Differential and Integral Calculus 1
MS-A0111
Hakula
Orlich/Ardiyansyah
Problem Sheet 2, 2020

NOTE[1]

The due date is published on the course pages. Homework can be submitted only digitally. Instructions on labeling the "papers" can be found on the course pages.

1 Introductory Problems (Home Exam)

INTRO 1 (a) Evaluate the limit $\lim_{x\to 0+} \log_x(1/2)$. (b) Find the derivative of $f(x) = x - \arcsin(\sin(x))$ for $-\pi \le x \le \pi$ and sketch the graph of f on that interval.

INTRO 2 [Modelling an epidemic] The number y of persons infected by a highly contagiuos virus is modelled by a logistic curve

$$y = \frac{L}{1 + Me^{-kt}},$$

where t is measured in months from the time the outbreak was discovered. A the time there were 200 infected persons, and the number grew to 1000 after one month. Eventually, the number levelled out at 10000. Find the values of the parameters L, M, and k of the model.

INTRO 3 Classify the critical points of the function $y = x \ln x$.

INTRO 4 Find the equation of the straight line of maximum slope tangent to the curve $y = 1 + 2x - x^3$.

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2 Introductory Problems

INTRO 5 Write the indicated case of Taylor's formula for the given function. What is the Lagrange remainder in this case?

$$f(x) = \sin x,\ a = \pi/4,\ n = 4.$$

INTRO 6 Find the nth-order Maclaurin polynomial of

$$f(x) = \frac{1}{1-x}$$

using the alternative definition of the Taylor polynomial.

INTRO 7 Express the given limit as a definite integral:

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{n}\sqrt{\frac{i-1}{n}}.$$

INTRO 8 If $a < b$ and f is continuous on $[a, b]$, show that

$$\int_a^b (f(x) - \bar{f})dx = 0.$$

3 Homework Problems

EXERCISE 1 Write the indicated case of Taylor's formula for the given function. What is the Lagrange remainder in this case?

$$f(x) = x,\ a = 1,\ n = 6.$$

EXERCISE 2 Find the nth-order Maclaurin polynomial of

$$f(x) = \frac{1}{(1-x)^2}$$

using the alternative definition of the Taylor polynomial.
(Hint: Take the derivative of the solution of Intro 6 and verify that the conditions are satisfied.)

EXERCISE 3   Express the given limit as a definite integral:

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{2}{n} \ln\left(1 + \frac{2i}{n}\right).$$

EXERCISE 4   Suppose that $a < b$ and f is continuous on $[a, b]$. Find the constant k that minimises the integral

$$\int_a^b (f(x) - k)^2 dx.$$