Aalto University
School of Electrical
Engineering

Lecture 7: Control of a DC Motor Drive

ELEC-E8405 Electric Drives (5 ECTS)

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Learning Outcomes

After this lecture and exercises you will be able to:

- Draw the block diagram of a cascaded control system and explain it
- Calculate the controller gains based on the model parameters and the desired bandwidth
- Implement the current and speed controllers in the Simulink software

Introduction

- Modern electric drives automatically identify motor parameters (auto-commissioning, identification run)
- Model-based controllers are preferred, since they can be automatically tuned based on the known (identified) motor parameters
- Cascaded control system is commonly used
  - Current or torque controller (fast)
  - Speed controller (slower), not always needed
- Various control methods exist: our approach is based on two-degrees-of-freedom (2DOF) PI controllers
- Control systems are typically implemented digitally (but can be often designed in the continuous-time domain)
- Controllers and tuning principles can be extended to AC drives

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A cascaded control system may also include a position controller, but we will not cover it in these lectures.

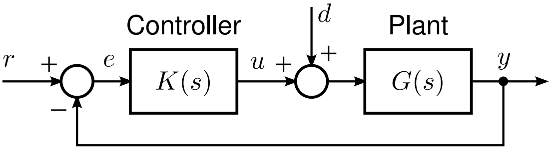
Outline

Closed-Loop Control

- Closed-loop transfer function

$$\frac{y(s)}{r(s)} = H(s) = \frac{L(s)}{1 + L(s)}$$

  where $L(s) = K(s)G(s)$ is the loop transfer function
- Typical control objectives
  - Zero control error in steady state
  - Well-damped and fast transient response



r = reference

e = control error

u = control output

d = load disturbance

y = output

The stability of the closed-loop system $H(s)$ is often evaluated indirectly via the loop transfer function $L(s)$. For example, the gain and phase margins can be read from a Bode plot or a Nyquist plot of $L(\mathrm{j}\omega)$. In these lectures, we mainly analyse the closed-loop transfer function $H(s)$ directly.

PI Controller

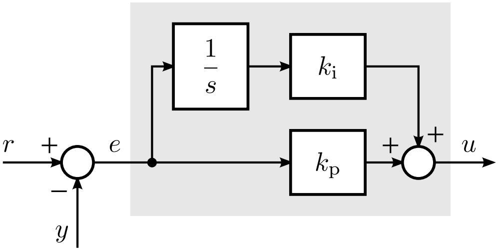
- Most common controller type
- Time domain

$$u = k_{\mathrm{p}} e + k_{\mathrm{i}} \int e \, \mathrm{d}t$$

- Transfer function

$$\frac{u(s)}{e(s)} = K(s) = k_{\mathrm{p}} + \frac{k_{\mathrm{i}}}{s}$$

PI controller

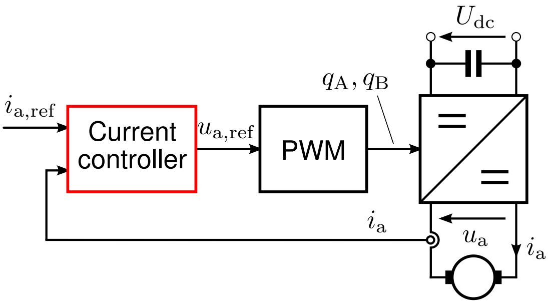


Outline

Current Control System

Closed-loop current control enables:

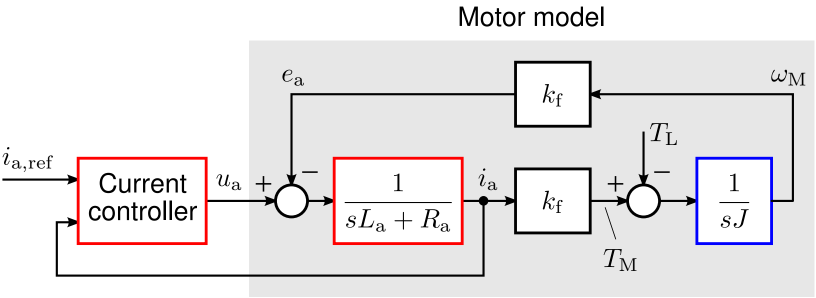
1. Current limitation
2. Precise and fast torque control



Block Diagram

Assumptions:

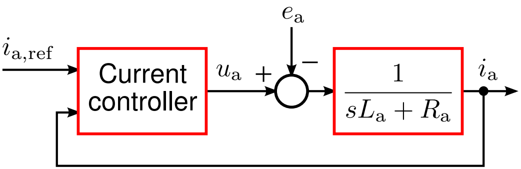
- Switching-cycle averaged quantities
- Ideal voltage production: $u_{\mathrm{a}} = u_{\mathrm{a,ref}}$



Switching-cycle averaged symbols will not be marked with overlining in these lectures.

Simplified Block Diagram

- Back-emf e_a is a slowly varying load disturbance
- P controller cannot drive the steady-state error to zero
- PI controller suffices



PI Current Controller

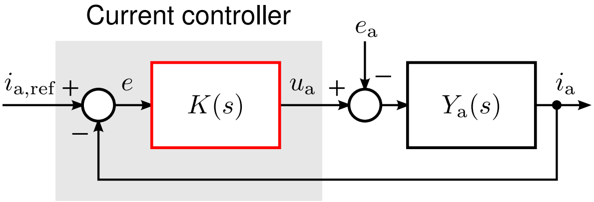
- Time domain

$$u_\mathrm{a} = k_\mathrm{p} e + k_\mathrm{i} \int e \,\mathrm{d}t$$

- Transfer function

$$\frac{u_\mathrm{a}(s)}{e(s)} = K(s) = k_\mathrm{p} + \frac{k_\mathrm{i}}{s}$$

- How to tune the gains k_p and k_i?



$$Y_\mathrm{a}(s) = \frac{1}{sL_\mathrm{a} + R_\mathrm{a}}$$

Closed-Loop Transfer Function

- Closed-loop transfer function

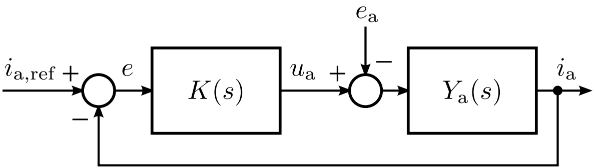
$$\frac{i_\mathrm{a}(s)}{i_\mathrm{a,ref}(s)} = H(s) = \frac{K(s)Y_\mathrm{a}(s)}{1 + K(s)Y_\mathrm{a}(s)}$$

- Desired closed-loop system

$$H(s) = \frac{\alpha_\mathrm{c}}{s + \alpha_\mathrm{c}}$$

where α_c is the bandwidth

- Time constant of the closed-loop system is $\tau_\mathrm{c} = 1/\alpha_\mathrm{c}$



Internal Model Control (IMC) Principle

- Let us equal the closed-loop transfer function with the desirable one

$$H(s) = \frac{K(s)Y_\mathrm{a}(s)}{1 + K(s)Y_\mathrm{a}(s)} = \frac{\alpha_\mathrm{c}}{s + \alpha_\mathrm{c}} \qquad \Rightarrow \qquad K(s)Y_\mathrm{a}(s) = \frac{\alpha_\mathrm{c}}{s}$$

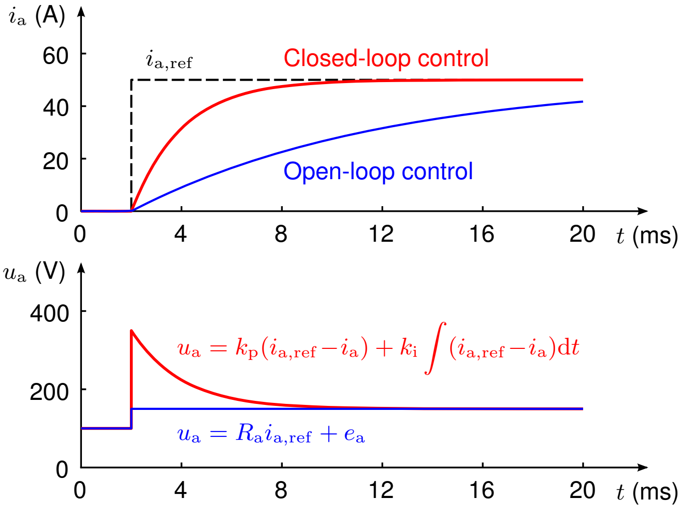
- Controller $K(s)$ can be solved

$$K(s) = \frac{\alpha_\mathrm{c}}{sY_\mathrm{a}(s)} = \frac{\alpha_\mathrm{c}}{s}(sL_\mathrm{a} + R_\mathrm{a}) = \alpha_\mathrm{c}L_\mathrm{a} + \frac{\alpha_\mathrm{c}R_\mathrm{a}}{s}$$

- Result is a PI controller with the gains $k_\mathrm{p} = \alpha_\mathrm{c}L_\mathrm{a}$ and $k_\mathrm{i} = \alpha_\mathrm{c}R_\mathrm{a}$
- Bandwidth α_c should be (at least) one decade smaller than the angular sampling frequency $2\pi/T_\mathrm{s}$

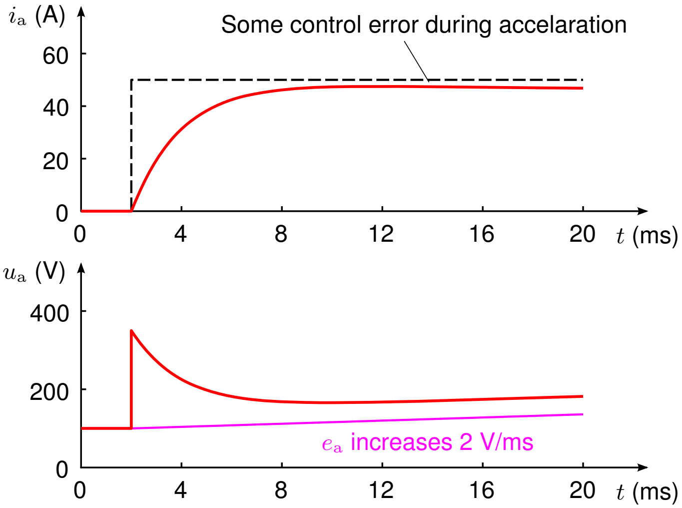
Example: Step Response

- $R_\mathrm{a} = 1\ \Omega$, $L_\mathrm{a} = 10$ mH
- Constant $e_\mathrm{a} = k_\mathrm{f}\omega_\mathrm{M} = 100$ V
- Open-loop controller
  - Slow ($\tau_\mathrm{a} = 10$ ms)
  - Inaccurate even in steady state (since R_a varies as a function of temperature)
- Closed-loop PI controller
  - Fast ($\tau_\mathrm{c} = 2$ ms)
  - Steady-state accuracy depends on the current measurement accuracy



Example: Step Response During Acceleration

- IMC-tuned PI controller is sensitive to the varying load disturbance $e_\mathrm{a} = k_\mathrm{f}\omega_\mathrm{M}$
- Causes control error during accelerations
- Load-disturbance rejection can be improved with an active resistance

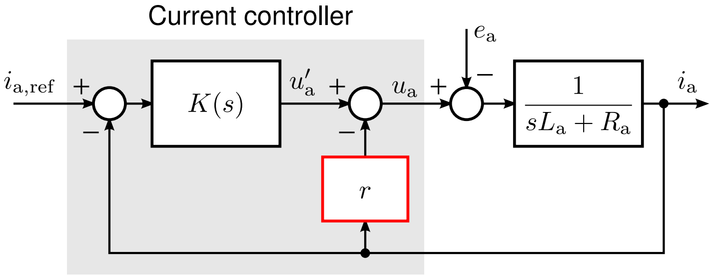


2DOF PI Current Controller: Improved Disturbance Rejection

- Active resistance r is a controller gain
- Affects like a physical resistance (without causing losses)

$$\frac{i_\mathrm{a}(s)}{u'_\mathrm{a}(s)} = Y'_\mathrm{a}(s) = \frac{1}{sL_\mathrm{a} + R_\mathrm{a} + r}$$

- Bandwidth of $Y'_\mathrm{a}(s)$ can be selected to equal α_c



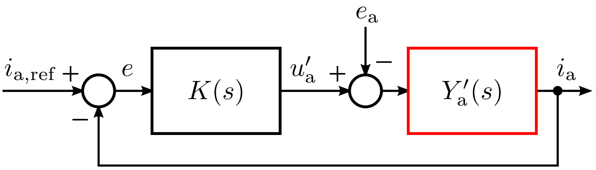
$$\frac{R_\mathrm{a} + r}{L_\mathrm{a}} = \alpha_\mathrm{c} \quad \Rightarrow \quad r = \alpha_\mathrm{c} L_\mathrm{a} - R_\mathrm{a}$$

This controller structure could also be represented as a state feedback controller with integral action and reference feedforward.

2DOF PI Current Controller

- IMC principle gives

$$K(s) = \frac{\alpha_c}{sY_a'(s)}$$
$$= \alpha_c L_a + \frac{\alpha_c(R_a + r)}{s}$$



$$Y_a'(s) = \frac{1}{sL_a + R_a + r}$$

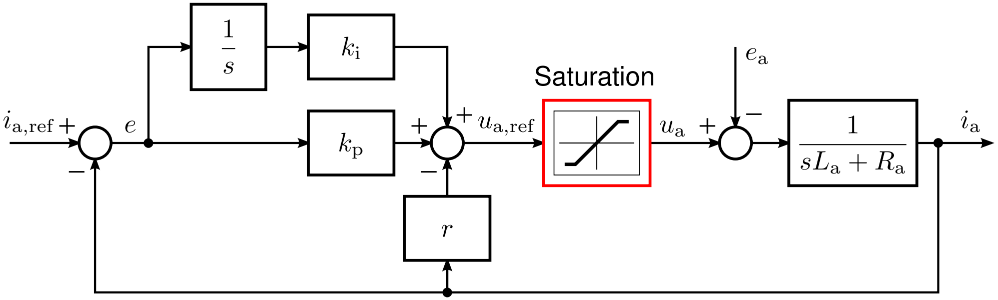
- Controller gains

$$r = \alpha_c L_a - R_a$$
$$k_p = \alpha_c L_a$$
$$k_i = \alpha_c(R_a + r) = \alpha_c^2 L_a$$

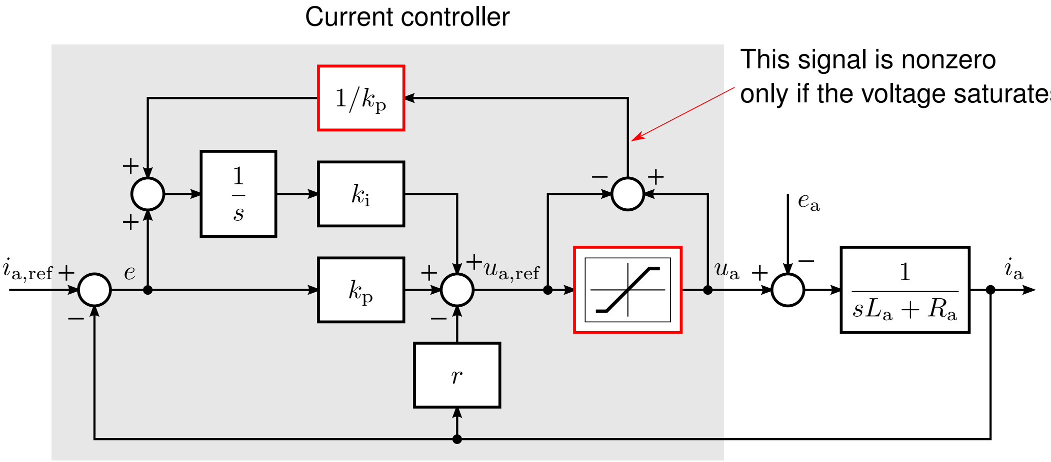
Outline

Voltage Saturation: Control Loop Becomes Nonlinear

- Maximum converter output voltage is limited: $-u_{\max} \leq u_{\mathrm{a}} \leq u_{\max}$
- Reference $u_{\mathrm{a,ref}}$ may exceed $u_{\max}$ for large $i_{\mathrm{a,ref}}$ steps (especially at high rotor speeds due to large back-emf e_{a})



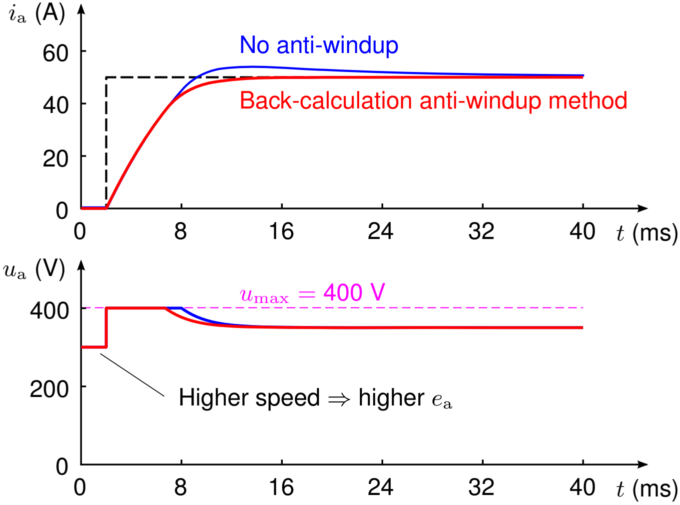
Back-Calculation Anti-Windup Method



This signal is nonzero only if the voltage saturates

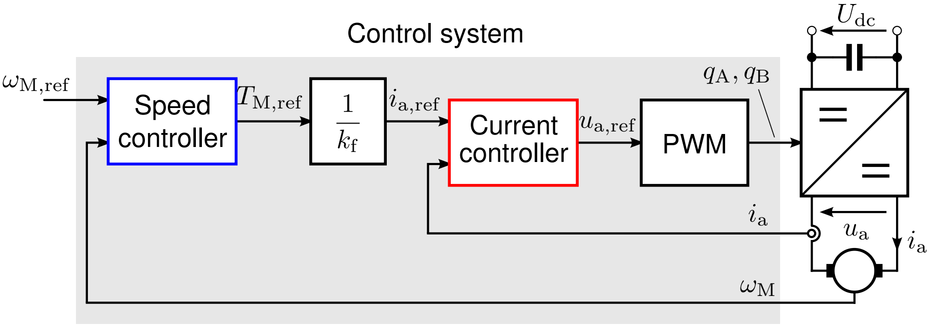
Example: Step Responses With and Without Anti-Windup

- Voltage saturates after the current reference step (due to high e_a)
- No overshoot if anti-windup is implemented
- Rise time is longer than the specified one (due to voltage saturation)



Outline

Cascaded Control System



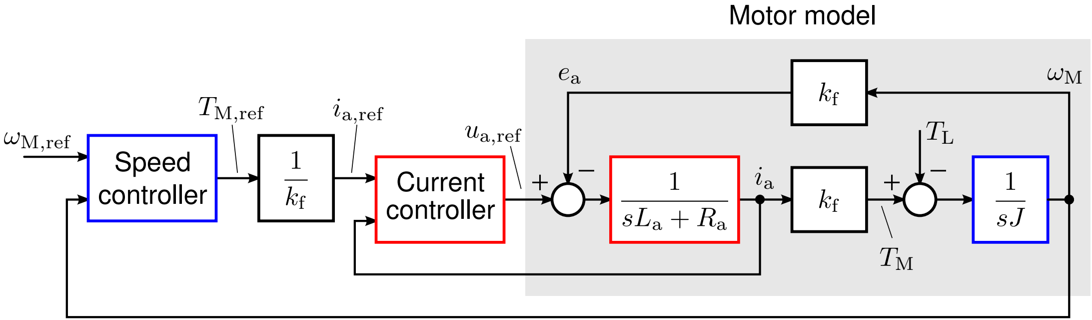
Current controller: Inner (faster) loop

Speed controller: Outer (slower) loop

Controllers can be designed separately due to different time scales

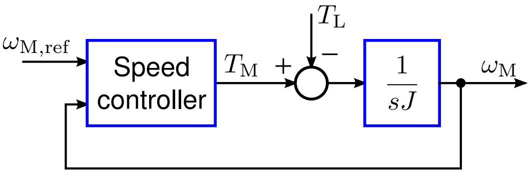
Block Diagram

- Ideal torque control can be assumed: $T_\mathrm{M} = T_\mathrm{M,ref}$
- What is the resulting system?



Simplified Block Diagram

- Assumption on ideal torque control $T_{\mathrm{M}} = T_{\mathrm{M,ref}}$
- Holds well if the current-control bandwidth is (at least) one decade faster than the speed-control bandwidth



- Mechanical and electrical systems are analogous

$$\frac{1}{sJ + B} \quad \leftrightarrow \quad \frac{1}{sL_{\mathrm{a}} + R_{\mathrm{a}}}$$

i.e. $J \leftrightarrow L_{\mathrm{a}}$, $B \leftrightarrow R_{\mathrm{a}}$, $T_{\mathrm{M}} \leftrightarrow u_{\mathrm{a}}$, $\omega_{\mathrm{M}} \leftrightarrow i_{\mathrm{a}}$, ...

- Same control structures and tuning principles can be directly used