Differential and Integral Calculus 1
MS-A0111
Hakula
Orlich/Ardiyansyah
Problem Sheet 3, 2020

NOTE[1]

The due date is published on the course pages. Homework can be submitted only digitally. Instructions on labeling the "papers" can be found on the course pages.

1 Introductory Problems

INTRO 1 Find the approximations M_8 and T_{16} for $\int_0^1 e^{-x^2}\,dx$. How do you justify the number of decimals you are reporting?

INTRO 2 [Trapezoid Rule] Compute the actual error in the approximation $\int_0^1 x^2\,dx \approx T_1$ and use it to show that the constant 12 in the error estimate cannot be improved.

INTRO 3 Find the area of the region bounded by

$$y = \frac{x}{x^2+16},\ y = 0,\ x = 0,\ x = 2.$$

INTRO 4 Using suitable trigonometric identities, calculate the following integrals:

$$\int \cos ax \cos bx\,dx, \quad \int \sin ax \sin bx\,dx, \quad \int \sin ax \cos bx\,dx.$$

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2 Homework Problems

EXERCISE 1 Find the approximations M_8 and T_{16} for

$$\int_0^{\pi/2} \frac{\sin x}{x} dx,$$

assuming that the integrand is $= 1$ at $x = 0$. How do you justify the number of decimals you are reporting?

EXERCISE 2 [Midpoint Rule] Compute the actual error in the approximation $\int_0^1 x^2\,dx \approx M_1$ and use it to show that the constant 24 in the error estimate cannot be improved.

EXERCISE 3 Find the area of the region bounded by

$$y = \frac{x}{x^4+16},\ y = 0,\ x = 0,\ x = 2.$$

EXERCISE 4 If n and m are integers, show that

(i)

$$\int_{-\pi}^{\pi} \cos mx \cos nx\,dx = 0,\ \text{if } m \neq n,$$

(ii)

$$\int_{-\pi}^{\pi} \sin mx \sin nx\,dx = 0,\ \text{if } m \neq n,$$

(iii)

$$\int_{-\pi}^{\pi} \sin mx \cos nx\,dx = 0.$$