Aalto University School of Science
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MS-C1350 Partial differential equations, fall 2020

Pre-lecture assignment for Mon 12 Oct 2020

Please answer YES or NO, unless otherwise stated.

1. Consider the Dirichlet problem for the Laplace equation in the upper half-space, see Section 3.9 in the lecture notes.

   (a) The Laplace equation is Fourier transformed with respect to all variables in the upper-half space.

   (b) The Laplace equation is Fourier transformed with the last coordinate fixed.

   (c) The Laplace equation becomes an ODE with respect to the last variable on the Fourier side.

   (d) The Laplace equation is satisfied if and only if it is satisfied on the Fourier side.

2. (Continuation to the previous problem)

   (a) $\widehat{\Delta u}(\xi, y) = \Delta \widehat{u}(\xi, y)$.

   (b) $\widehat{u}(\xi, y) = \widehat{g}(\xi) e^{-|\xi| y}$ is a solution to the original problem on the Fourier side.

   (c) $u(x, y) = (2\pi)^{-n} \displaystyle\int_{\mathbb{R}^n} e^{-|\xi| y} \widehat{g}(\xi) e^{ix\cdot\xi} \, d\xi$ is a solution to the original problem in the upper half-space.

   (d) $u(x, y) = (P_y * g)(x)$, with $\widehat{P_y}(\xi) = e^{-|\xi| y}$, is a solution to the original problem in the upper half-space.

3. Consider the initial value problem for the heat equation in the upper half-space, see Section 3.10 in the lecture notes.

   (a) The initial condition is used to determine free parameters in the solution of the ODE.

   (b) The initial condition can be verified by inserting $t = 0$ in the formula $u(x, t) = (H_t * g)(x, t)$.

   (c) The initial condition can be verified on the Fourier side by inserting $t = 0$ in the formula for $\widehat{u}(\xi, t)$.

   (d) If $g \in C_0^\infty(\mathbb{R}^n)$, then the initial values are attained in the limit sense $\lim\limits_{t \to 0} u(x, t) = g(x)$ for every $x \in \mathbb{R}^n$.

4. (a) One advantage of the Fourier transform is that a PDE becomes an ODE.

   (b) One advantage of the Fourier transform is that representation formulas for solution are convolutions.

   (c) One advantage of the Fourier transform is that it also proves the uniqueness of a solution.

   (d) One advantage of the Fourier transform is that it also applies to nonhomogeneous problems.

5. (a) Fundamental solutions are solutions to the corresponding PDEs in the upper half-space.

   (b) Fundamental solutions have zero initial and boundary values.

   (c) General solutions can be represented by the fundamental solution and the data of the problem.

   (d) Fundamental solutions are radial functions in the spatial variable.