Differential and Integral Calculus 1
MS-A0111
Hakula
Orlich/Ardiyansyah
Problem Sheet 1, 2020

NOTE[1]

The due date is published on the course pages. Homework can be submitted only digitally. Instructions on labeling the "papers" can be found on the course pages.

1 Introductory Problems

INTRO 1 Evaluate the limit or explain why it doesn't exist:

$$\lim_{h\to 0} \frac{\sqrt{4+h}-2}{h}.$$

INTRO 2 If $2 - x^2 \leq g(x) \leq 2\cos x$ for all x, find $\lim_{x\to 0} g(x)$.

INTRO 3 If $\lim_{x\to a} g(x) = M$, show that there exists a number $\delta > 0$ such that

$$0 < |x-a| < \delta \implies |g(x)| < 1 + |M|.$$

(Hint: Take $\epsilon = 1$ in the definition of a limit.)

INTRO 4 Evaluate, if possible, the limit of the sequence $\{a_n\}$.

$$a_n = \sqrt{n+1} - \sqrt{n}.$$

INTRO 5 Use the definition of derivative to calculate

$$\frac{d}{dx}\left(\frac{x}{x^2+1}\right)\bigg|_{x=3}.$$

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INTRO 6 Calculate the derivative of $f(x) = x^{1/3}$ using only the definition.
(Hint: Revise factoring of cubes $a^3 - b^3$.)

2 Homework Problems

EXERCISE 1 Evaluate the limit or explain why it doesn't exist:

$$\lim_{x \to 1/2} \frac{1}{\sqrt{x - x^2}}.$$

EXERCISE 2 Evaluate

$$\lim_{x \to 0} \frac{1}{|x - 1] - |x + 1|}.$$

EXERCISE 3 If $\lim_{x \to a} f(x) = L$ and $\lim_{x \to a} g(x) = M$, prove that

$$\lim_{x \to a} f(x)g(x) = LM.$$

(At some point Intro 3 will prove to be useful.)

EXERCISE 4 Evaluate, if possible, the limit of the sequence $\{a_n\}$.

$$a_n = \sqrt{n^2 + n} - \sqrt{n^2 - 1}.$$

EXERCISE 5 How should the function $g(x) = x^2 \operatorname{sgn} x$ be defined at $x = 0$ so that it is continuous there? Is it then differentiable there?
(sgn is the sign function or signum.)

EXERCISE 6 Calculate the derivative of $f(x) = x^{1/n}$, where n is a positive integer, using the definition.