

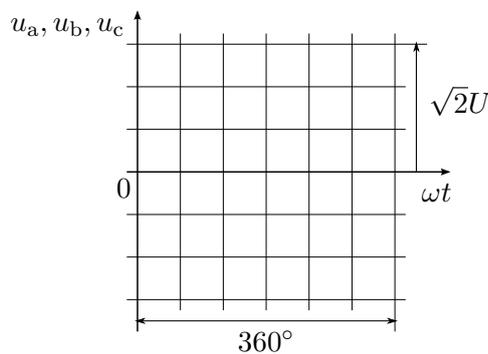
**Problem 1: Design of a PI current controller**

The parameters of a DC motor are  $R_a = 0.87 \Omega$  and  $L_a = 16 \text{ mH}$ . An ordinary PI current controller is used. The current-control bandwidth is required to be  $\alpha_c = 2\pi \cdot 300 \text{ rad/s}$ .

- (a) Derive the expressions for the controller gains according to the principle of internal model control.
- (b) Derive the expressions for the controller gains by cancelling the pole of the open-loop system and by requiring that the 0-dB crossover angular frequency of the loop transfer function is  $\alpha_c$ .

**Problem 2: Waveforms in a balanced three-phase system**

Sketch the waveforms of balanced three-phase voltages on the squared paper (or the grid below).



**Hint:** It is convenient to use a  $6 \times 6$  grid to draw these waveforms. You can first mark the points corresponding to zero crossings and peak values. It is also worth noticing that  $\sin(\pi/6) = 1/2$  and mark these points on the grid.

**Problem 3: Power in single-phase and three-phase systems**

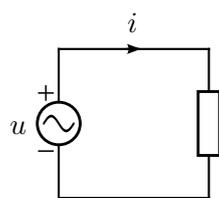
The purpose of this problem is to demonstrate that the instantaneous power in a balanced three-phase system is constant in steady state.

- (a) A single-phase load is fed with the voltage  $u(t) = \sqrt{2}U \sin(\omega t)$  and it draws the current  $i(t) = \sqrt{2}I \sin(\omega t - \varphi)$ . Derive the expressions for the instantaneous power and the average power.

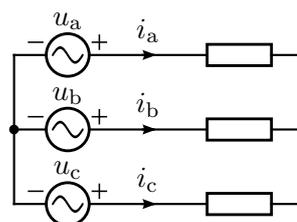
**Hint:** The trigonometric product-to-sum identity may be useful:

$$\sin \alpha \sin \beta = \frac{1}{2} [\cos(\alpha - \beta) - \cos(\alpha + \beta)]$$

- (b) A balanced three-phase load is fed with balanced three-phase voltages. Derive the expressions for the instantaneous power and the average power.



(a)



(b)

**Problem 4: Transmission losses**

The figure below shows two different configurations to feed the same power to the resistive load:

- (a) The balanced three-phase supply feeds the three resistors  $R_{load}$ . The resistance of the transmission wires is  $R$ .
- (b) The single-phase supply, taken from one phase of the previous three-phase supply, feeds the load resistor  $R_{load}/3$ . The same transmission wires are also used as in the previous case.

The resistance  $R$  of the wires can be assumed to be much smaller than the resistance  $R_{load}$ . Compare the transmission losses in these two cases.

