Aalto University School of Science
Department of Mathematics and Systems Analysis Korte

MS-C1350 Partial differential equations, fall 2020

Pre-lecture assignment for Mon 21 Sept 2020

Please answer YES or NO, unless otherwise stated.

1. (a) Separation of variables is based on finding special solutions to the PDE as a product of functions so that each function depends only on one variable.
   (b) Separation of variables applies to PDE problems on rectangular domains.
   (c) Separation of variables is based on transforming a PDE to a system of ODEs.
   (d) Separation of variables is based on finding solutions to the corresponding system of ODEs.
2. Consider the Dirichlet problem for the Laplace equation in the unit disc, see Section 2.11 in the lecture notes.
   (a) All functions $r^{|j|}e^{ij\theta}$, $j \in \mathbb{Z}$, are solutions to the Laplace equation.
   (b) All functions $r^{|j|}e^{ij\theta}$, $j \in \mathbb{Z}$, have zero boundary values on the unit circle.
   (c) The solution of the original problem is obtained as a countable linear combination of the special solutions above.
   (d) The coefficients in the linear combination are determined by the Fourier coefficients of the boundary function.
3. Continuation of the previous problem.
   (a) The $L^1([-\pi,\pi])$-norm of the function $\theta \mapsto P_r(\theta)$ is equal to one for every $r \in (0,1)$.
   (b) The solution to the original problem can be represented as a convolution of the Poisson kernel with the boundary function.
   (c) The solution of the original problem on the boundary of the unit disc can be obtained by inserting $r = 1$ in the convolution formula.
   (d) The solution of the original problem is always zero at the origin.
4. Consider the space-time model of the initial value problem for the heat equation on the unit circle, see Section 2.12 in the lecture notes.
   (a) The initial value function corresponds to the boundary values at the time zero.
   (b) The boundary values can also be given on the lateral boundaries $\{-\pi\} \times \mathbb{R}_+$ and $\{\pi\} \times \mathbb{R}_+$.

(c) The heat distribution on a circle at the moment t is given by the solution on the line segment $[-\pi, \pi] \times \{t\}$.

(d) The temperature at a given point θ at all moments of time $t > 0$ are given by the solution on the line $\{\theta\} \times \mathbb{R}_+$.

5. Continuation of the previous problem.

   (a) The functions $e^{-j^2 t} e^{ij\theta}$, $j \in \mathbb{Z}$, are solutions to the heat equation in $(-\pi, \pi) \times \mathbb{R}_+$

   (b) The functions $e^{-j^2 t} e^{ij\theta}$, $j \in \mathbb{Z}$, have zero initial values.

   (c) The functions $e^{-j^2 t} e^{ij\theta}$, $j \neq 0$, decay to zero as $t \to \infty$.

   (d) The solution of the problem is given by a convolution of the initial value with the heat kernel.