

Shapes in Action 2020

Sept 15th

*Some tools to analyze
symmetries*



Program schedule for Sept 15th

15:15 Feedback from picture analysis exercise

How to look planar patterns from math perspective ?

16:00 Break

16:15 Some tools to analyze regular tilings

17:00 Break

17:15 Laura: Group Work Instructions & testing Breakout rooms

Essential concepts we had so far

Symmetry (group) : = **actions** that preserve the given pattern in the given **geometry**.

Consider today only **patterns in the plane**.

Given **geometry** = standard Euclidean plane.

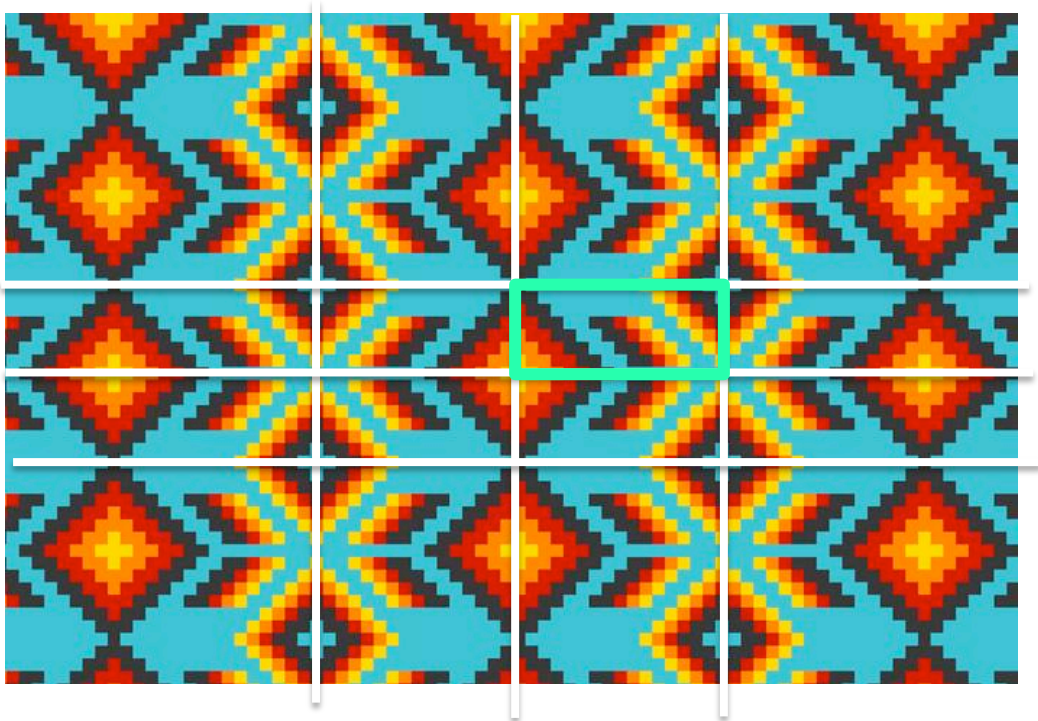
Actions := rigid motions (= *isometries*) of the plane:

- Translations
- Rotations with respect to a point
- Reflections with respect to a line

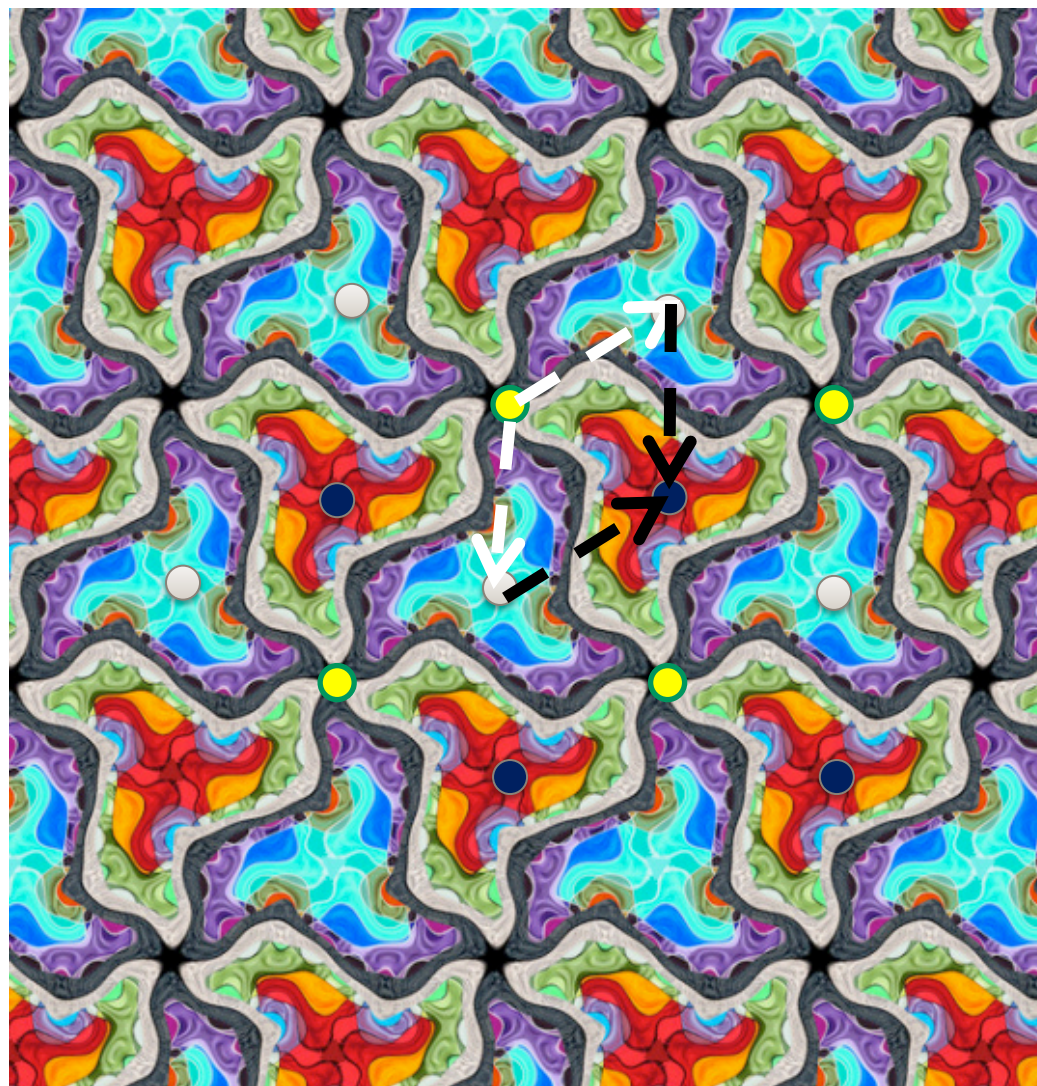
And all possible *combinations* of those.

How to count each operation only once?

Ex: Two consecutive reflections create rotation

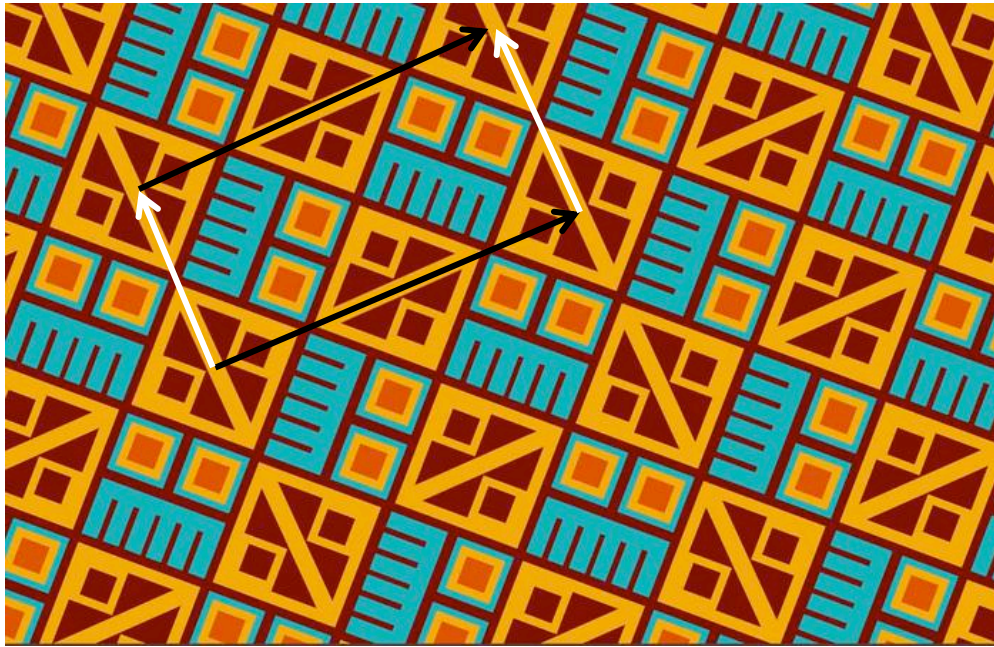


- Only reflection lines explain the whole symmetry pattern
 - all edges of the highlighted rectangle are different
 - All vertices of the highlighted rectangle are different
- 1.

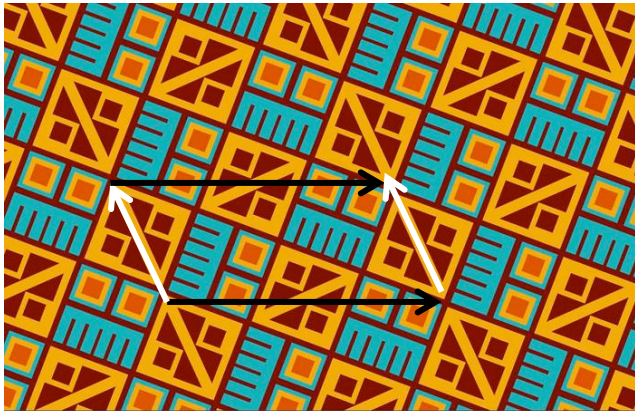


- Three different types of (genuine) rotation points of order three (=120 degree rotations)
- No reflection lines
- Parallelogram in the picture has only two different edges

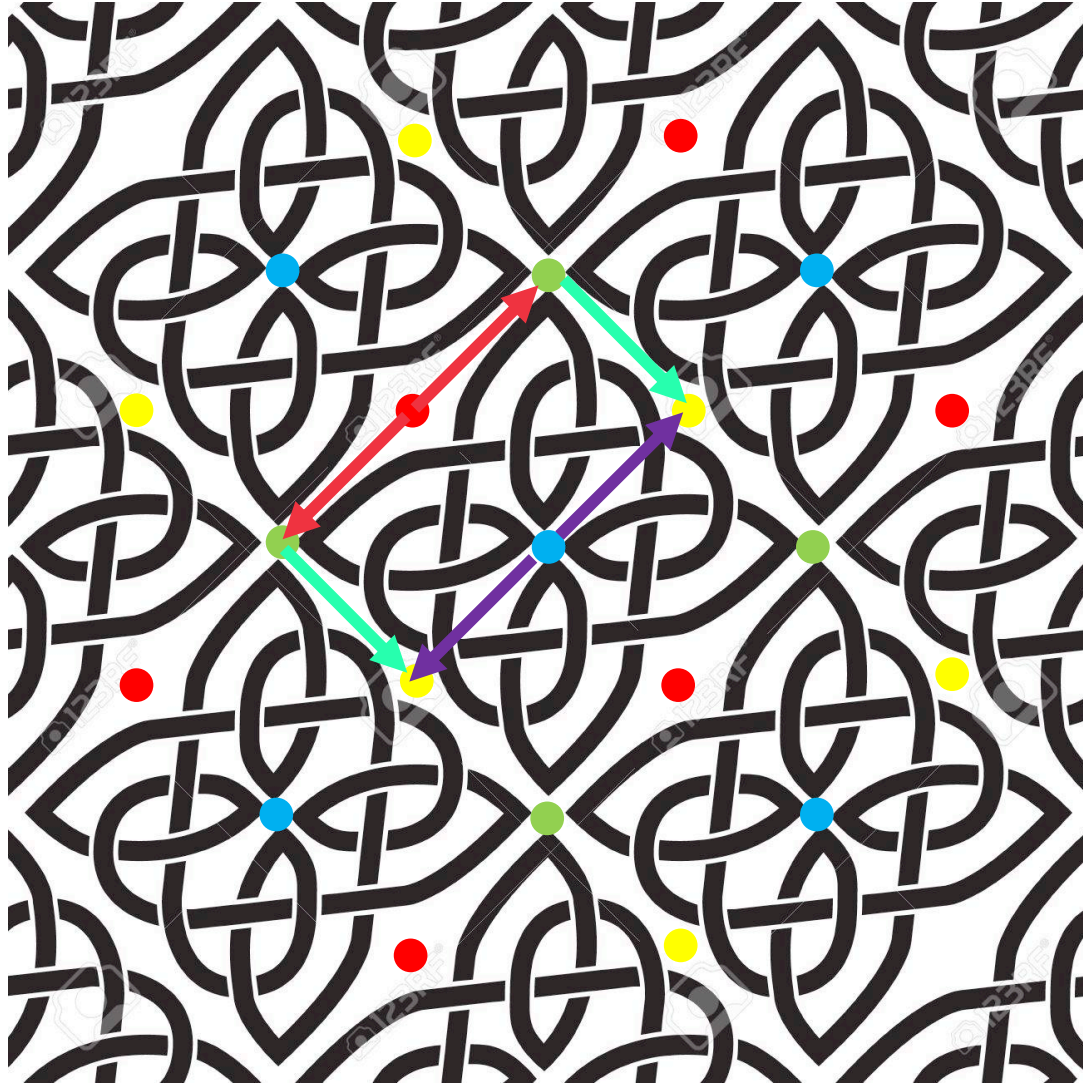
2.



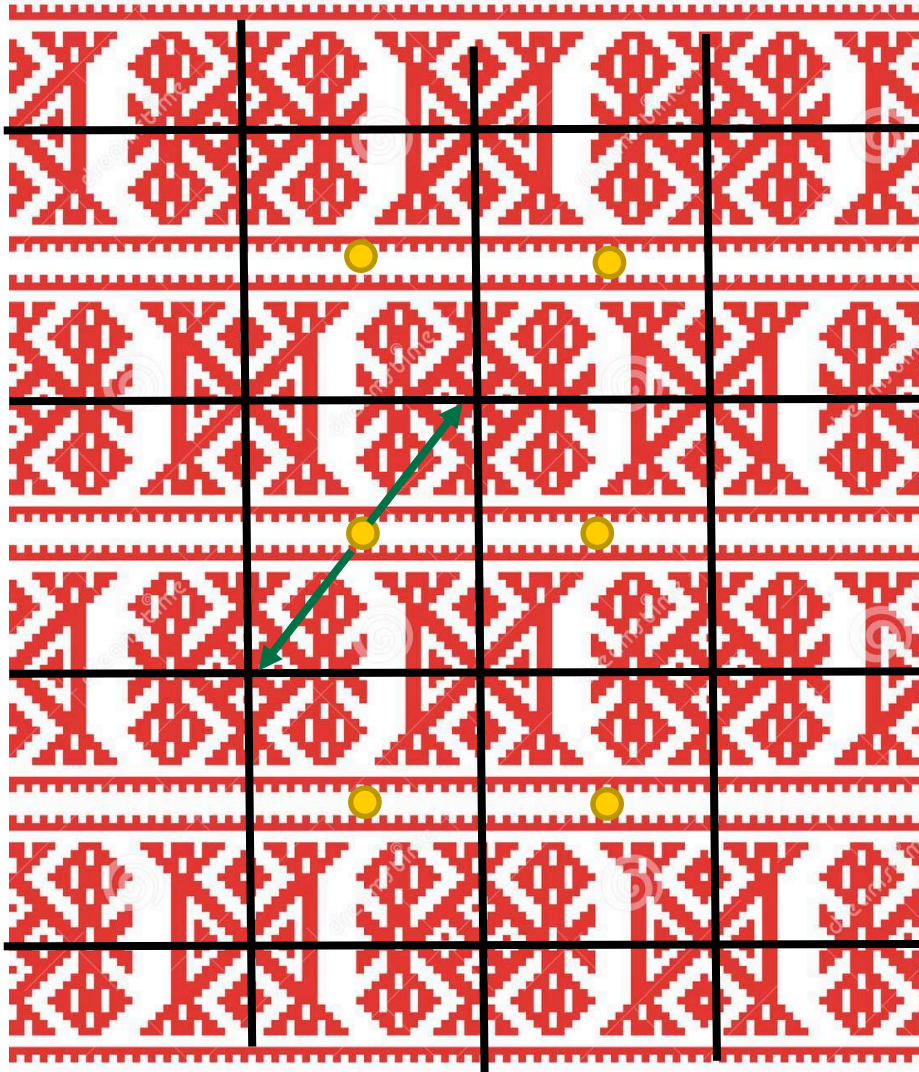
- No rotation points
- No reflection lines
- Two different translations generate the whole pattern
- Highlighted rectangle has only two different edges



- For this type of symmetry no unique way to choose a representative for a parallelogram 'spanning' the pattern

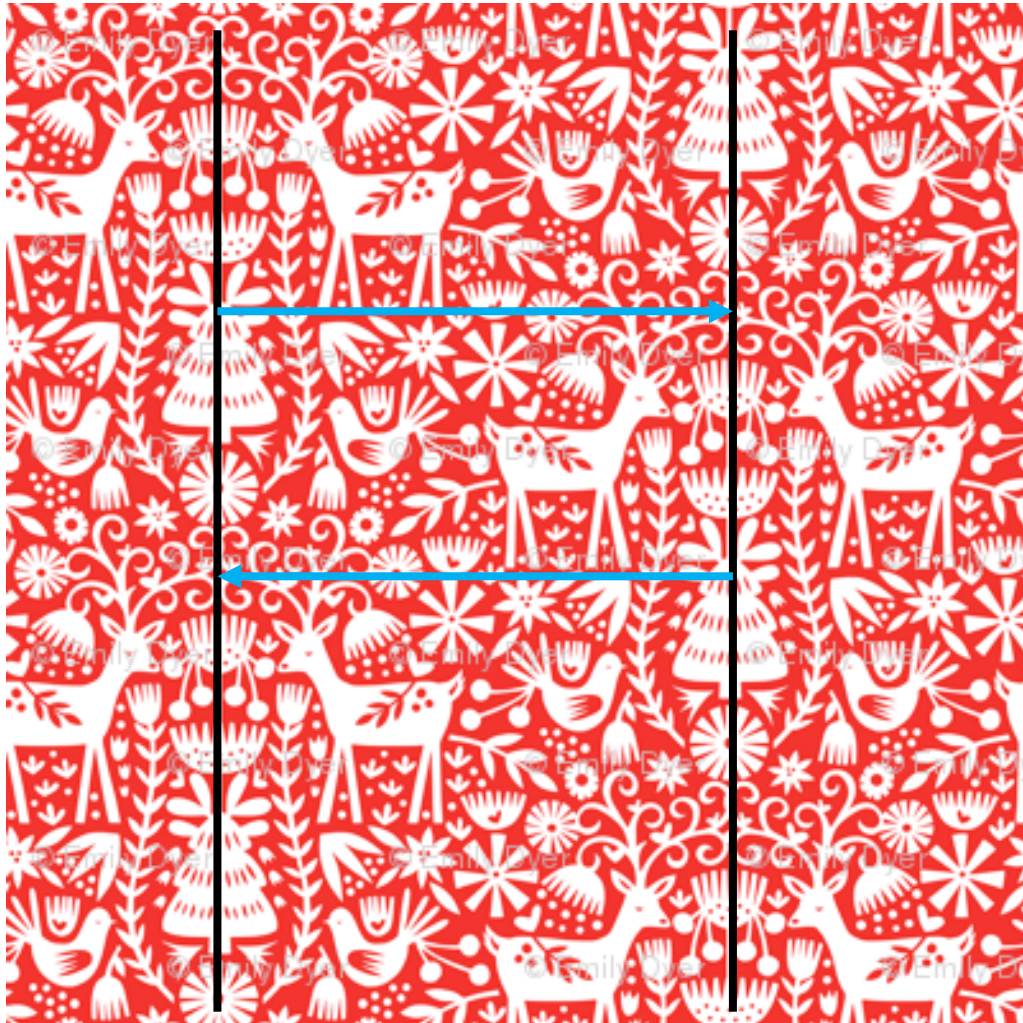


- Four different rotation points of order two (=180 degree rotations)
- No reflection lines



- Two types of reflection lines
- One (genuine =not produced by consecutive reflections)
rotation point of order two

5.



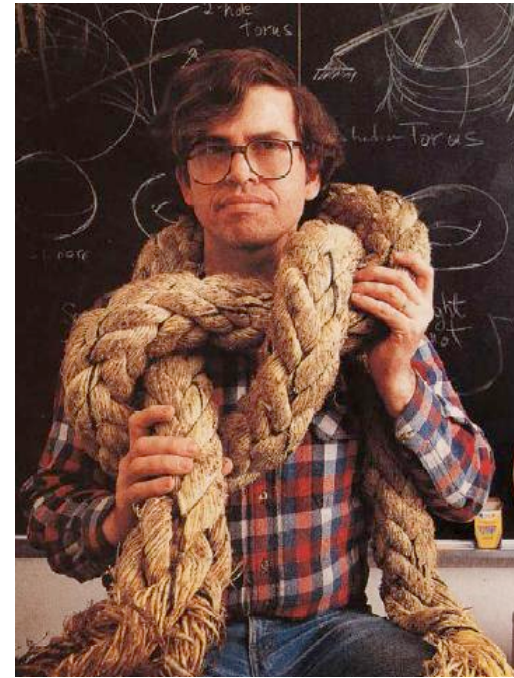
- Only one type of reflection line: Two black lines in the picture are the *same up to rigid motion*
 - Between the lines also mirror images that are not caused by a mirror line
- => Blue horizontal arrows cutting the shape have opposite orientation

Symmetries of planar patterns

Goal : *Signature/Orbifold notation due to B. Thurston and J.H. Conway (90 ´)*

- What are the patterns in plane that are covered ?
- Intro to the notation
- Concepts '***fundamental domain***' and '***orbifold***' of a tiling

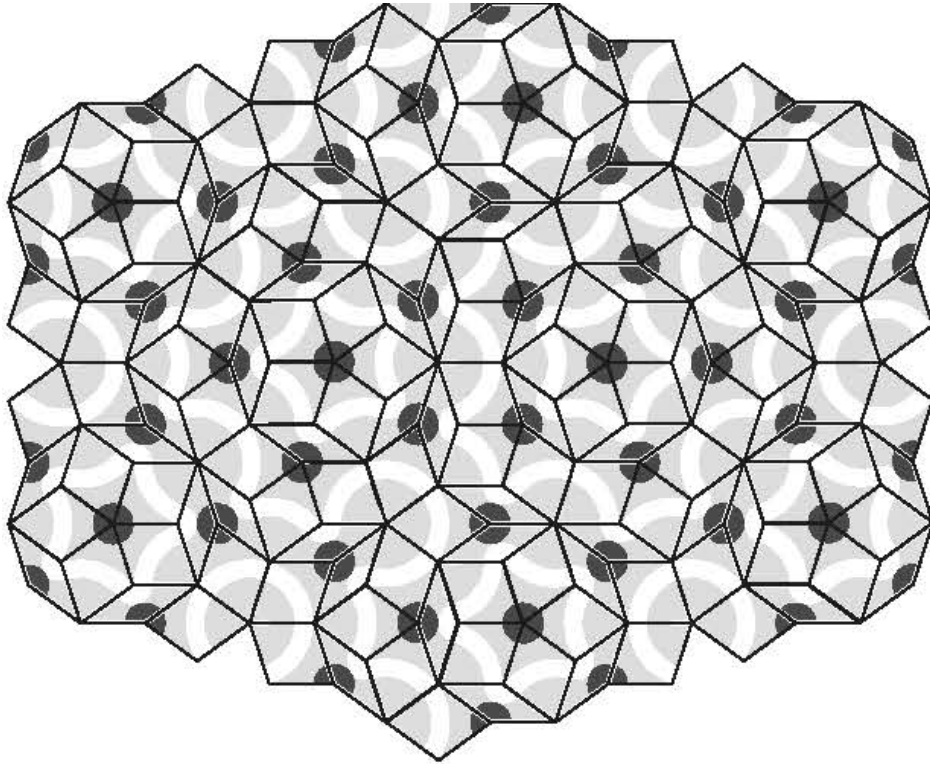
Classification up to symmetry = rigid motions acting on given geometry



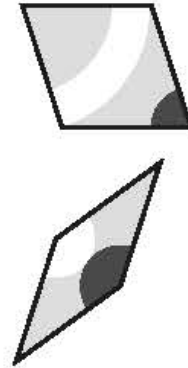
Classification of infinite repeating patterns. Need two (linearly) independent directions.



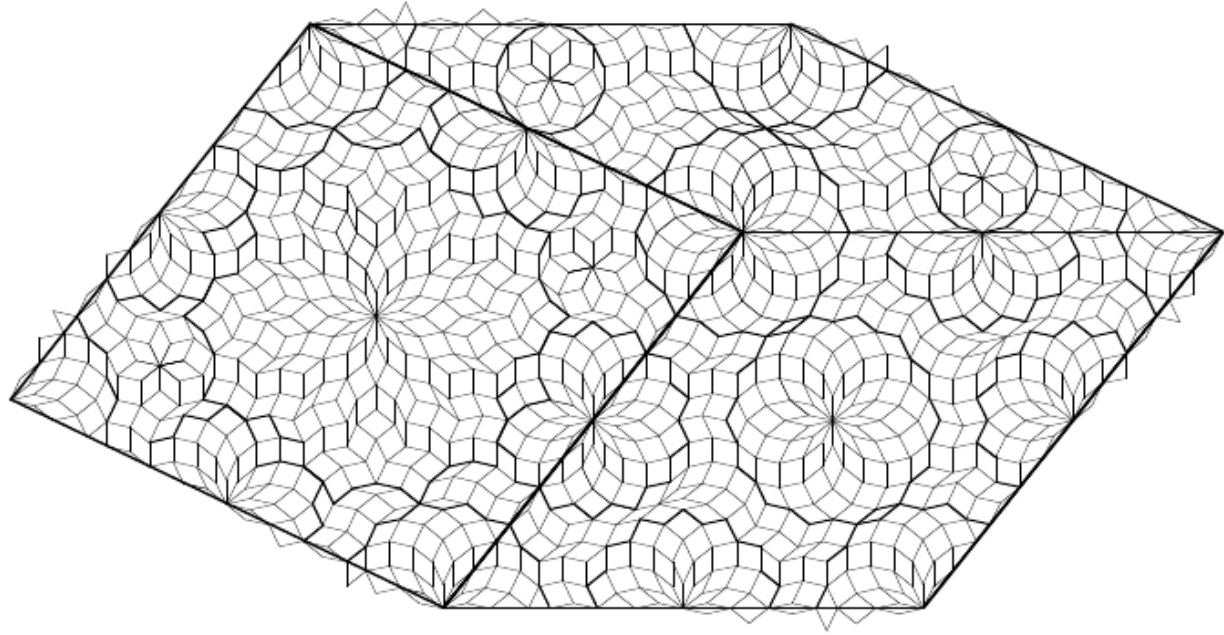
Planar patterns that are **not** covered by Thurston & Conway classification



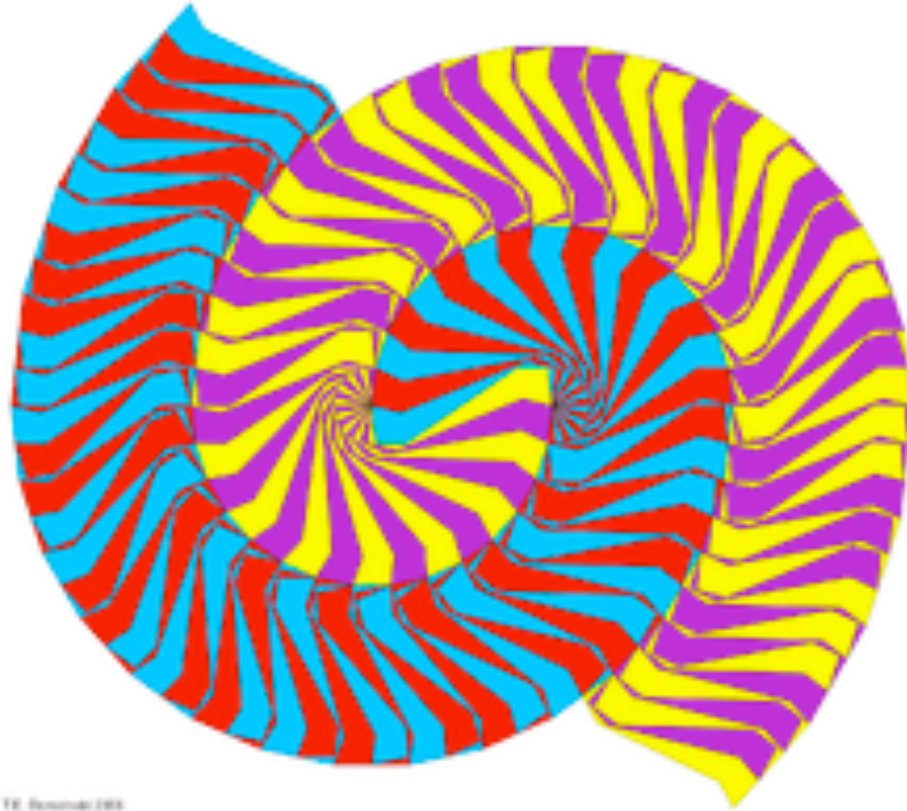
- Penrose rhombuses
- Non-periodic tiling



Sub Rosa by Markus Rissanen

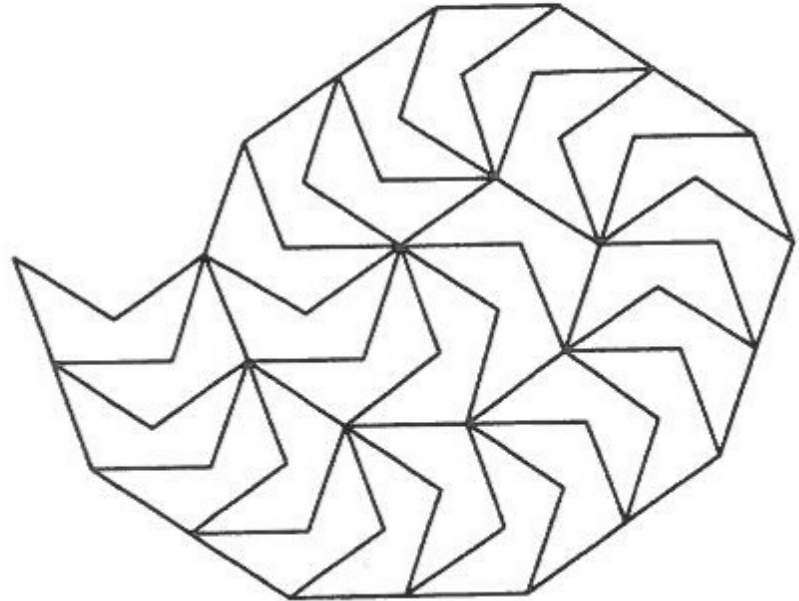
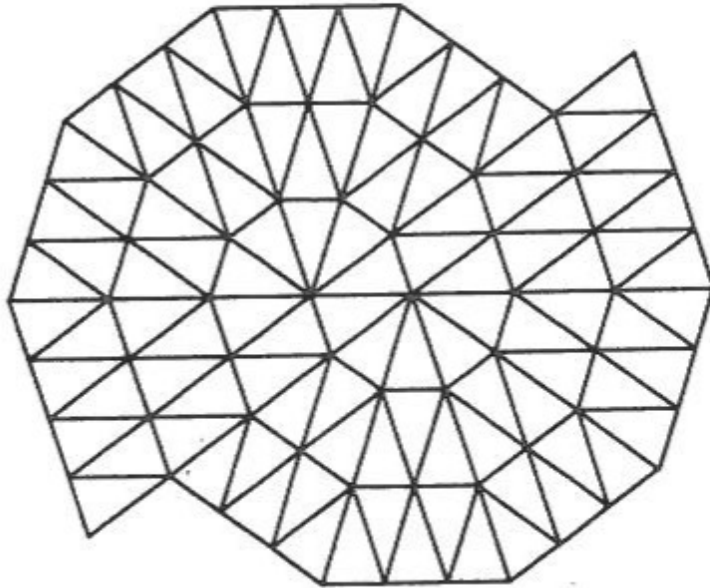


Voderberg's monohedral tiling (1936)

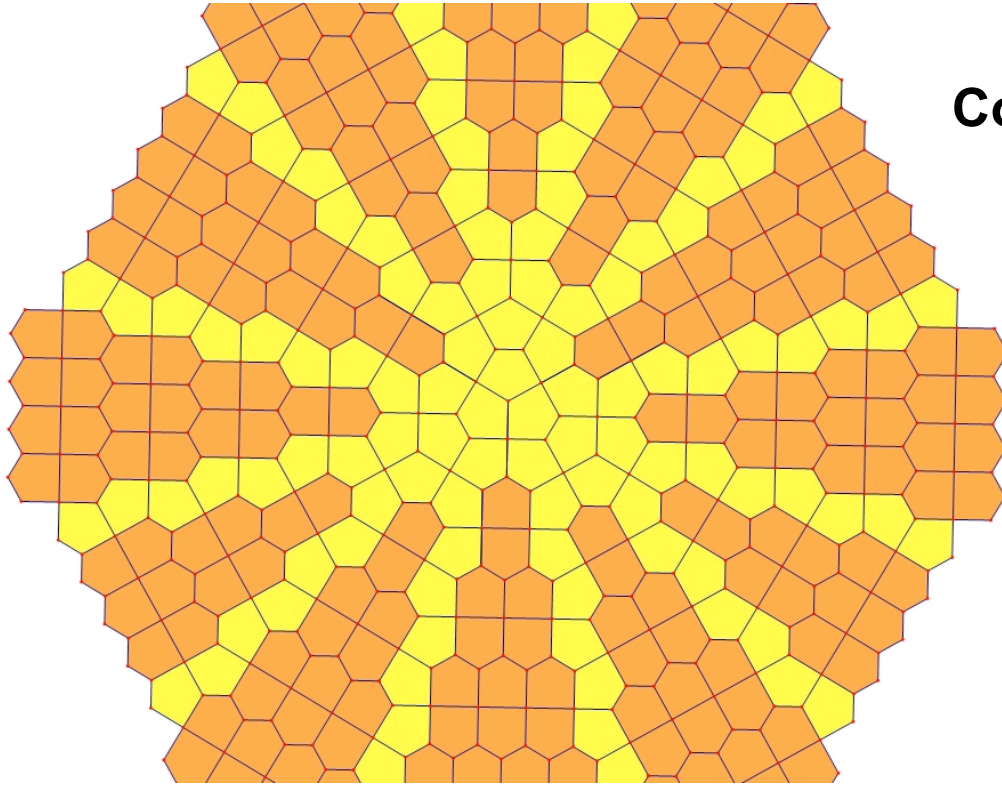


TE, December 2008

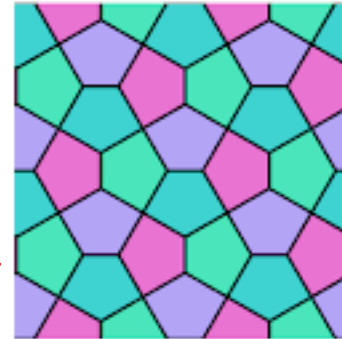
Other spiraling monohedral tilings



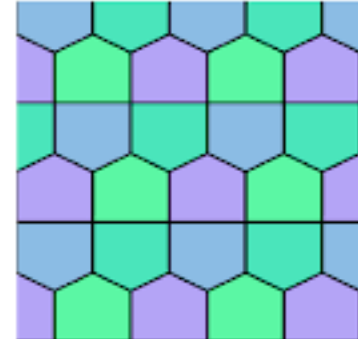
Marjorie Rice non-periodic tiling (70')



Combination of two pentagons



Cairo Pentagonal
Tiling



Prismatic Pentagonal
Tiling

Ingredients of the Thurston-Conway Signature/Orbifold notation

**Goal: find unique names for the regular
pattern classes that can be found by
analysing pictures**

Star *

Star * (in the signature notation) denotes a *mirror* or *kaleidoscopic symmetry* = reflection with respect to a line.

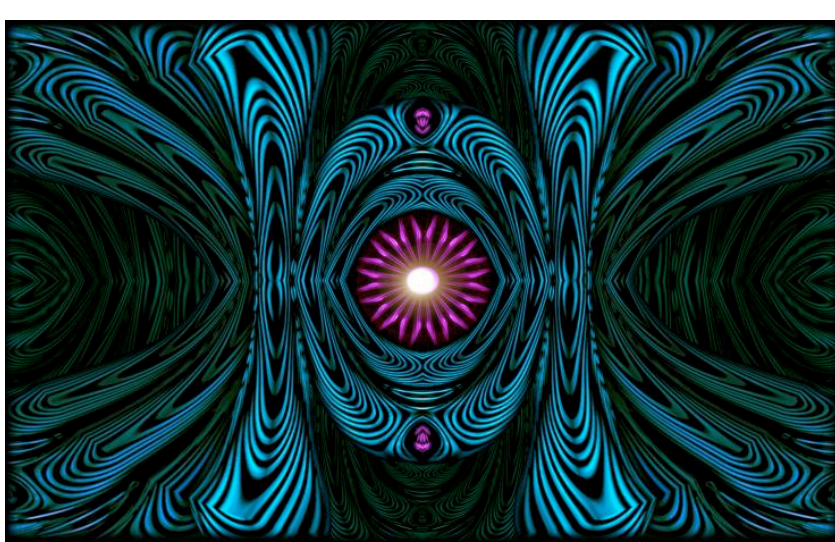
Star alone means: there is one (and only one) single line of mirror symmetry.



Point •

Point • indicates that all the symmetries fix a point

‘Star two point symmetry’ *2• (=period two kaleidoscopic symmetry fixing a point) *8•, *1• = *•, • = no symmetry (except trivial)



Gyrations=rotations with respect to a point

'Period three gyrational point symmetry' = $3\bullet$, $2\bullet$, $4\bullet$, $6\bullet$, $1\bullet = \bullet =$ no symmetry (except trivial)



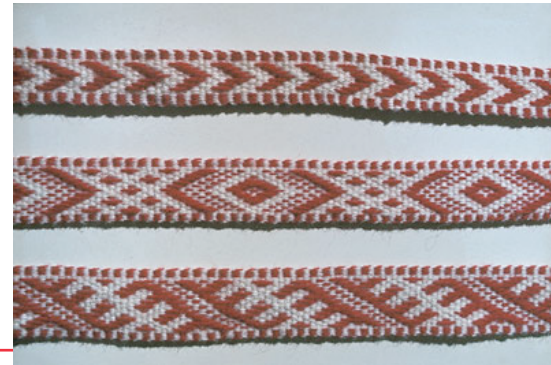
Finite rosette patterns (2D point groups)

Classification: Can be described by *signatures* $*N\bullet$ and $N\bullet$,
 $N=1,2,3,\dots$



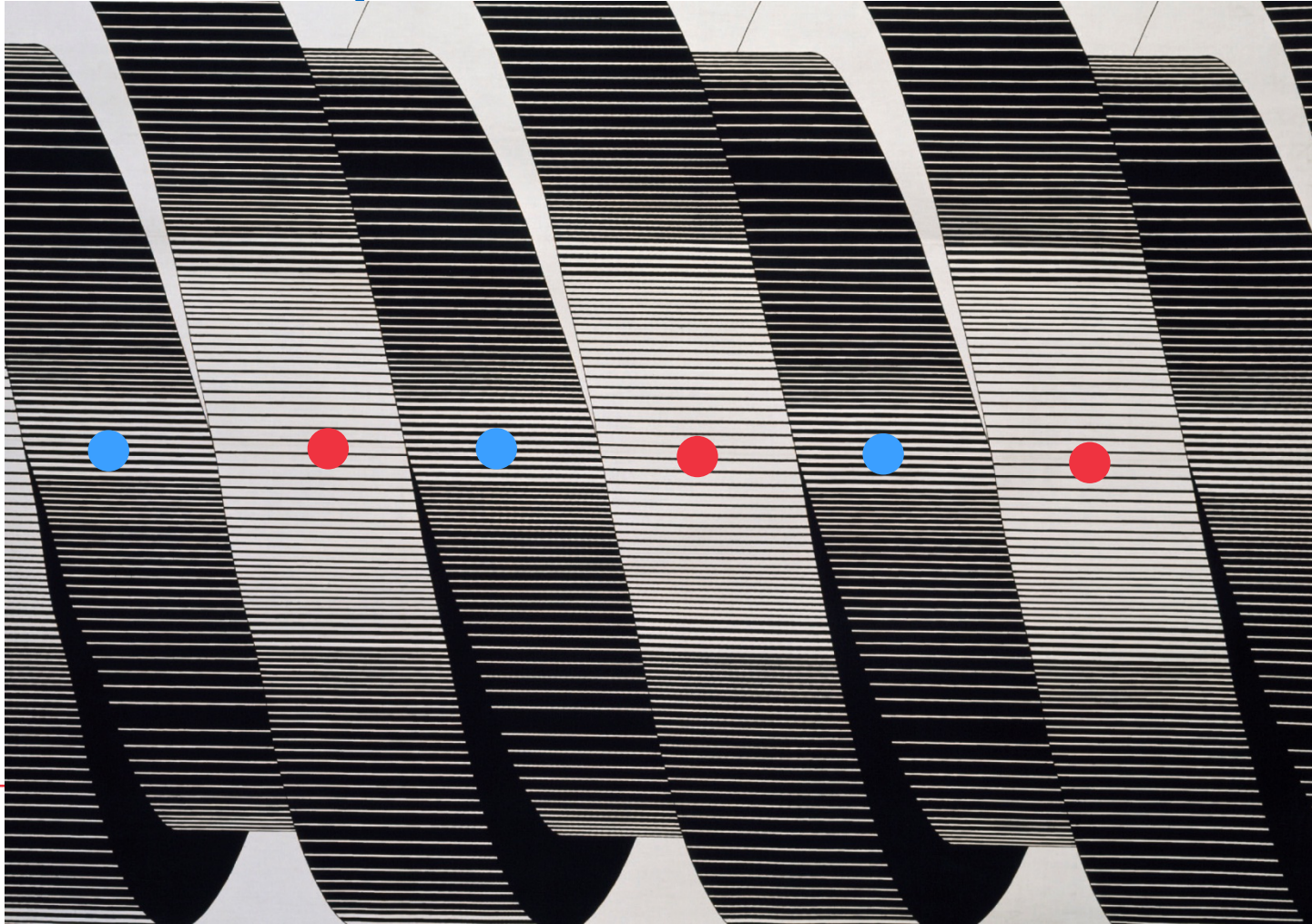
Frieze patterns

Classification: 7 types (will see a bit later)



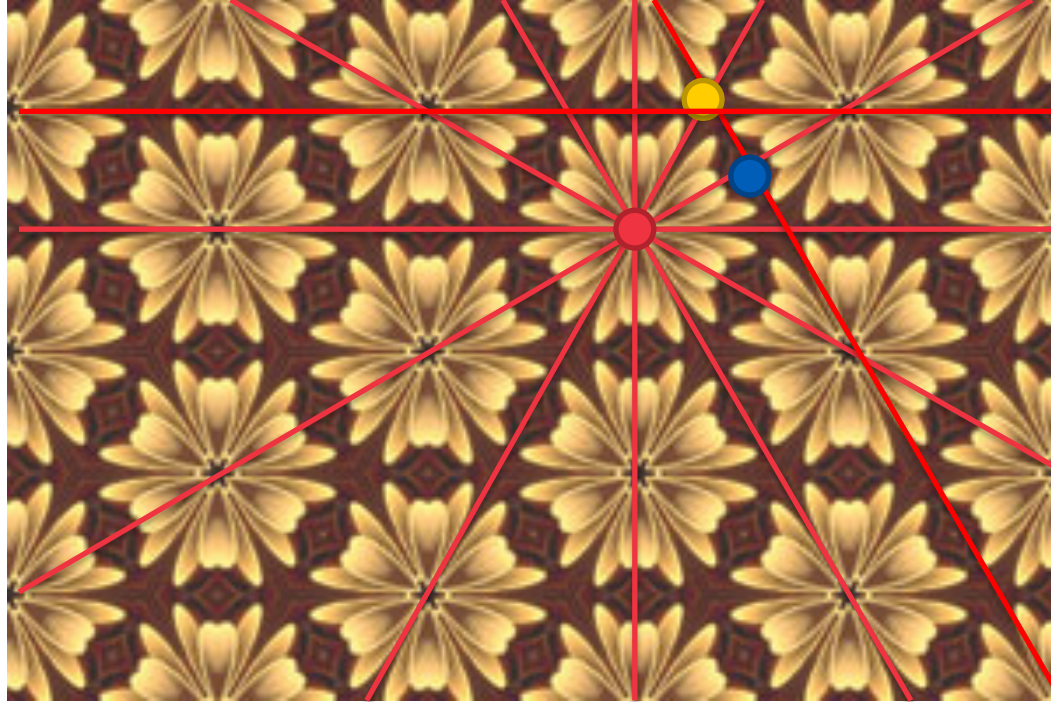
Interpretation as a frieze pattern (from Laura's earlier slides)

2 rotation
points of order 2



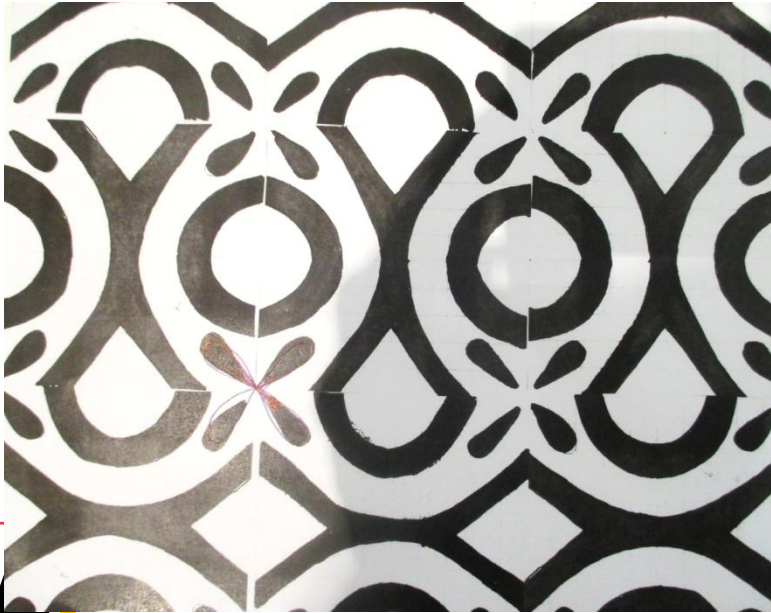
Signatures for plane patterns through *local* symmetries: *632

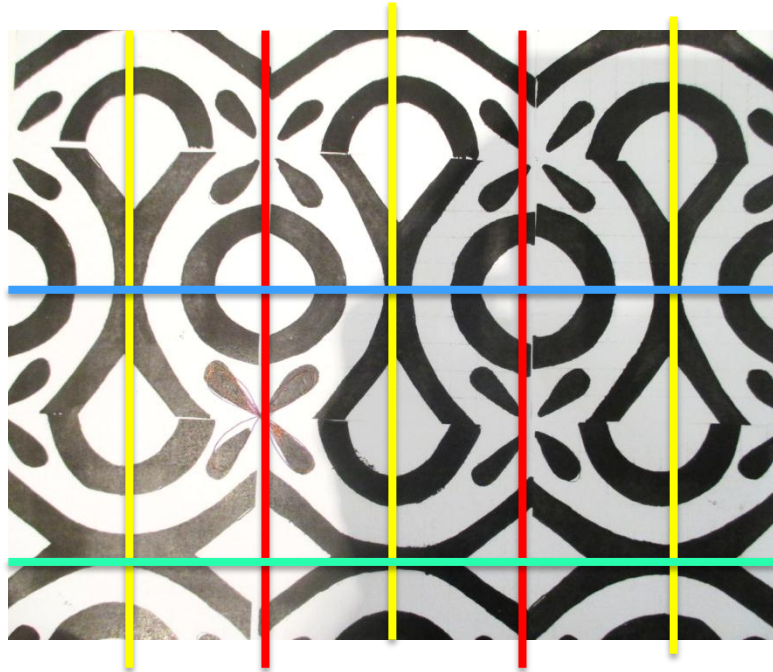
- 6 lines of reflection
- 3 lines of reflection
- 2 lines of reflection



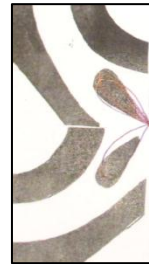
An example from Laura's presentation Sept 8th

'Designers module/tool' to construct patterns

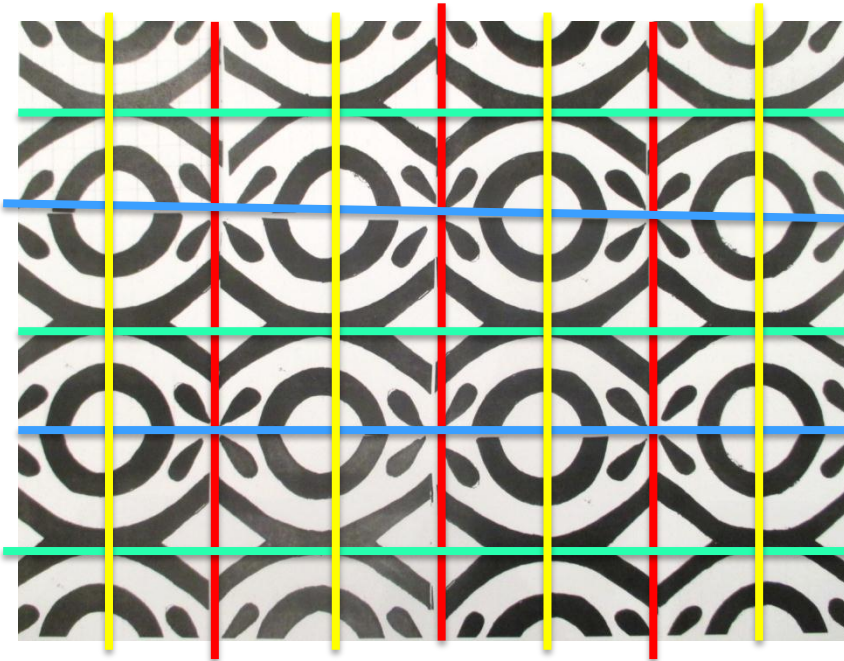




- reflection lines bound the **fundamental domain** (parallelogram below) meeting in four different types of vertices
=> *2222

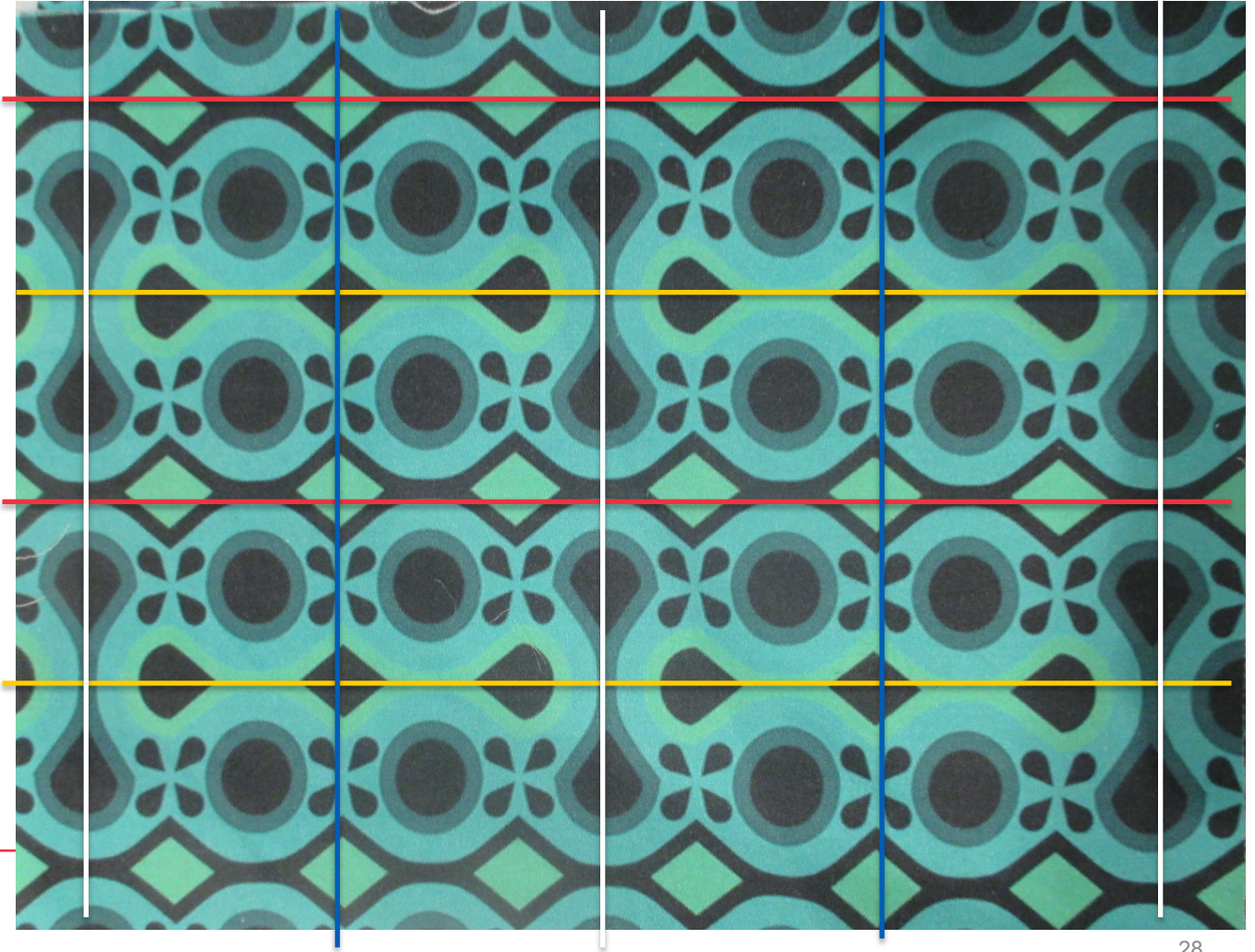


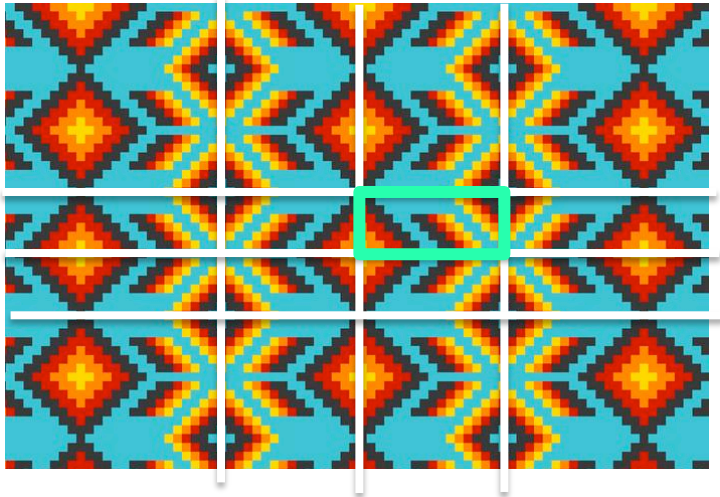
Different pattern - exactly the same symmetry as the previous one



***2222**

Again *2222

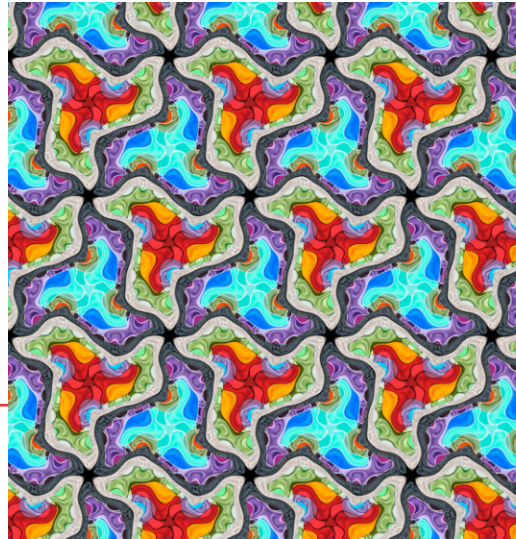




***2222**



2222

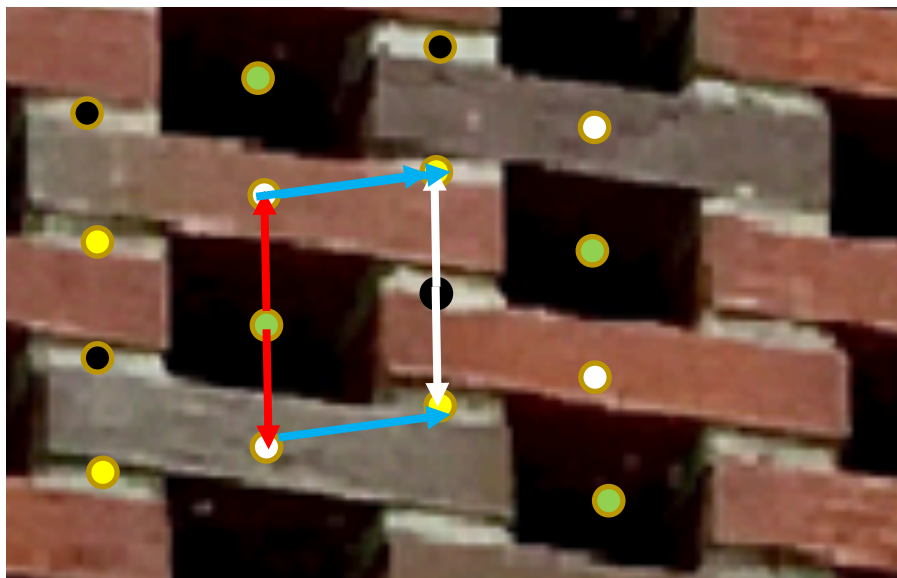


333

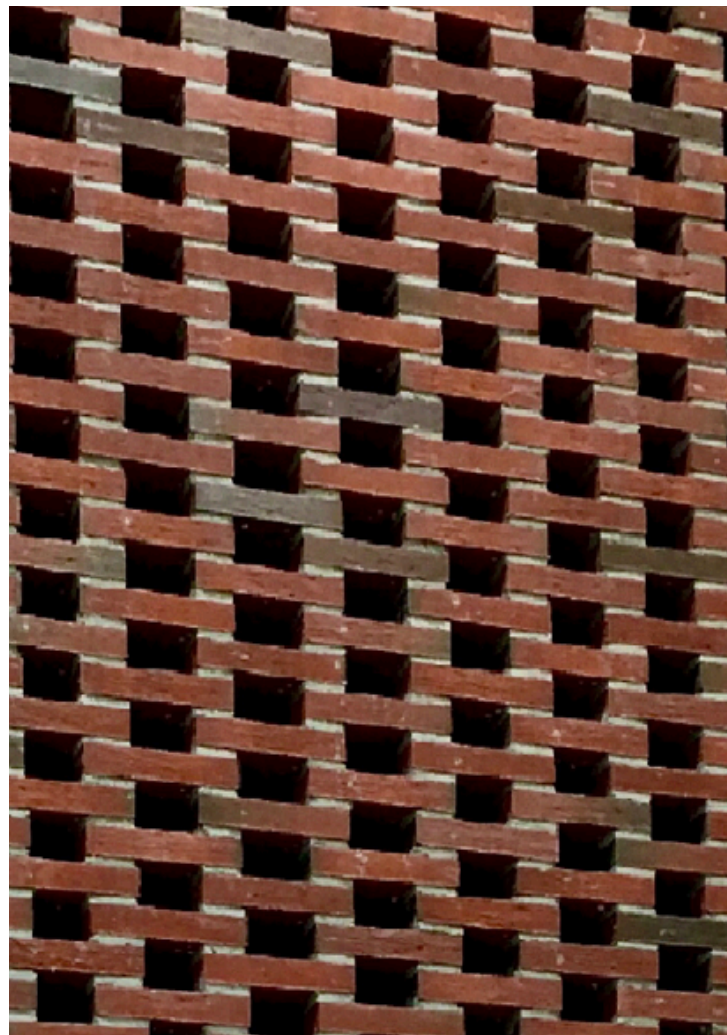
Picture by Toni Monahan: Detail from Väre

Ignore colour differences/details in bricks and cement

4 different rotation points of order two



2222

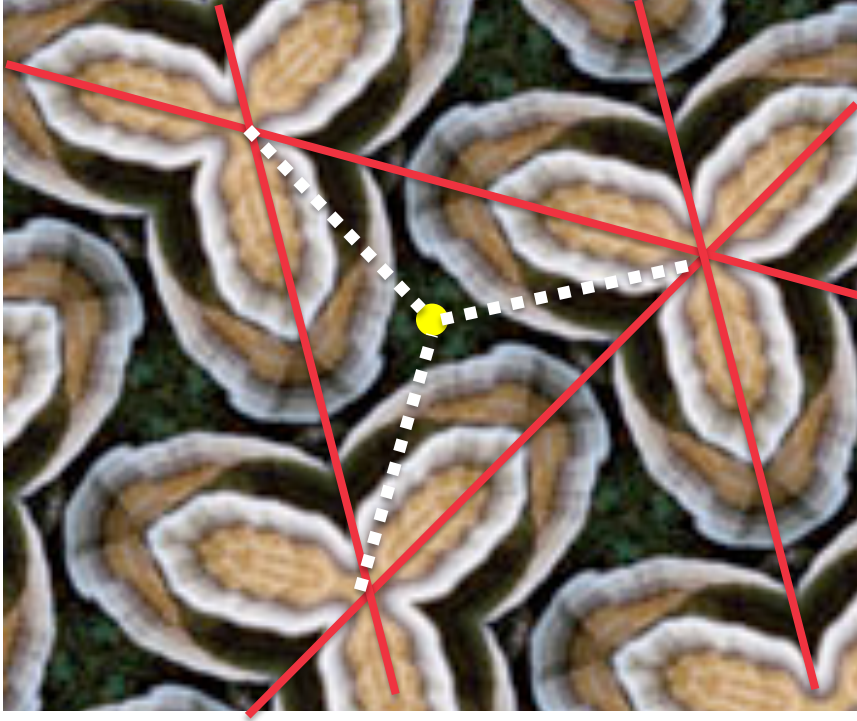


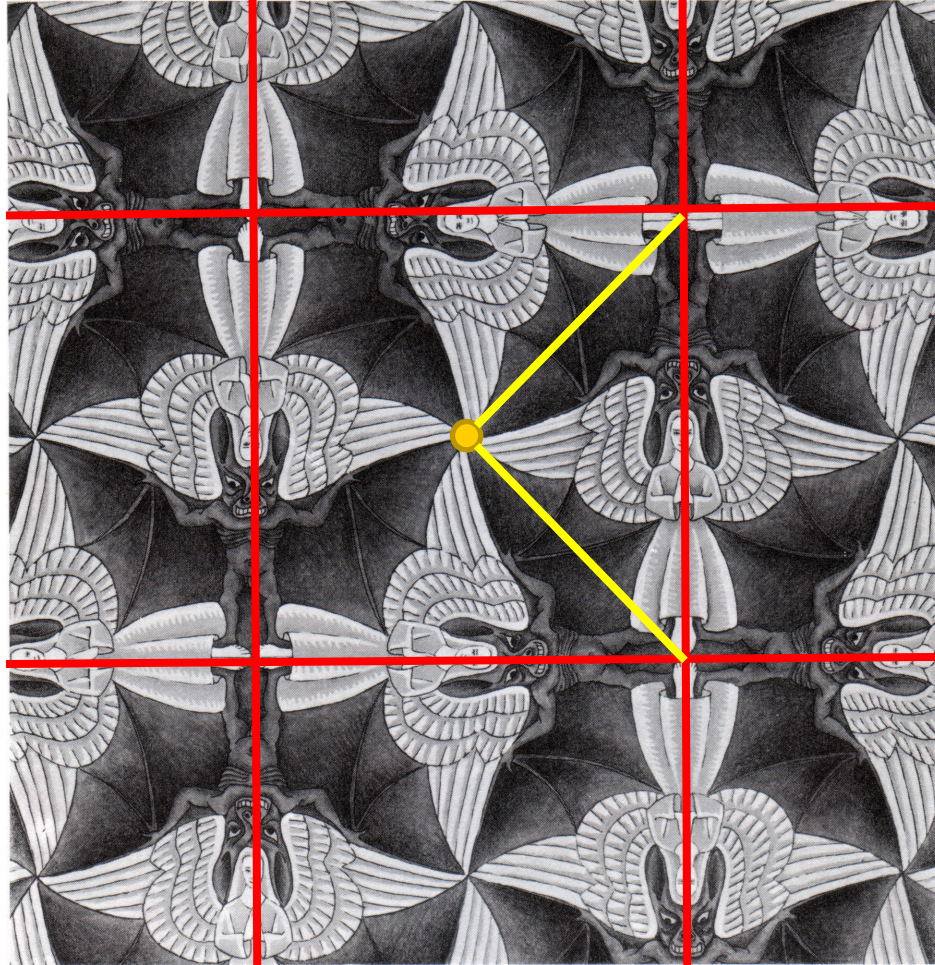
Describing kaleidoscopes with possible rotations

- Draw all mirror lines (=lines of reflection)
- Find the fundamental domain of the kaleidoscope
- How many lines meet at each vertex? => Local symmetries of form ***N**
- Find rotationally symmetric point (non-kaleidoscopic)

Kaleidoscopic vertices are of the *same type* if they have same number of reflection lines **and** they can be interchanged by a rigid motion (=translation, reflection)

3*3 means: one rotation point of order 3,
one vertex with **3 reflection lines**

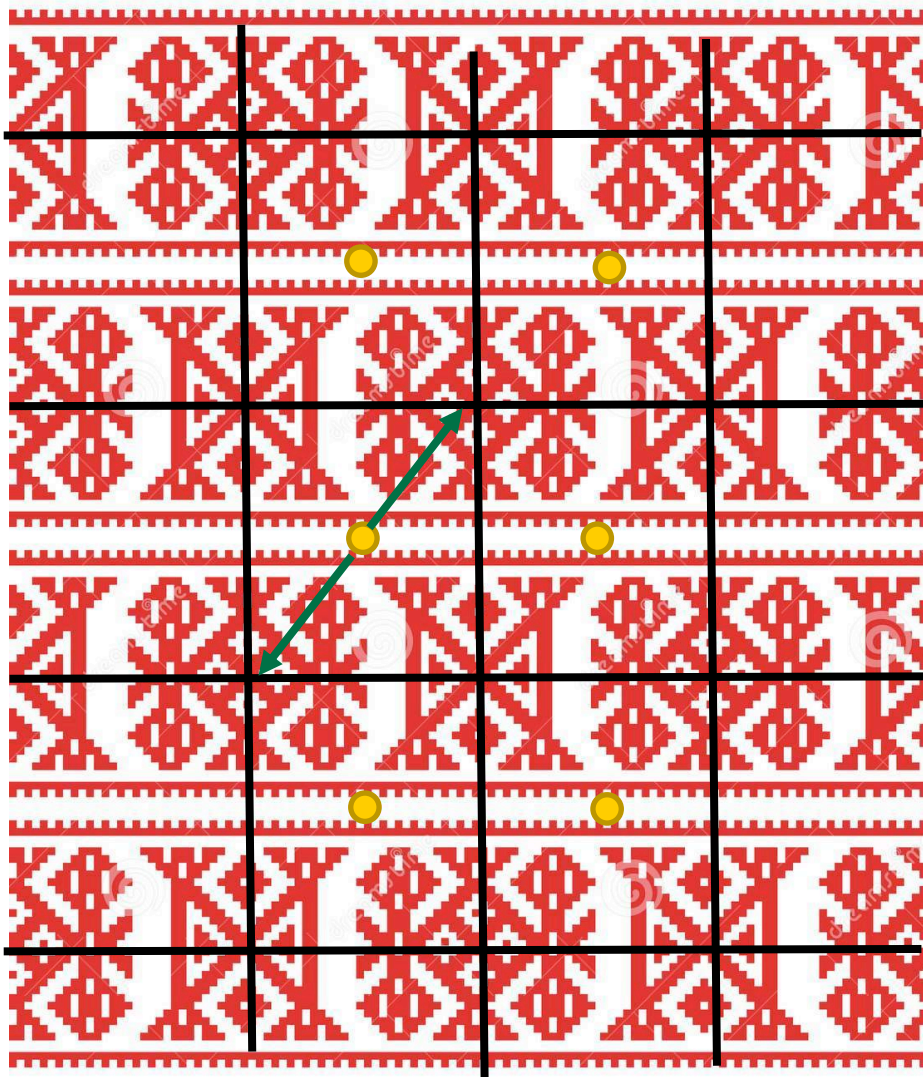




$4*2$

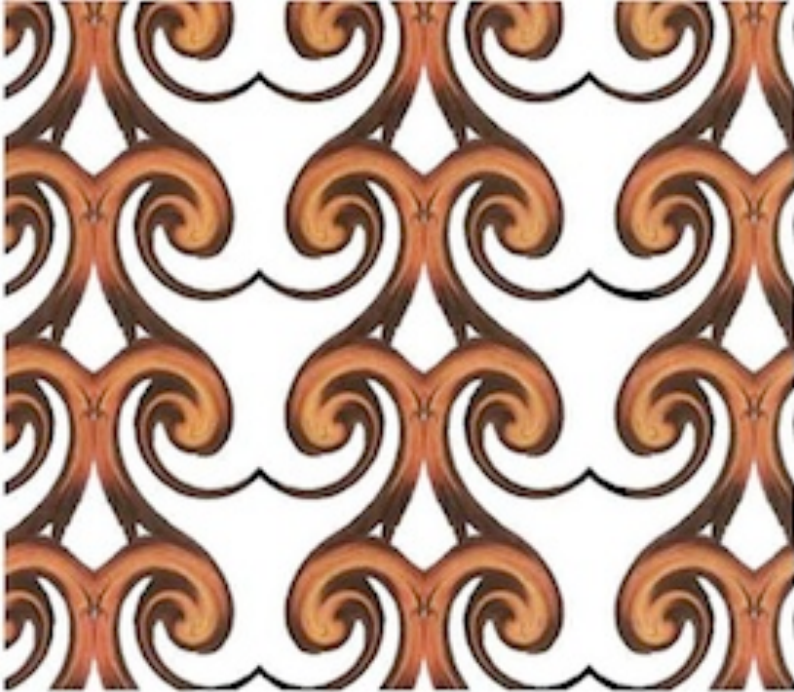
Fundamental domain spanned by

- A segment of a red reflection line
- one rotation point of order 4



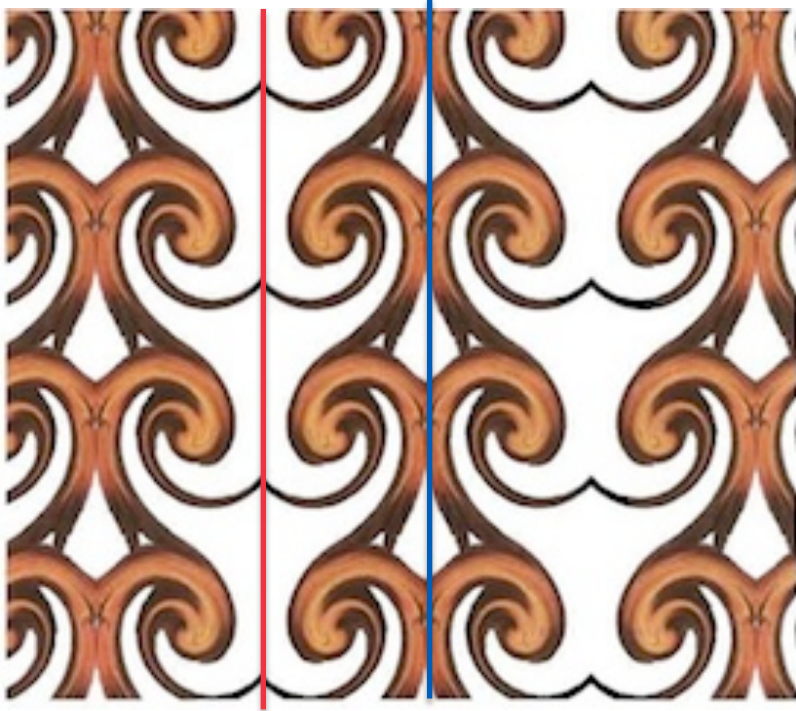
$2^{*}22$

More mirrors * : What is the number of different kinds of reflection lines?



What is the fundamental domain of these ?

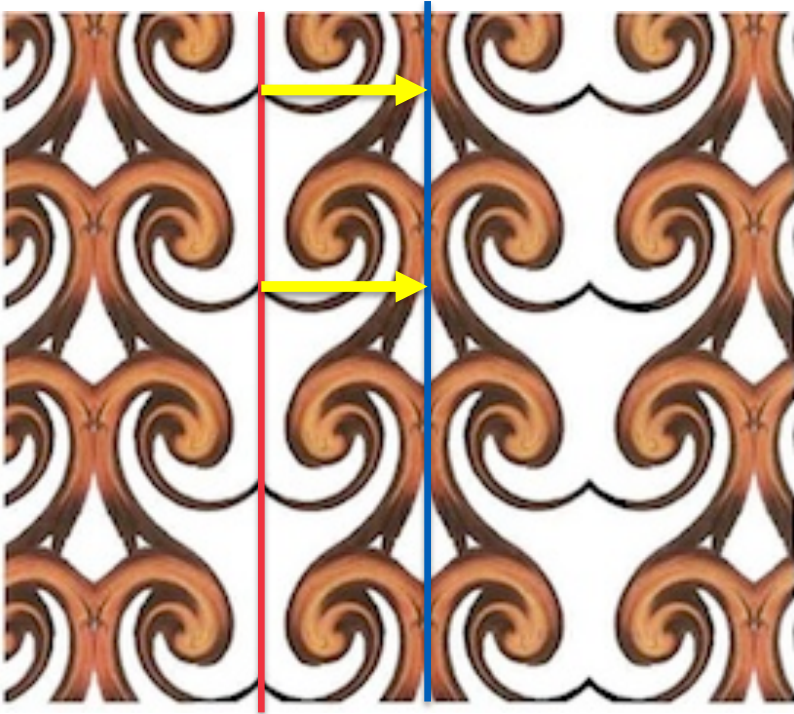
Signature **



What is the signature of this ?



Find a piece that is repeated between the reflection lines

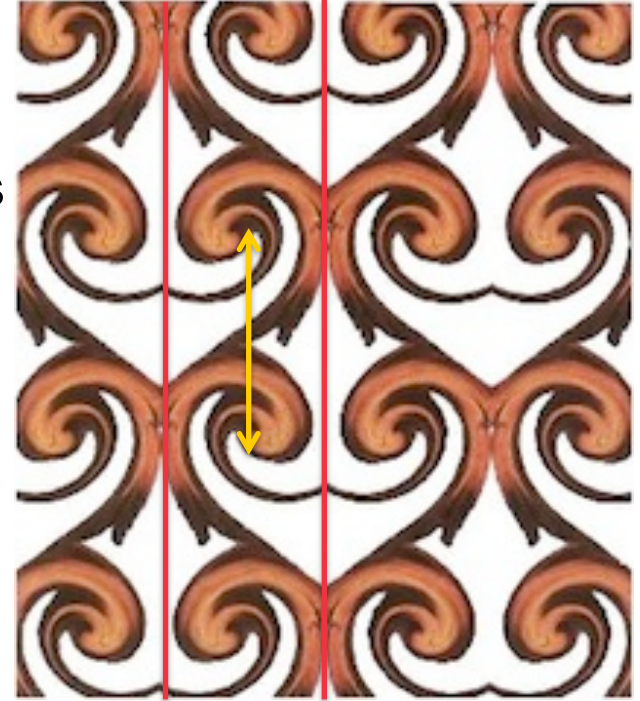


Note the info on the boundary to get the whole tiling from the fundamental domain

One reflection line and a Miracle **x**

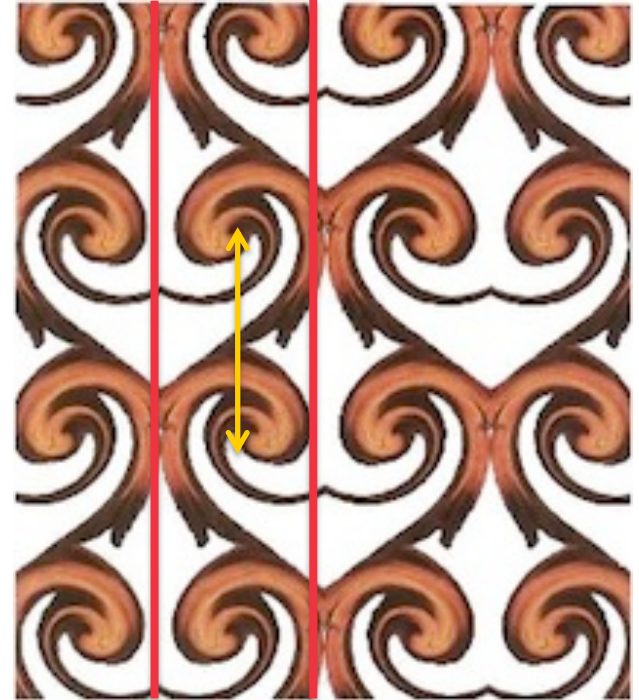
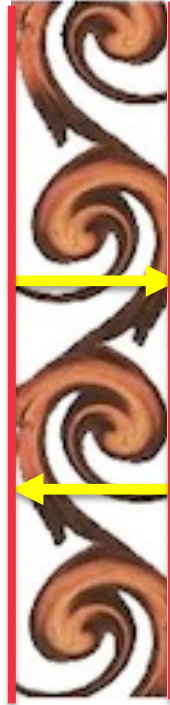
x : between the reflection lines (of the same type) two oppositely oriented patterns that can be connected with a path without crossing the lines

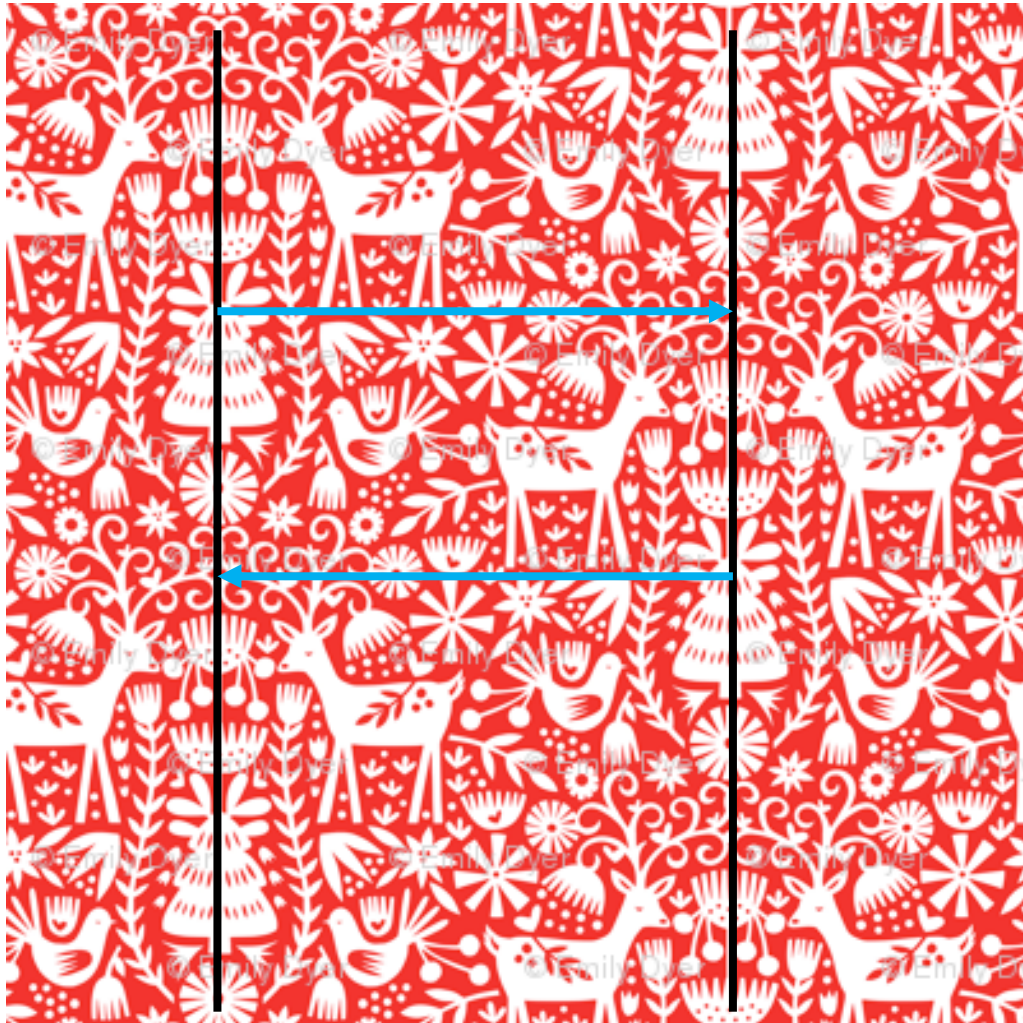
The signature of this pattern
Is ***x**



How to choose the fundamental domain ?

Note again the info on the boundary (compare to the previous pattern)

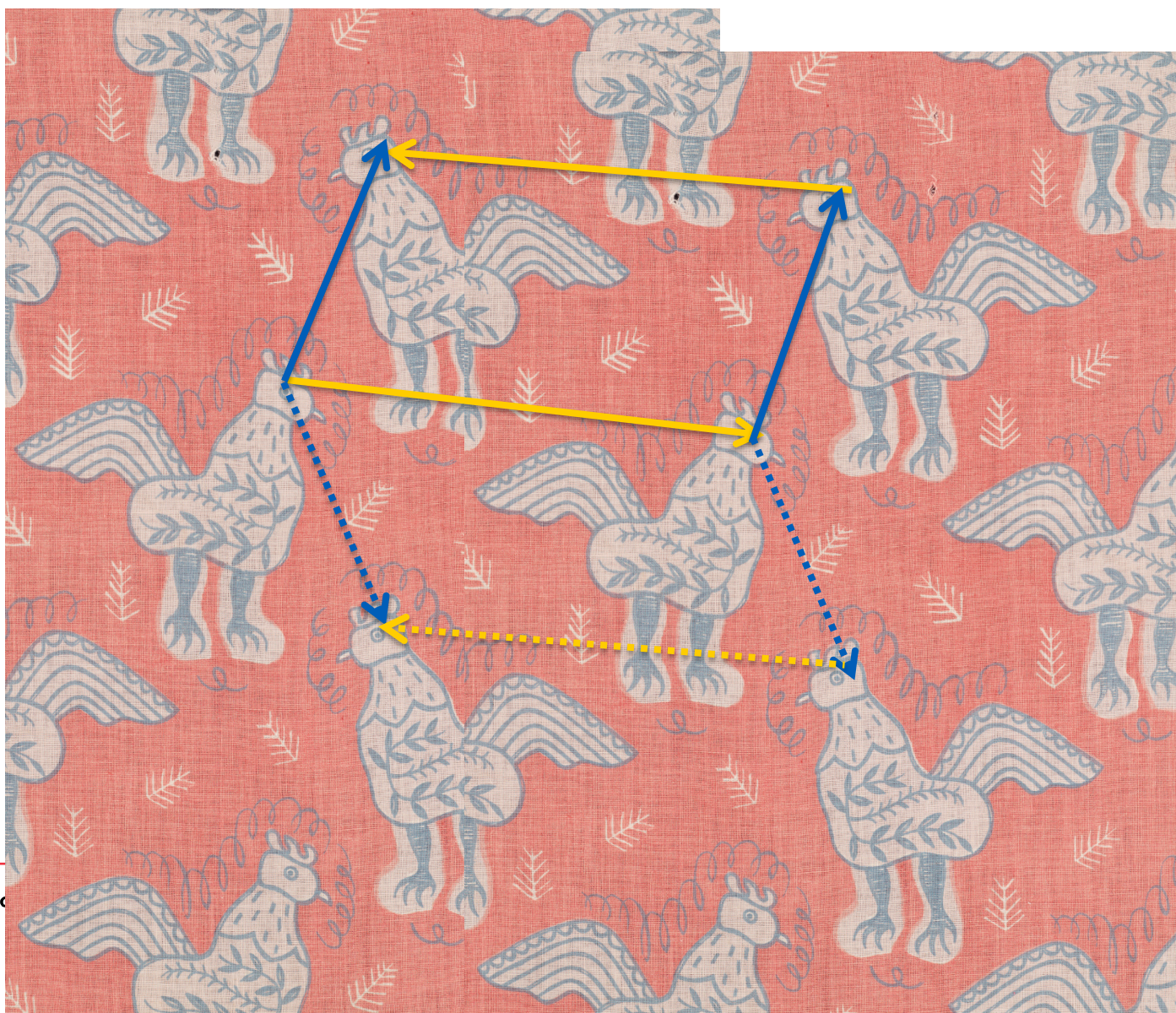


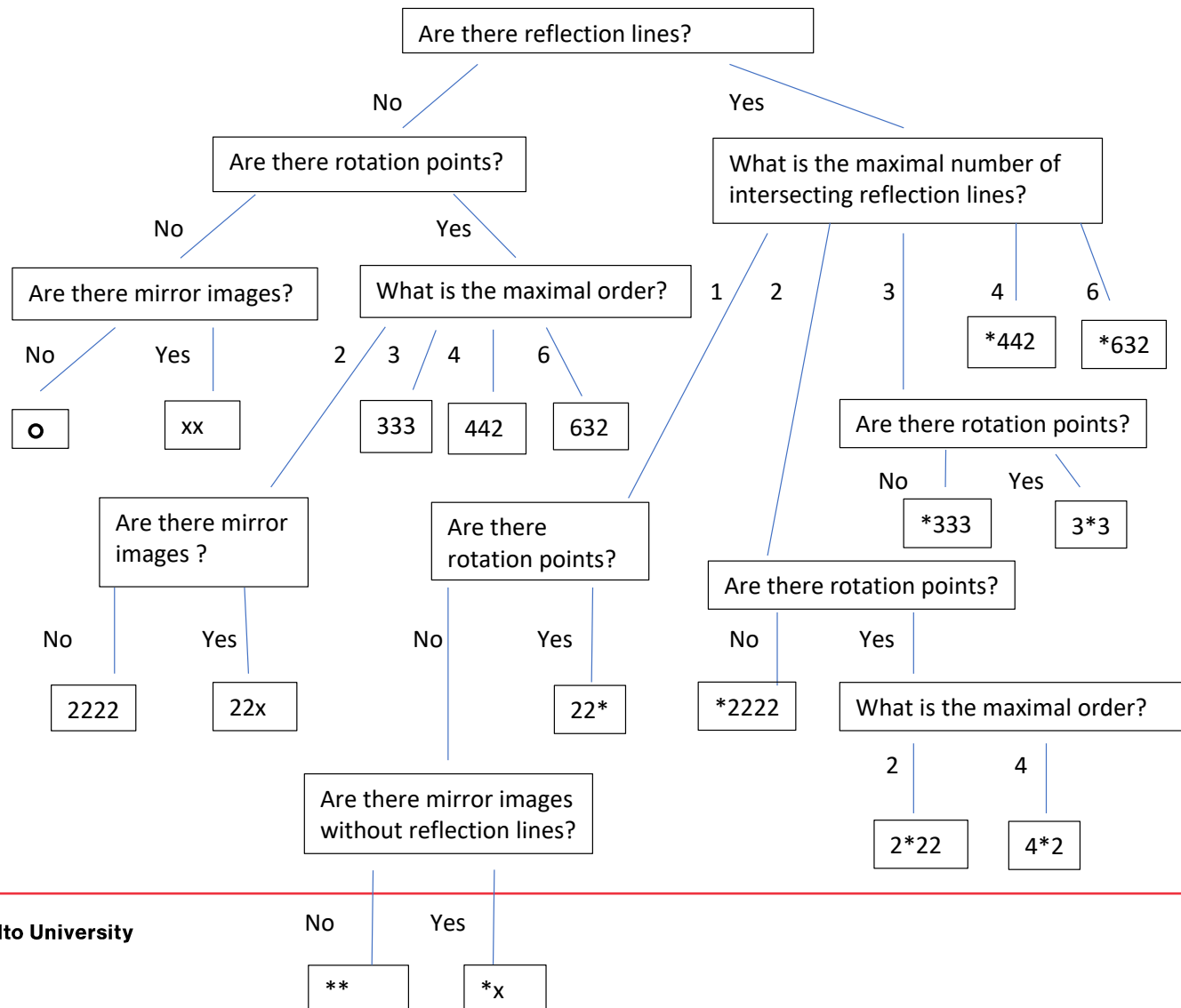


Signature *x

Two Miracles **xx** . What is the fundamental domain?







Home work by Tue 22nd

New pictures in file Pictures220920.pdf

Please

- 1. Find the signature of the pattern**
- 2. Find a Fundamental domain for the pattern**

Possible group work after Laura's instructions

- 1. Introduce yourself to your group mates**
- 2. Discuss about your experience so far**
- 3. Share your textile workshop outcomes and thoughts about them**

Note: Updated (14.9) group list in the file in MyCourses front page