Aalto University School of Science
Department of Mathematics and Systems Analysis — Korte

MS-C1350 Partial differential equations, fall 2020

Pre-lecture assignment for Mon 5 Oct 2020

Please answer YES or NO, unless otherwise stated.

1. (a) $L^1(\mathbb{R}^n) \subset L^2(\mathbb{R}^n)$.
   (b) $L^2(\mathbb{R}^n) \subset L^1(\mathbb{R}^n)$.
   (c) The Fourier transform of a function in $L^1(\mathbb{R}^n)$ is a bounded function.
   (d) The Fourier transform of a function in $L^1(\mathbb{R}^n)$ is a continuous function.

2. (a) The Fourier transform of a real function is a real valued function.
   (b) The Fourier transform of a compactly supported function is a compactly supported function.
   (c) If $f \in L^1(\mathbb{R}^n)$, then $\widehat{f} \in L^1(\mathbb{R}^n)$.
   (d) The Fourier transform at zero equals to the integral of the function over $\mathbb{R}^n$.

3. (a) If $f \in C_0^\infty(\mathbb{R}^n)$, then $\frac{\partial \widehat{f}}{\partial \xi_j}(\xi) = \widehat{\frac{\partial f}{\partial x_j}}(\xi)$, $j = 1, 2, \dots, n$.
   (b) The jth partial derivative of a function becomes multiplication by $i\xi_j$ on the Fourier side.
   (c) The smoothness of a function is reflected in the decay of its Fourier transform.
   (d) If $f \in C_0^\infty(\mathbb{R}^n)$, then $\widehat{f} \in C^\infty(\mathbb{R}^n)$ (challenging).

4. (a) Convolution becomes multiplication on the Fourier side.
   (b) $\|\widehat{f}\|_{L^2(\mathbb{R}^n)} = \|f\|_{L^2(\mathbb{R}^n)}$.
   (c) The $L^2(\mathbb{R}^n)$ norm of the Fourier transform is the same as the $L^2(\mathbb{R}^n)$ norm of the function up to a multiplicative constant.
   (d) The $L^1(\mathbb{R}^n)$ norm of the Fourier transform is the same as the $L^1(\mathbb{R}^n)$ norm of the function up to a multiplicative constant.

5. (a) Fourier inversion theorem applies to all functions in $C_0^\infty(\mathbb{R}^n)$.
   (b) Fourier inversion theorem applies to all functions in $L^1(\mathbb{R}^n)$.
   (c) Fourier inversion theorem applies to all functions in $C(\mathbb{R}^n)$.
   (d) The Fourier transform of a Gaussian function is a Gaussian function.