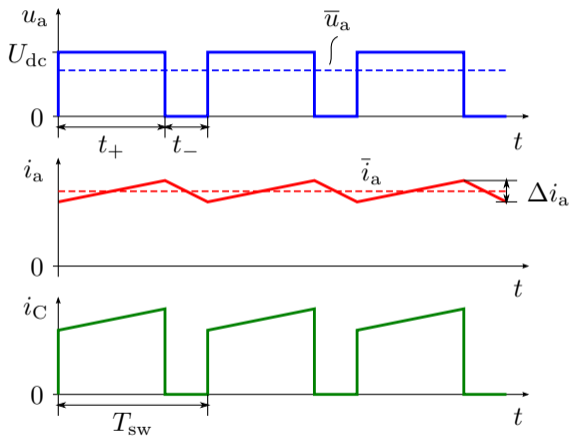


# Buck Converter

- ▶ Low power loss  $u_{CE}i_C$  in the transistor
  - On-state:**  $u_{CE} \approx 0$
  - Off-state:**  $i_C = 0$
- ▶ Motor is an inductive load
  - ▶ Current  $i_a$  must flow even when the transistor is switched off
  - ▶ Freewheeling diode is needed
- ▶ Next we will consider short time periods
  - ▶  $e_a = \text{constant}$  and  $R_a = 0$  can be assumed







► **Duty cycle**

$$d = \frac{t_+}{T_{sw}} \quad 0 \leq d \leq 1$$

- $t_+$  is the on-time
- $T_{sw}$  is the switching period
- **Average** of the voltage  $u_a$

$$\begin{aligned} \bar{u}_a &= \frac{1}{T_{sw}} \int_0^{T_{sw}} u_a dt \\ &= dU_{dc} \end{aligned}$$

over the period  $T_{sw}$

# Current Ripple

- Voltage equation

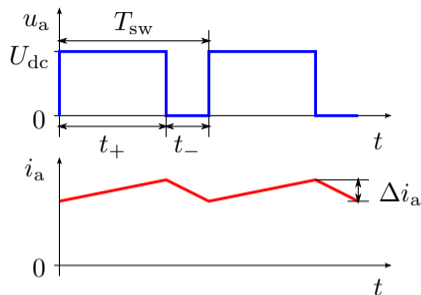
$$L_a \frac{di_a}{dt} = u_a - e_a$$

- On-state:  $u_a = U_{dc}$

$$\begin{aligned} \Delta i_a &= \frac{1}{L_a} \int_0^{t_+} (U_{dc} - e_a) dt \\ &= \frac{(U_{dc} - e_a)t_+}{L_a} \end{aligned}$$

- Off-state:  $u_a = 0$

$$-\Delta i_a = \frac{1}{L_a} \int_0^{t_-} (-e_a) dt = -\frac{e_a t_-}{L_a}$$



- Duty ratio in steady state

$$D = \frac{t_+}{T_{sw}} = \frac{U_a}{U_{dc}} = \frac{E_a}{U_{dc}}$$

- Current ripple in steady state

$$\Delta i_a = \frac{D(1-D)U_{dc}}{f_{sw}L_a}$$

# Maximum Current Ripple

- ▶ Maximum ripple for  $D = 1/2$  (at about half base speed)

$$\Delta i_{a,\max} = \frac{U_{\text{dc}}}{4f_{\text{sw}}L_a}$$

- ▶ Example parameter values for a 1-kW DC motor

$$L_a = 50 \text{ mH} \quad U_{\text{dc}} = 100 \text{ V} \quad f_{\text{sw}} = 5 \text{ kHz} \quad \Rightarrow \quad \Delta i_{a,\max} = 0.1 \text{ A}$$

- ▶ If the rated current is 10 A, the current ripple is only 1% (5 kHz is not a high switching frequency at 1-kW power level)
- ▶ Current ripple  $\Delta i_a$  and torque ripple  $\Delta T_M = k_f \Delta i_a$  are typically insignificant

# Outline

Introduction

Buck Converter

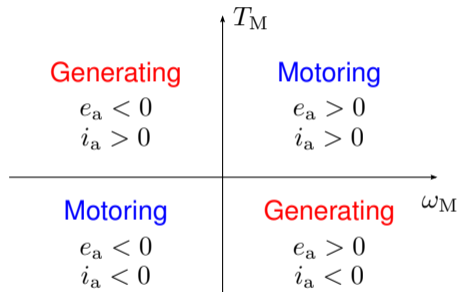
**4-Quadrant DC-DC Converter**

Unipolar Pulse-Width Modulation

Synchronous Sampling

# Four Quadrants

- ▶ Back-emf  $e_a = k_f \omega_M$
- ▶ Torque  $T_M = k_f i_a$
- ▶ Mechanical power  
 $p_M = \omega_M T_M = e_a i_a$
- ▶ Converter should allow both its output voltage  $u_a \approx e_a$  and current  $i_a$  to reverse in 4-quadrant operation





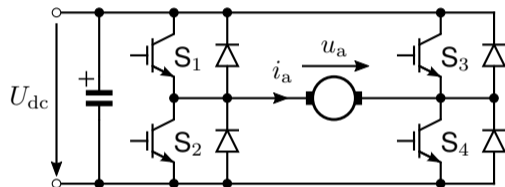
# 4-Quadrant DC-DC-Converter

**S<sub>1</sub> and S<sub>4</sub> switched ON:**  $u_a = U_{dc}$

**S<sub>2</sub> and S<sub>3</sub> switched ON:**  $u_a = -U_{dc}$

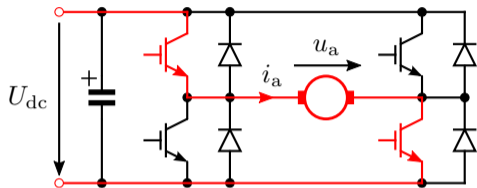
**S<sub>1</sub> and S<sub>3</sub> switched ON:**  $u_a = 0$

**S<sub>2</sub> and S<sub>4</sub> switched ON:**  $u_a = 0$

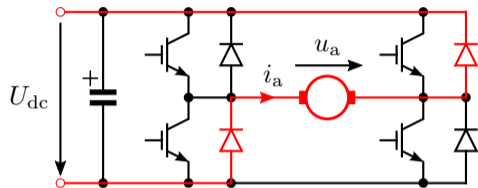


# Operation Modes

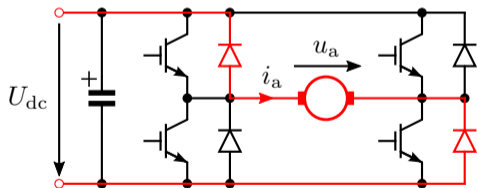
Only Nonzero Voltage Switching States Are Shown



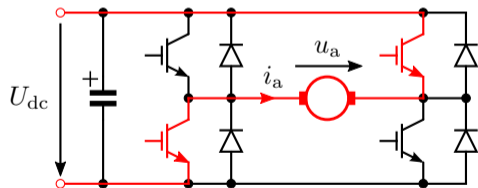
$$u_a = U_{dc}, \quad i_a > 0$$



$$u_a = -U_{dc}, \quad i_a > 0$$



$$u_a = U_{dc}, \quad i_a < 0$$

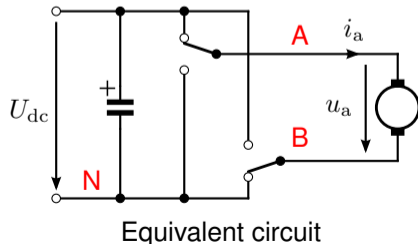
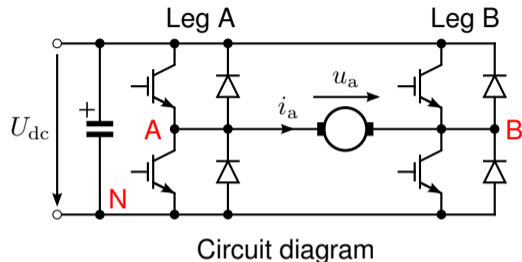


$$u_a = -U_{dc}, \quad i_a < 0$$

# Notation of Potentials and Voltages

- ▶ Legs can be modelled as bi-positional switches
- ▶ Negative DC-bus potential **N**
- ▶  $u_{AN}$  is the voltage between potentials **A** and **N**
- ▶  $u_{BN}$  is the voltage between potentials **B** and **N**
- ▶ Converter output voltage

$$u_a = u_{AN} - u_{BN}$$



# Switching States of the Bi-Positional Switches

- ▶ Switching state  $q$ 
  - ▶  $q = 0$  if the switch is connected to **N**
  - ▶  $q = 1$  if the switch is connected to **P**

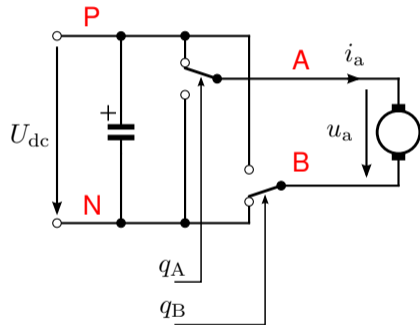
- ▶ Pole voltages

$$u_{AN} = q_A U_{dc} \quad u_{BN} = q_B U_{dc}$$

- ▶ Converter output voltage

$$u_a = (q_A - q_B) U_{dc}$$

- ▶ Figure:  $q_A = 1$  and  $q_B = 0$ , giving  $u_a = U_{dc}$



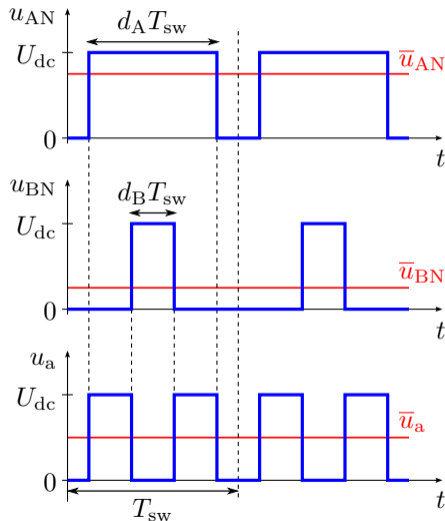
# Switching-Cycle Averaged Quantities

- ▶ Average pole voltage over  $T_{sw}$

$$\bar{u}_{AN} = \frac{1}{T_{sw}} \int_0^{T_{sw}} u_{AN} dt = d_A U_{dc}$$

- ▶ Average voltage  $\bar{u}_{BN}$  is obtained similarly
- ▶ Average output voltage

$$\bar{u}_a = (d_A - d_B) U_{dc}$$



# Outline

Introduction

Buck Converter

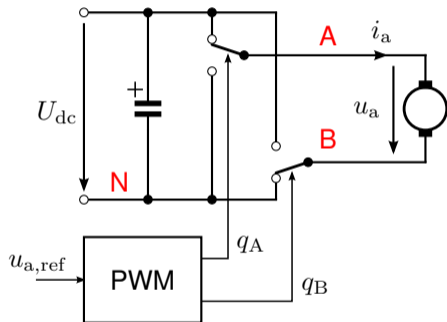
4-Quadrant DC-DC Converter

**Unipolar Pulse-Width Modulation**

Synchronous Sampling

# Pulse-Width Modulation

- ▶ PWM generates the control signals  $q_A$  and  $q_B$  for the power switches
- ▶ Goal: switching-cycle averaged voltage  $\bar{u}_a$  equals the reference voltage  $u_{a,\text{ref}}$
- ▶ Various PWM methods exist: they all give  $\bar{u}_a = u_{a,\text{ref}}$  but produce different pulse patterns
- ▶ **Unipolar PWM** will be considered in the following



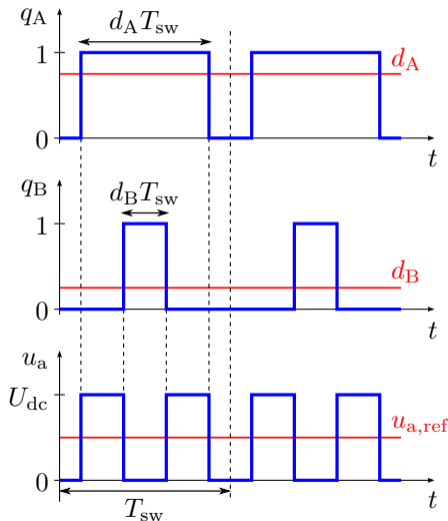
# Duty Cycles

- Conditions  $\bar{u}_a = u_{a,\text{ref}}$  and  $d_A + d_B = 1$  lead to the duty cycles

$$d_A = \frac{1}{2} \left( 1 + \frac{u_{a,\text{ref}}}{U_{\text{dc}}} \right)$$

$$d_B = \frac{1}{2} \left( 1 - \frac{u_{a,\text{ref}}}{U_{\text{dc}}} \right)$$

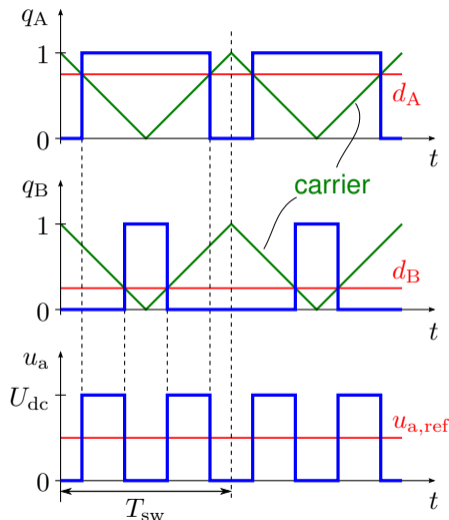
- Example in the figure:  $u_{a,\text{ref}} = 0.5U_{\text{dc}}$
- What are the duty cycles  $d_A$  and  $d_B$ ?
- How to generate the control signals  $q_A$  and  $q_B$ ?





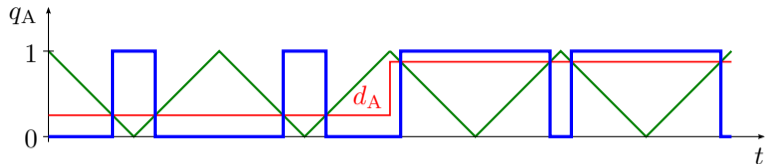
# Carrier Comparison

- ▶ Carrier comparison is often used for generating the control signals
  - ▶ Triangular carrier with the period  $T_{sw}$
  - ▶ Magnitude varies between 0 and 1
- ▶ If  $d$  is higher than the carrier, then  $q = 1$  (otherwise  $q = 0$ )
- ▶ Same carrier for both  $d_A$  and  $d_B$
- ▶ Next slide: step change in the voltage reference ( $-0.5U_{dc} \rightarrow 0.75U_{dc}$ )

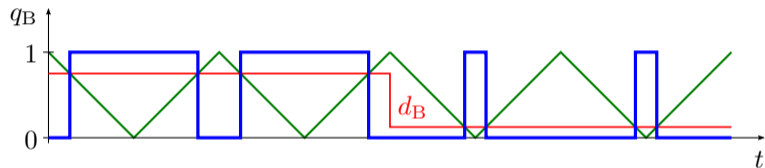


There are various ways to scale the carrier waveform and the reference quantities. Using the carrier varying between 0 and 1 together with the duty cycle references is convenient in digital implementation.

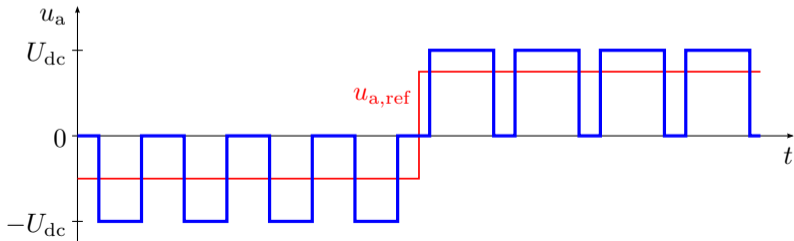
$$d_A = \frac{1}{2} \left( 1 + \frac{u_{a,\text{ref}}}{U_{\text{dc}}} \right)$$



$$d_B = \frac{1}{2} \left( 1 - \frac{u_{a,\text{ref}}}{U_{\text{dc}}} \right)$$



$$\bar{u}_a = (d_A - d_B)U_{\text{dc}}$$



# Outline

Introduction

Buck Converter

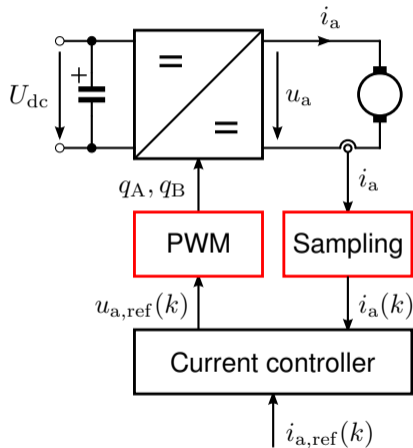
4-Quadrant DC-DC Converter

Unipolar Pulse-Width Modulation

**Synchronous Sampling**

# Digital Controller

- ▶ Current is measured for the feedback of the current controller
- ▶ Digital controllers are used nowadays
- ▶ Sampling of the current is typically synchronized with the PWM
- ▶ Synchronised sampling effectively removes the current ripple from the samples

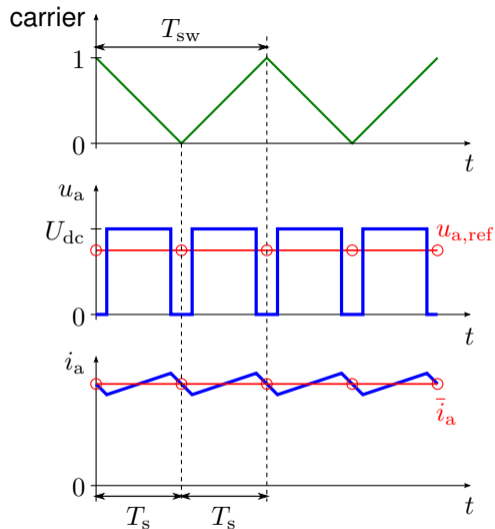


# Synchronous Sampling

- ▶ Voltage reference  $u_{a,\text{ref}}$  can be updated in the beginning and in the middle of the carrier (marked with the circles)
- ▶ Current samples (circles) can be taken at these same time instants
- ▶ Next slide: Current response is governed by

$$L_a \frac{di_a}{dt} = u_a - e_a$$

where  $R_a = 0$  is assumed



Different variants of sampling synchronized with the PWM exist, while only one is presented here. Furthermore, it can be noticed that actually four current samples per carrier period could be taken without the current ripple in the case of the unipolar PWM.

