How to obtain pressure values from the impulse response of a loudspeaker

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1 Calibration of the measuring chain

In the exercise, a calibrator was used to measure the sensitivity of the measurement microphone. The calibration signal provides 94 dB SPL at 1 kHz thus if we divide the measured signals by the *rms* value of the calibrator signal we obtain a sensitivity of $1V/Pascal$. Therefore,

$$\frac{x(n)}{V_{cal}} = y(n)\ (Pascal) \tag{1}$$

where $x(n)$ is whatever signal coming from the microphone in *Volts*; V_{cal} is the *rms* value of the calibration signal; and $y(n)$ the calibrated signal in *Pascal*.

2 Pressure values from impulse responses

2.1 The theory behind (in discrete domain[1])

For any signal, it holds that

$$s(n) \bigotimes s^{-1}(n) = \delta(n) \tag{2}$$

where $s(n)$ is the original signal with frequency response $S(k)$; s^{-1} is a signal with frequency response $S^{-1}(k) = 1/S(k)$; $\otimes$ is the convolution operator;

[1]Notice that the indexes in this text starts from 1 and not from 0 as in the theoretical equations. This is because the Matlab *effect* has been taken into account.

and $\delta(n)$ is the Kronecker delta (Dirac delta in discrete time). It also holds that

$$\mathrm{g} \cdot s(n) \bigotimes s^{-1}(n) = \mathrm{g} \cdot \delta(n) \tag{3}$$

where g is a gain factor applied to $s(n)$.

Let's assume that $s(n)$ has been modified by a linear temporally invariant (LTI) system (like a loudspeaker) and that our measuring channel have been already calibrated

$$y(n) = \mathrm{g} \cdot s(n) \bigotimes h(n) \tag{4}$$

where $h(n)$ is the impulse response of the LTI system. Furthermore, $h(n)$ can be decomposed into

$$h(n) = \mathrm{S} \cdot h_N(n) \tag{5}$$

where S is a gain factor that belong to the LTI system; and $h_N(n)$ is the energy normalized impulse response of the system:

$$\sum_{n=1}^{\infty} h_N^2(n) = 1 \tag{6}$$

Now, let's rewrite Eq.(2) adding all parameters

$$\mathrm{g} \cdot s(n) \bigotimes h_N(n) \cdot \mathrm{S} \bigotimes s^{-1}(n) = y(n) \bigotimes s^{-1}(n) = \mathrm{g} \cdot \mathrm{S} \cdot h_N(n) \tag{7}$$

In frequency domain, Eq.(7) is written as

$$\mathrm{g} \cdot S(k) \cdot H_N(k) \cdot \mathrm{S} \cdot S^{-1}(k) = Y(k) \cdot S^{-1}(k) = \mathrm{g} \cdot \mathrm{S} \cdot H_N(k) \tag{8}$$

2.2 Calculating the pressure

For the case of a loudspeaker, the result of Eq.(7) and Eq.(8) is the measured impulse response thus we can assume that g is the *rms* voltage at the entrance of the loudspeaker and S is the sensitivity of the loudspeaker measured at the receiver point (thus attenuated by distance). Therefore, the product g $\cdot$S represents the *rms* pressure value in $Pascal$ and $|H_N(k)|$ the variations around the *rms* pressure at each frequency.
Notice that this is true only when the sensitivity of the measurement channel

is equal to the inverse of the *rms* value of s^{-1},$V_{s^{-1}}$. For instance, if the sensitivity of the measurement channel is 2 $V/Pascal$ then, $V_{s^{-1}}$ should be 0.5 V_{rms}.

For the case of the measurement exercise, $V_{s^{-1}} = 1\ V_{rms}$ so the full process is written then as

$$h(n) = \frac{w(n)}{V_{cal}} \tag{9}$$

where $w(n)$ is the measured impulse response before calibration and $h(n)$ is the measured response after calibration. Doing the Fourier transform of $h(n)$,

$$H(k) = \mathcal{FFT}\{h(n)\} \tag{10}$$

we can calculate the pressure at the measuring point for a specific frequency as

$$L_p(k) = 20 \cdot \log_{10}\left(\frac{|H(k)|}{p_{ref}}\right) \tag{11}$$

Since k is the frequency beam of the FFT, we can also calculate the pressure in a band using

$$L_{p_{band}} = 20 \cdot \log_{10}\left(\frac{\sqrt{\frac{\sum_{k=k_1}^{k_2} |H(k)|^2}{\Delta k}}}{p_{ref}}\right); \quad \Delta k = k_2 - k_1 + 1 \tag{12}$$

or using Parseval's theorem we can arrive to the same result in time-domain with

$$L_{p_{band}} = 20 \cdot \log_{10}\left(\frac{\sqrt{\sum_{n=1}^{N} h_{band}^2(n)}}{p_{ref}}\right) \tag{13}$$

where $h_{band}(n)$ is the impulse response $h(n)$ bandpass filtered at the frequencies of interest.