Statistical Mechanics
E0415

Fall 2020, lecture 2
Random walks

Take-home 1

Take-home task 1

Study the material for the week: Sethna Chapter 1. Then, answer the following question(s) and return your answer to MyCourses **deadline on the day before the lecture: September 17 at noon.**

1.t

Have a look at https://royalsocietypublishing.org/doi/10.1098/rsos.200307 and write a short (1 paragraph) essay commenting on two points: which issues of the book do you recognize in the study, what do you think you would have done in the study?

To the first part of the question:

" From the point of view of the book, the relevant quantities in the study were the concept of ensembles, as was the case with the group, Monte Carlo in case of the simulations and fluctuations and correlationts in the case of the behaviour of individuals in the crowd. "

"-- the article is related with Random Walk and the characterization, for its gist on human crowd movement trajectories, Monte Carlo, for its use of simulation, and interactions between constituents, for its consideration of avoidance and other possible interactions between humans in a crowd when simulatingthe trajectories and headings.“

"Criticality and/or phase transition behaviours would also be a potentially relevant topic, for example if crowding behaviours change significantly in nature when crowds become tightly packed"

More answers

Second part:

"-- one of the possible future directions of the
study, could be to devise more representative statistics. This may be approached
by better understanding the dynamics (e.g., putting more abrupt changes to the
patterns), or by fine graining the trajectories into a mixture of modalities(e.g., Lévy
flightvs.Brownian motion), to have more than a scalar mean for each simulation
parameter. Further, going from model-based methods, we could also consider data-
driven methods to learn the latent representations of the real trajectories then generate
the simulations."

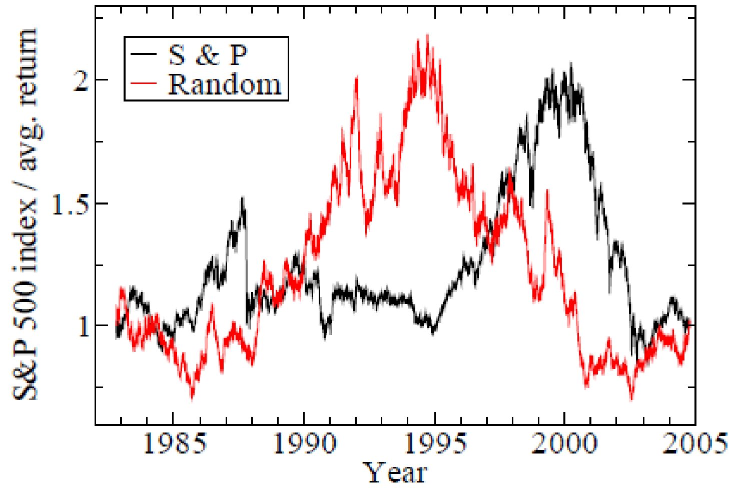
"Personally, I would have had participants from other regions and of different ages and backgrounds, diversifying the participants."

"-- I would have tried to see how experienced people (someone that has before observed a real crowd) could do in the test."

Universality, scale invariance

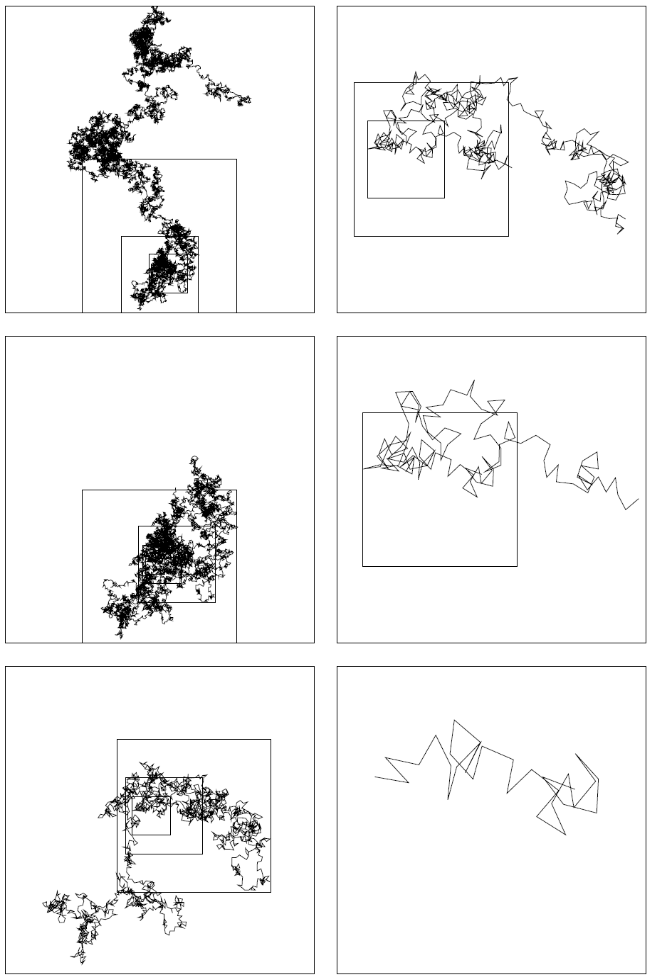
- **Three main points with random walks**: scaling (scale-free) behavior, universality (small details do not matter), probability distributions (and the governing equation(s)).
- Example: coin flips/tosses – do heads or tails win? Square-root law with N.
- Example II: drunkard's walk (on a 2D plane).
- Universality – compare with polymers (self-avoidance). "Entropic repulsion", walk exponent becomes ϒ = ¾ (2D), 0.59 (3) – exact and numerical values. "Universal critical exponent" (Self-Avoiding Walks, SAW).

Sorts of RWs



Stock Exchange Index vs.
a simple RW

Fig. 2.2 Random walk: scale invariance. Random walks form a jagged, fractal pattern which looks the same when rescaled. Here each succeeding walk is the first quarter of the previous walk, magnified by a factor of two; the shortest random walk is of length 31, the longest of length 32 000 steps. The left side of Fig. 1.1 is the further evolution of this walk to 128 000 steps.



The Diffusion Equation

An equation for ρ: two interpretations – density of a cloud, pdf of a single RW.

$$\frac{\partial \rho}{\partial t} = D\nabla^2 \rho = D\frac{\partial^2 \rho}{\partial x^2}.$$

Derivation of DE: separation of scales (RW step against the gradient of ρ).

Relation of the diffusion constant D > 0 to the microscopic RW details:

$$D = a^2/2\Delta t.$$

Step size a, timescale Δt.

Currents & external forces

Remember: DE conserves particles, thus the density – "conservation law".

$$\frac{\partial \rho}{\partial t} = -\frac{\partial J}{\partial x}. \qquad J_{\text{diffusion}} = -D\frac{\partial \rho}{\partial x},$$

In the presence of external forces the particles have a deterministic drift.

$$x(t+\Delta t) = x(t) + F\gamma\Delta t + \ell(t).$$

This shows up in the current, and in the equation for ρ.

$$J = \gamma F \rho - D\frac{\partial \rho}{\partial x}.$$

$$\frac{\partial \rho}{\partial t} = -\gamma F\frac{\partial \rho}{\partial x} + D\frac{\partial^2 \rho}{\partial x^2},$$

Case study: density profile with gravity ("atmosphere").

$$\rho^*(x) = A\exp\left(-\frac{\gamma}{D}mgx\right).$$

Solving the diffusion equation

Example: Fourier method. FT in space, substitute: reveals the diffusive timescale and the role of D.

General solution as superposition of the FT of the initial profile or condition $\widetilde{\rho_k}(0)$.

$$\frac{\partial \rho}{\partial t} = \frac{\mathrm{d}\widetilde{\rho}_k}{\mathrm{d}t}\mathrm{e}^{\mathrm{i}kx} = D\frac{\partial^2 \rho}{\partial x^2} = -Dk^2\widetilde{\rho}_k\mathrm{e}^{\mathrm{i}kx},$$

$$\frac{\mathrm{d}\widetilde{\rho}_k}{\mathrm{d}t} = -Dk^2\widetilde{\rho}_k,$$

$$\widetilde{\rho}_k(t) = \widetilde{\rho}_k(0)\mathrm{e}^{-Dk^2t}.$$

$$\rho(x,t) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\widetilde{\rho}_k(0)\mathrm{e}^{\mathrm{i}kx}\mathrm{e}^{-Dk^2t}\,\mathrm{d}k.$$

$$\widetilde{\rho}_k(0) = \int_{-\infty}^{\infty}\rho(x,0)\mathrm{e}^{-\mathrm{i}kx}\,\mathrm{d}x,$$

Homework

1.2 Waiting times (Sethna 1.3 p. 6) HOMEWORK (5 points)

Take-home task 2

Study the material for the week: Sethna Chapter 2. Then, answer the following question(s) and return your answer to Mycourses **deadline on the day before the lecture: September 24 at noon.**

2.t

Let us turn this into an exercise in gambling. You play heads and tails (toss a coin, and guess the outcome: win or lose the coin). Three questions: you start with 10 coins. Give an argument how the distribution of times it takes for you to lose all your coins looks like. What happens if you play till you have zero, or until you won all the 10 coins of your friend? Let us now consider the case where the coin is not fair: the fractional Brownian motion, where the subsequent outcomes are correlated (positively or negatively). How does that influence qualitatively those outcomes?

Projects, presentation

Group	Paper	Project
Andersson Mika, Kaskela Vilja, Utriainen Laura	Negative temperatures I	The classical Heisenberg model in 2D
Kilpeläinen Joel, Rabensteiner Samuel	Negative temperatures II	Calculation of the Lyapunov spectrum
Harmat Adam, Lin Ziyuan	Jarzynski inequality	Machine learning the phase transition of Ising model
Manninen Ilkka, Nguyen Huy	Entropy & information	Domain growth kinetics
Arjas Kristian, Harnist Bent,	Negative Representation and Instability...	Models of opinion formation
Ihalainen Erkko, Lamponen Eeli, Rantanen Riku	Quantum phase transition	The minority game

Use the course Zoom to agree with your group (and maybe to meet)

Other scheduling

Project “show”; probably 4^{th} of Dec.

Paper presentations: TBD

Friday lectures: when 12.15 is out of questions, do you prefer 9.15 or 14.30? (**9.15 selected, next 25.9**)