Aalto University
School of Business

Intermediate Microeconomics

Costs

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Fall 2020

ECON-2100

Economies of Scale

What is the relation of average costs and level of output?

$\mathrm{AC}(q) = \mathrm{TC}(q)/q$

- Decreasing Returns to Scale (DRS) aka diseconomies of scale
  $\partial \mathrm{AC}(q)/\partial q > 0 \longleftrightarrow \mathrm{MC}(q) > \mathrm{AC}(q)$
- Constant Returns to Scale (CRS)
  $\partial \mathrm{AC}(q)/\partial q = 0 \longleftrightarrow \mathrm{MC}(q) = \mathrm{AC}(q)$
- Increasing Returns to Scale (IRS)
  $\partial \mathrm{AC}(q)/\partial q < 0 \longleftrightarrow \mathrm{MC}(q) < \mathrm{AC}(q)$

Economies of scale vs shifts in cost curve

- Tech progress shifts TC down

- Changes in input prices shift TC

- IRS vs learning-by-doing

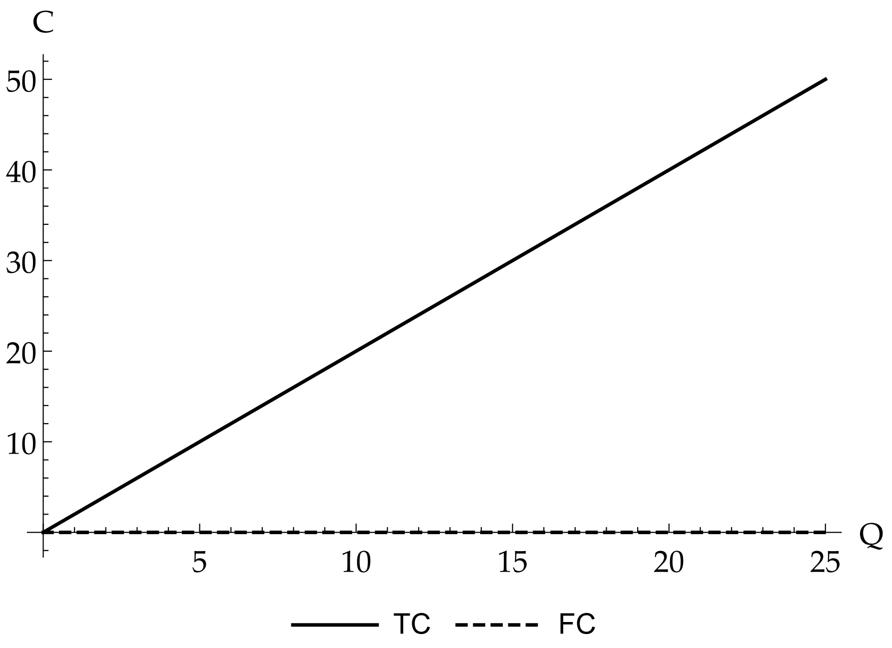
Economies of Scale

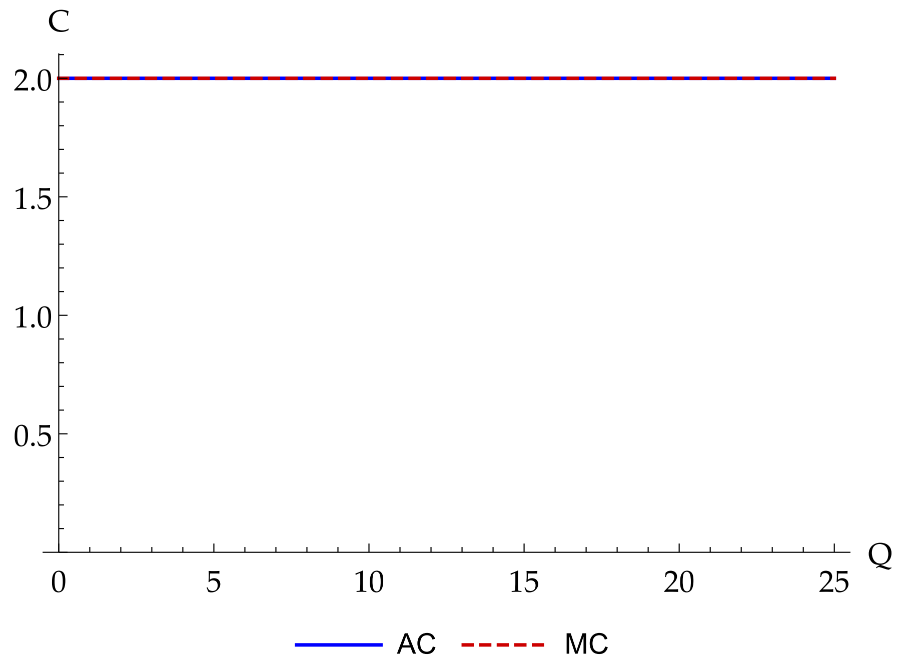
What happens to total cost, when output increased by factor $k > 1$

- DRS: $\mathrm{TC}(kq) > k\mathrm{TC}(q)$
- CRS: $\mathrm{TC}(kq) = k\mathrm{TC}(q)$
- IRS: $\mathrm{TC}(kq) > k\mathrm{TC}(q)$

Similarly: what happens to total output, when the quantity of all inputs are multiplied by same factor $k > 1$ (“all” is tricky)

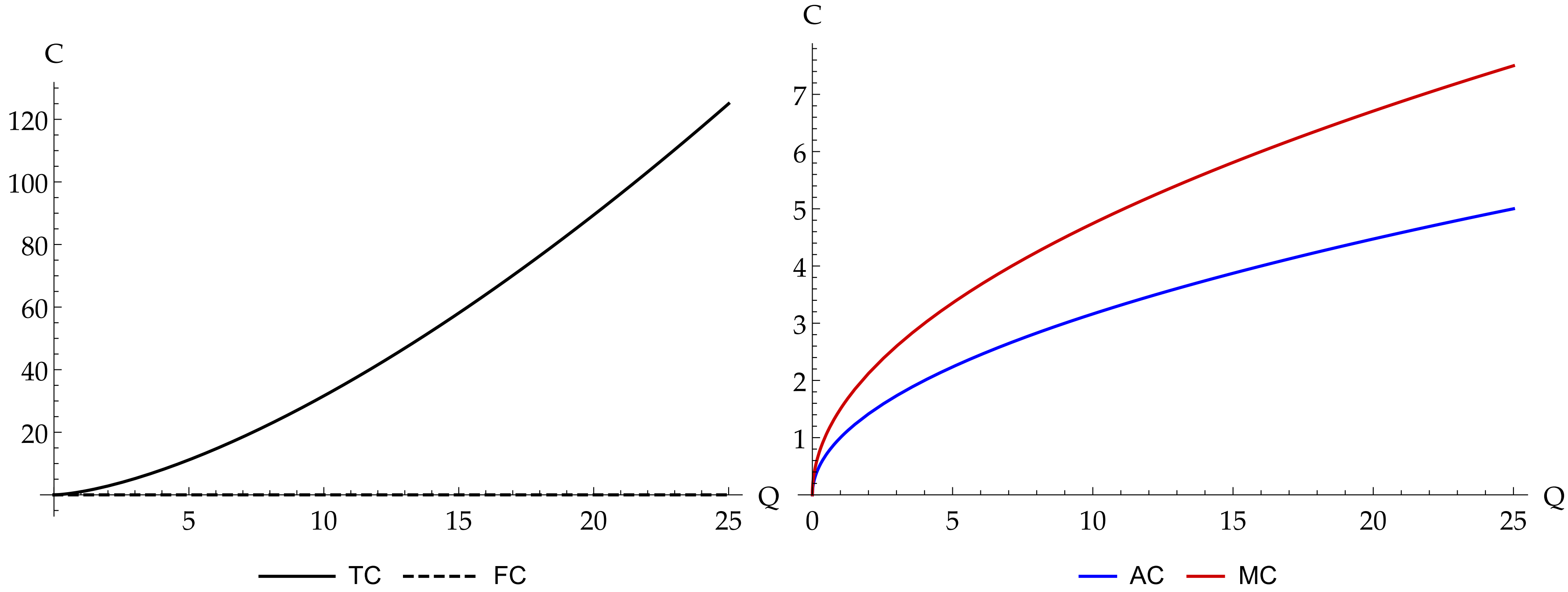
Constant Returns to Scale: Example





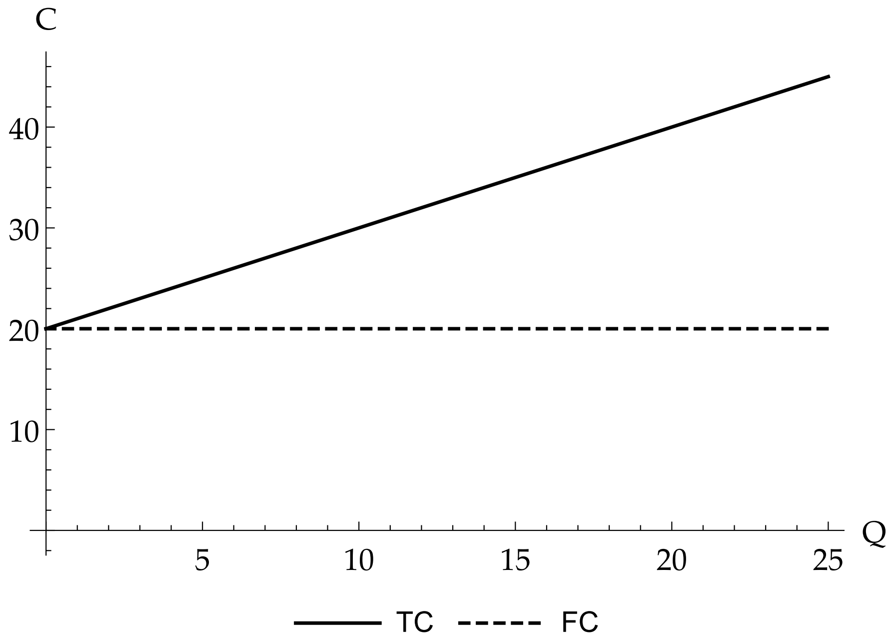
$$\text{TC}(q) = 2q \Longrightarrow \text{AC}(q) = 2, \text{MC}(q) = 2$$

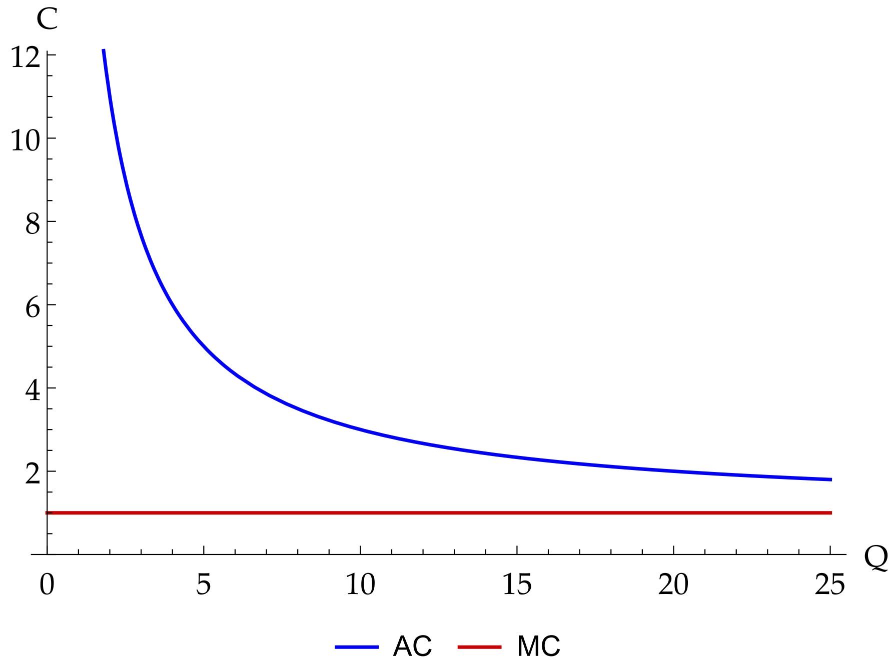
Decreasing Returns to Scale: Example



$$\mathrm{TC}(q) = \sqrt{q^3} \Longrightarrow \mathrm{AC}(q) = \sqrt{q}, \mathrm{MC}(q) = \frac{3}{2}\sqrt{q}$$

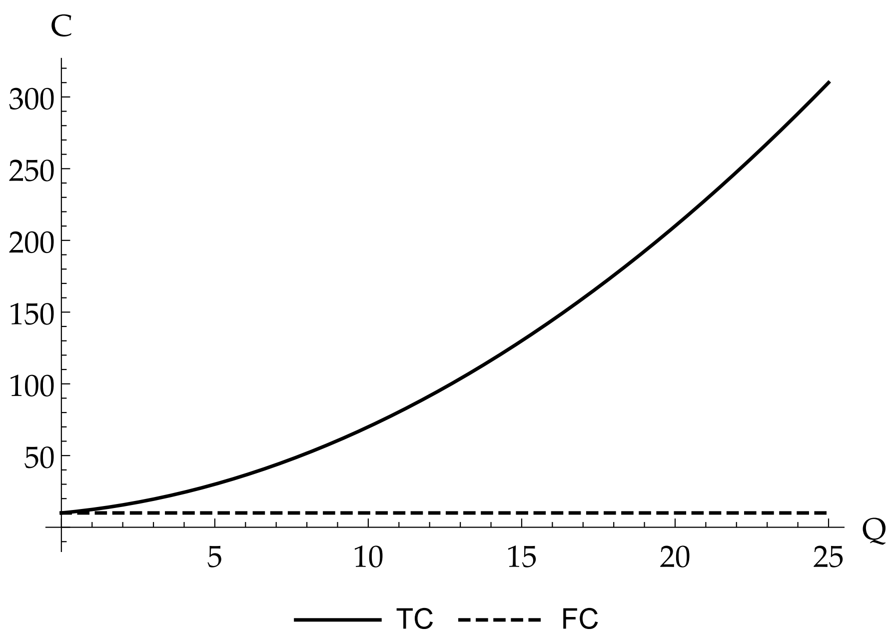
Increasing Returns to Scale: Example

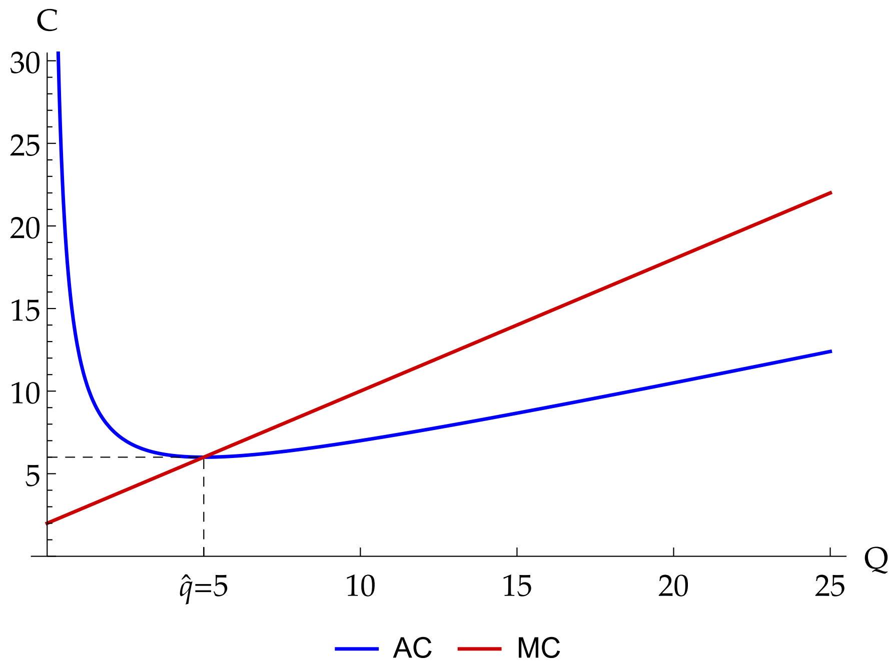




$$\mathrm{TC}(q) = 20 + q \Longrightarrow \mathrm{AC}(q) = \frac{20}{q} + 1, \mathrm{MC}(q) = 1$$

First IRS, then DRS: Example





$$\mathrm{TC}(q) = 10 + 2q + \tfrac{2}{5}q^2 \implies \mathrm{AC}(q) = 2 + \tfrac{10}{q} + \tfrac{2}{5}q, \mathrm{MC}(q) = 2 + \tfrac{4}{5}q$$

$$\text{AC minimized where } \mathrm{AC}(q) = \mathrm{MC}(q) \implies \hat{q} := 5, \quad \mathrm{AC}(\hat{q}) = 6$$

Economies of Scope

aka Synergies

Is total cost of production lower if different goods are produced in one firm (or establishment)

$$\mathrm{TC}(q_1, q_2) < \mathrm{TC}_1(q_1) + \mathrm{TC}_2(q_2)$$

Good reason for mergers and spin-offs

Examples: Chicken and eggs, cable TV and broadband, department store/mall?

Organizational economies and diseconomies of scope, limits to firm size

Shared overhead

Fixed cost aka overhead

A firm with multiple products, operations, orders... may need to "allocate" its fixed costs across those parts for accounting purposes. This allocation should not affect real decisions.

Hierarchies of shared overheads

Example:

Company, Factory, Department, Machine, Product, Order

Shared overhead between orders

Shared overhead and average cost pricing

One machine dedicated to one part:

$\mathrm{FC} = 120$ paid once per time-period, for any quantity > 0

$\mathrm{MC} = 2.0$

1. Suppose we have only one order, for 100 units.

At what price should we accept the order?

Answer: if $P \geq \mathrm{AC}(100)$

$P \geq (120 + 2 \times 100)/100 = 3.2$

e.g. if we can get $P = 3.5$, profits are $100 \times (3.5 - 3.2) = 30$

Shared overhead between orders

2. Suppose we know we'll produce the 100 unit order anyway. Then a chance comes to fill another (independent) order of 50 units. What is the lowest price at which this order is profitable?

Correct answer: if $P' \geq \mathrm{MC} = 2.0$

Common wrong answer: if

$P' \geq \mathrm{AC}(150) = (120 + 2 \times 150)/150 = 2.8$

Why should we accept the second order at $P' = 2.5$?

Profit if we don't accept $= 3.5 \times 100 - (120 + 2 \times 100) = 30$

Profit if we accept $= (3.5 \times 100 + 2.5 \times 50) - (120 + 2 \times 150) = 55$

The additional profit is $(P' - \mathrm{MC}) \times 50 = (2.5 - 2) \times 50 = 25$

Requiring all orders to cover "their share" of FC reduces profit!

Shared overhead between orders

Why do we calculate economic costs? To inform our decisions of what to do.

Different orders are usually not really "first" orders and "seconds."

What do we do when we know that we can get two orders per year, call them order *a* and order *b*?

3. Suppose the prices we could get are some P_a, P_b.
Order sizes are fixed at 100 for a and 50 for *b*.
Costs are still $\mathrm{FC} = 120$, $\mathrm{MC} = 2$.

At what combination of prices should we accept one or both deals?

One product, two deals

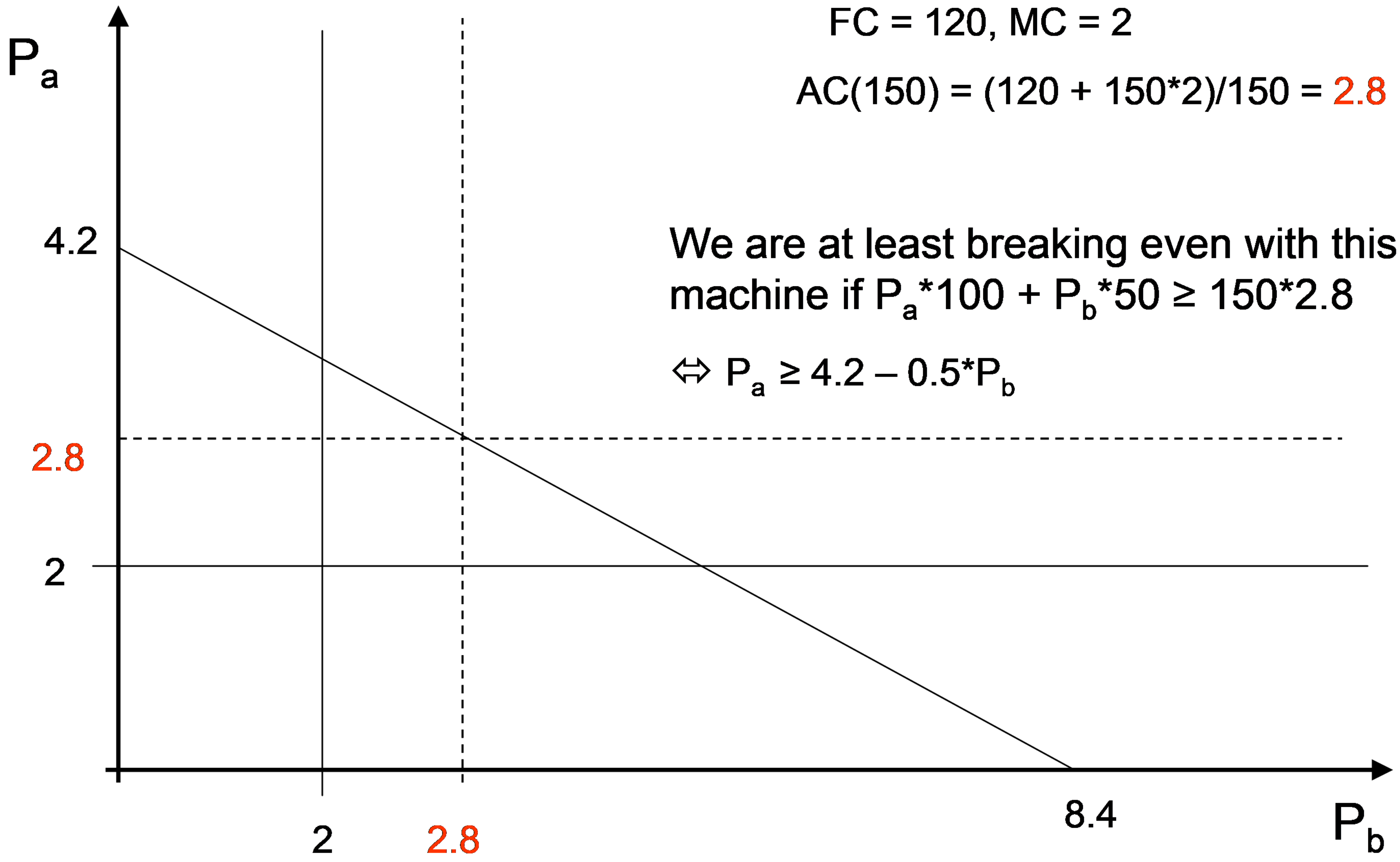
At what prices should we operate?

Order a: 100 units

Order b: 50 units

FC = 120, MC = 2

AC(150) = (120 + 150*2)/150 = 2.8



We are at least breaking even with this machine if $P_a*100 + P_b*50 \geq 150*2.8$

$\Leftrightarrow P_a \geq 4.2 - 0.5*P_b$

One product, two deals

Order a: 100 units

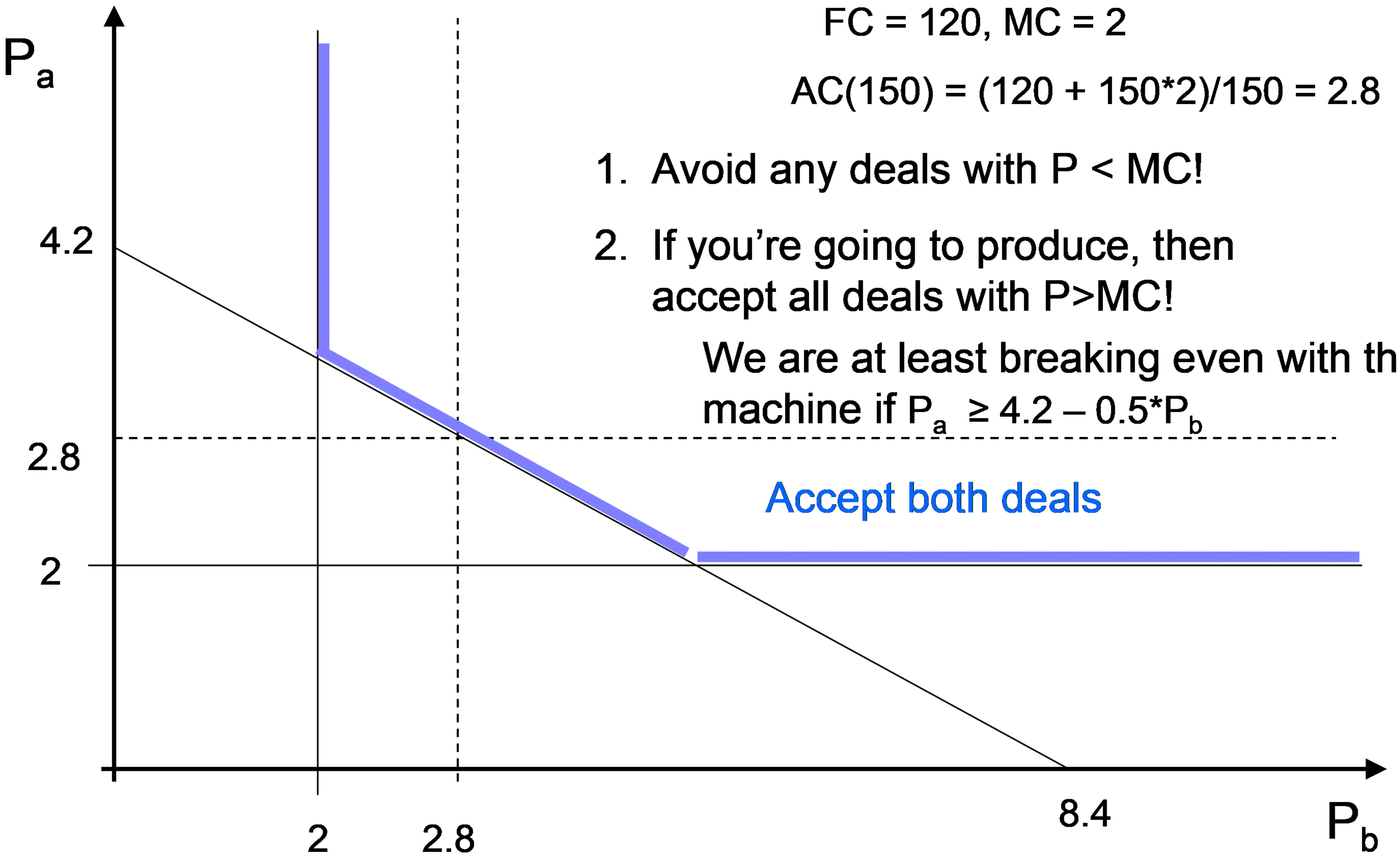
Order b: 50 units

FC = 120, MC = 2

AC(150) = (120 + 150*2)/150 = 2.8

1. Avoid any deals with P < MC!
2. If you're going to produce, then accept all deals with P>MC!

   We are at least breaking even with this machine if $P_a \geq 4.2 - 0.5 * P_b$



One product, two deals

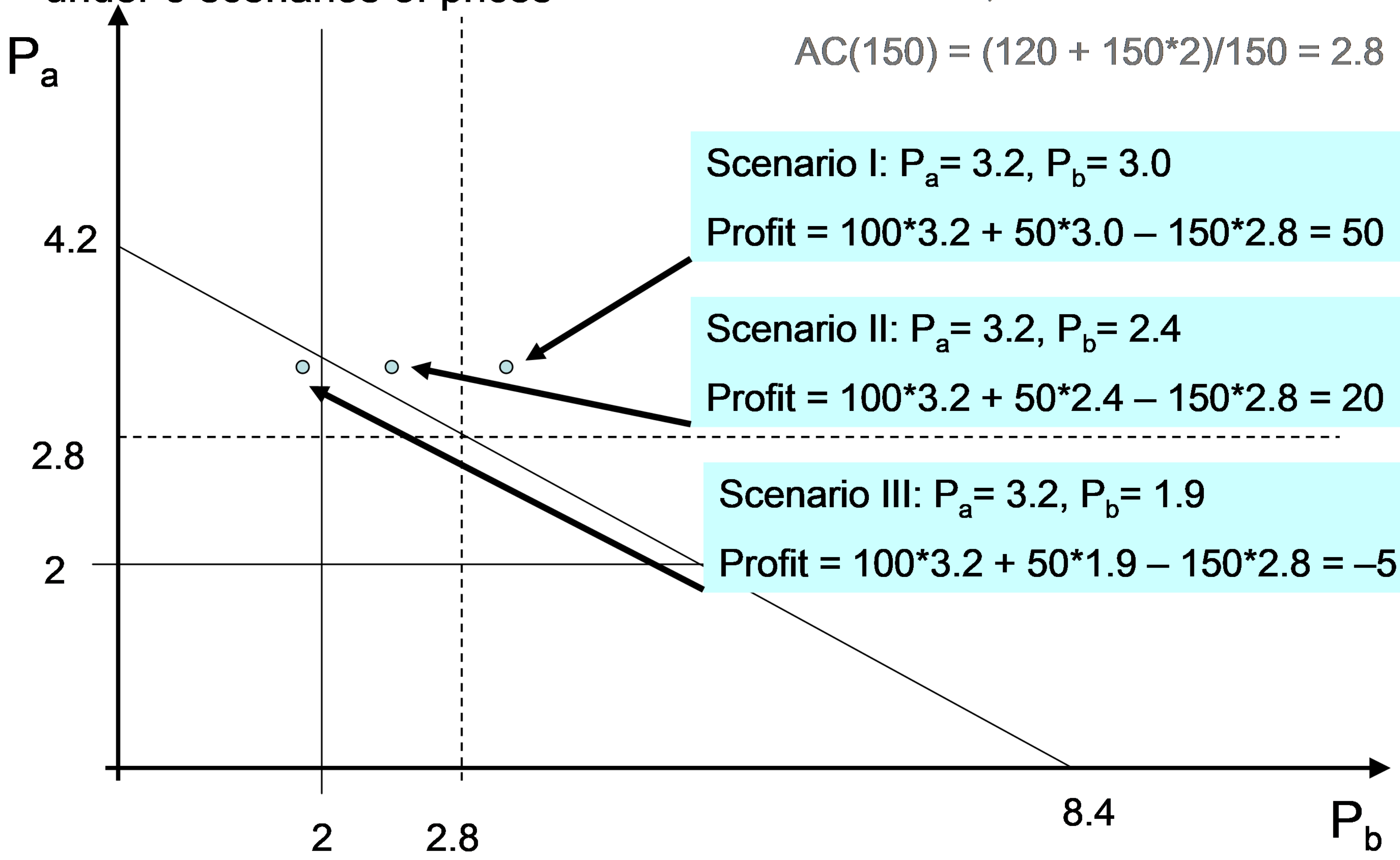
Profit from accepting both deals under 3 scenarios of prices

Order a: 100 units

Order b: 50 units

FC = 120, MC = 2

AC(150) = (120 + 150*2)/150 = 2.8



Scenario I: P_a= 3.2, P_b= 3.0

Profit = 100*3.2 + 50*3.0 – 150*2.8 = 50

Scenario II: P_a= 3.2, P_b= 2.4

Profit = 100*3.2 + 50*2.4 – 150*2.8 = 20

Scenario III: P_a= 3.2, P_b= 1.9

Profit = 100*3.2 + 50*1.9 – 150*2.8 = –5

One product, two deals

What about the possibility to accept one deal only?

Order a: 100 units

Order b: 50 units

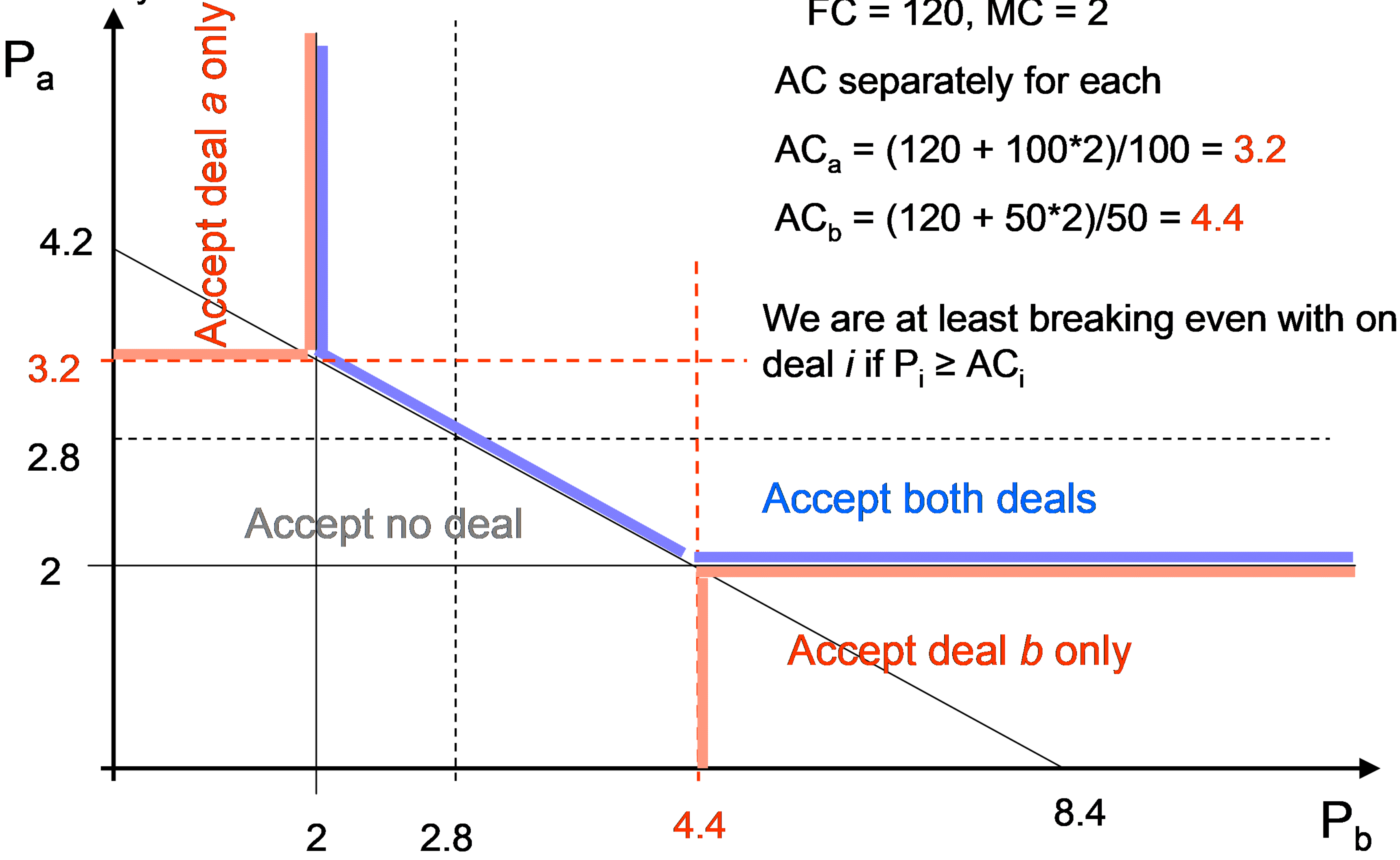
FC = 120, MC = 2

AC separately for each

$AC_a = (120 + 100*2)/100 = 3.2$

$AC_b = (120 + 50*2)/50 = 4.4$

We are at least breaking even with only deal *i* if $P_i \geq AC_i$



Costs and pricing

AC crucial for decision to produce at all

Profits are positive $\longleftrightarrow P > \mathrm{AC}$

MC crucial for decision how much to produce and how to price

The AC/MC rule of overhead:

On average, all orders have to cover the AC – or we should not produce at all. Individual orders only need to cover the MC.

If we produce, and face potential orders at take-it-or-leave-it prices P_i, we should accept those with $P_i > MC$.

Cost types

Economic costs are those that should be taken into account in decision-making: they affect welfare and they can be affected

- Fixed vs variable costs
- Sunk costs `https://dilbert.com/strip/2018-02-05`
- Opportunity cost vs accounting costs

Cost type often depends on time horizon