Worksheet 2

MS-E1621, Algebraic Statistics

September 16, 2020

Group members: Write your names here.

Discuss the questions in the group. You may use any materials (course textbook, other books, Internet etc). The group can decide whether to write down solutions or only discuss them.

1 Your questions

Copy your questions here.

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2 Further questions

1. Page 12: Why do the following properties follow from Definition 2.1.1?

   - $P(\emptyset) = 0$
   - $P(A_1 \cup A_2) = P(A_1) + P(A_2)$ if $A_1 \cap A_2 = \emptyset$
   - $P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 \cap A_2)$
   - $P(A_1) \leq P(A_2)$ if $A_1 \subseteq A_2$

2. Page 14: What is the Axiom of Choice? How is it used in the proof of Proposition 2.1.3?

3. Page 14: What is meant by the nonexistence of a "uniform distribution" on $\mathcal{P}([0, 1))$?

4. Page 14: What is the motivation for defining σ-algebras?

5. Page 15: Why does a density function define a probability measure?

6. Page 15: Is a density function of a probability measure unique?

7. Page 16: Is there a simpler form for the Bayes' rule?

8. Page 17: How does one derive

$$P(\bigcap_{i \in I} A_i | \bigcap_{j \in J} A_j) = P(\bigcap_{i \in I} A_i)$$

from Definition 2.1.13?

9. Page 18: A random variable is informally described as a variable whose values depend on the outcome of a random phenomenon. How does this relate to the definition of a random variable as a function in Definition 2.2.1?

10. Page 19: What is meant by the monotonocity and continuity properties that a cdf exhibits?

11. Page 19: Does every random vector have a density function?

12. Page 20: What is the difference of Definition 2.2.6 and Definition 2.1.13?

13. Page 24: Why do the equalities in (2.3.1) and (2.3.2) hold?

14. Page 25: If two random variables have zero covariance, are they independent?

15. Page 25: How does one derive equation (2.3.4)?

16. Page 26: Why $E[X_1^2] = E[X_1]$ in Example 2.3.10?

17. Page 26: What is the motivation for defining correlation?

18. Page 26: Why does the equality on the last line of the page 26 hold?

19. Page 27: What is the Cauchy-Schwarz inequality? Why the inequality in Remark 2.3.15 is equivalent to it?

20. Page 29: What is the moment generating function $E[\exp(tX)]$?