Aalto University School of Science
Department of Mathematics and Systems Analysis — Korte

MS-C1350 Partial differential equations, fall 2020

Pre-lecture assignment for Mon 14 Sept 2020

Please answer YES or NO, unless otherwise stated.

1. (a) $f \in L^2([-\pi,\pi]) \implies f \in C([-\pi,\pi])$.
   (b) $f \in C([-\pi,\pi]) \implies f \in L^2([-\pi,\pi])$.
   (c) $\|f-g\|_{L^2([-\pi,\pi])} = 0 \iff f(t) = g(t)$ for every $t \in \mathbb{R}$.
   (d) Assume that $f, g \in C([-\pi,\pi])$. Then $\|f-g\|_{L^2([-\pi,\pi])} = 0 \iff f(t) = g(t)$ for every $t \in \mathbb{R}$.

2. (a) The definition of the Fourier series applies to all functions in $L^1([-\pi,\pi])$.
   (b) The definition of the Fourier series applies to all functions in $L^2([-\pi,\pi])$.
   (c) The definition of the Fourier series applies to all functions in $C([-\pi,\pi])$.
   (d) If the Fourier series converges pointwise everywhere, the obtained function is 2π-periodic.

3. (a) A trigonometric polynomial belongs to a subspace of $L^2([-\pi,\pi])$ spanned by the functions $e_j(t) = e^{ijt}$, $j = -n,\dots,n$ for some $n \in \mathbb{N}$.
   (b) The partial sum $S_n f$ of a Fourier series is an orthogonal projection of the function f to the subspace of $L^2([-\pi,\pi])$ spanned by $\{e_j\}_{j=-n}^{n}$.
   (c) The partial sum of a Fourier series gives the best approximation of a function in $L^2([-\pi,\pi])$ with trigonometric polynomials.
   (d) The statement that the partial sum of a Fourier series gives the best approximation of a function in $L^2([-\pi,\pi])$ with trigonometric polynomials means that there does not exist a function in $L^2([-\pi,\pi])$, which would be closer to the original function in L^2-norm.

4. (a) $L^2([-\pi,\pi])$ is an infinite dimensional vector space.
   (b) The Fourier series of every function $f \in L^2([-\pi,\pi])$ converges with respect to the L^2-norm.
   (c) The statement that $S_n f$ approximates a function f in $L^2([-\pi,\pi])$ means that, the error term $\|f - S_n f\|_{L^2([-\pi,\pi])}$ converges to zero as $n \to \infty$.
   (d) Fourier coefficients are coordinates of the function with respect to the basis $\{e_j\}_{j\in\mathbb{Z}}$.

5. (a) Every element $f \in L^2([-\pi,\pi])$ is uniquely determined by its Fourier coefficients $\widehat{f}(j)$, $j \in \mathbb{Z}$.
   (b) The Fourier coefficients $\widehat{f}(j)$ of a function $f \in L^2([-\pi,\pi])$ converge to zero as $|j| \to \infty$.

(c) The Fourier coefficients $\widehat{f}(j)$ of a function $f \in L^2([-\pi, \pi])$ converge faster to zero as $|j| \to \infty$ if the function f is smoother.

(d) The zero function is the only element in $L^2([-\pi, \pi])$, whose all Fourier coefficients are zero.