

MS-C1350 Partial differential equations, fall 2020

Pre-lecture assignment for Mon 28 Sept 2020

Please answer YES or NO, unless otherwise stated.

1. Consider the one-dimensional initial value problem for the wave equation in Section 2.13 of the lecture notes.
 - (a) Initial values describe the shape and velocity of the string at time zero.
 - (b) The boundary values can also be given on the lateral boundaries in the space-time model of the problem.
 - (c) The solution has its maximum and minimum values at the endpoints of the interval.
 - (d) The solution decays to zero as $t \rightarrow \infty$.
2. Continuation of the previous problem.
 - (a) If the profile of the string is horizontal at time zero, then all coefficients a_j are zero.
 - (b) If the velocity of the string is zero at time zero, then all coefficients b_j are zero.
 - (c) If the profile of the string is horizontal at time zero, the solution of the problem is identically zero.
 - (d) If the profile of the string is horizontal and the velocity is zero at time zero, the solution of the problem is identically zero.
3. Which of the following kernels satisfy the conditions in Definition 2.53 in the lecture notes?
 - (a) The Poisson kernel $P_{1-\varepsilon}(\theta)$.
 - (b) The heat kernel on the unit circle $H_t(\theta)$.
 - (c) The Dirichlet kernel $D_n(\theta)$.
 - (d) The integral average $\frac{1}{\varepsilon} \mathbf{1}_{[-\pi, \pi]} \left(\frac{x}{\varepsilon} \right)$.
4. The following claims are related to Definition 2.53 in the lecture notes.
 - (a) Conditions (1)–(3) are related to kernels that can be used in the convolution approximation of functions.
 - (b) The parameter $\varepsilon > 0$ gives the scale of the approximation.
 - (c) Condition (1) asserts that the total mass of the kernel equals zero at any scale of approximation.
 - (d) Condition (3) asserts that the total mass of the kernel is concentrated near the origin at small scales of approximation.

5. (a) Approximations of the identity can be used to approximate functions with smoother functions.
- (b) Approximations of the identity can be used to study boundary and initial values in PDE problems.
- (c) The pointwise convergence of the Dirichlet series can be concluded directly using approximations of the identity.
- (d) The pointwise value of a continuous function is obtained as a limit of weighted integral averages in approximations of the identity.