Exercise session 1

Aalto University
School of Business

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Sep 18, 2020

Exercise 1

A look at real world data on Gapminder.org

Exercise 1

https://www.gapminder.org/tools/#$chart-type=bubbles

a) Is life expectancy positively or negatively correlated with income?

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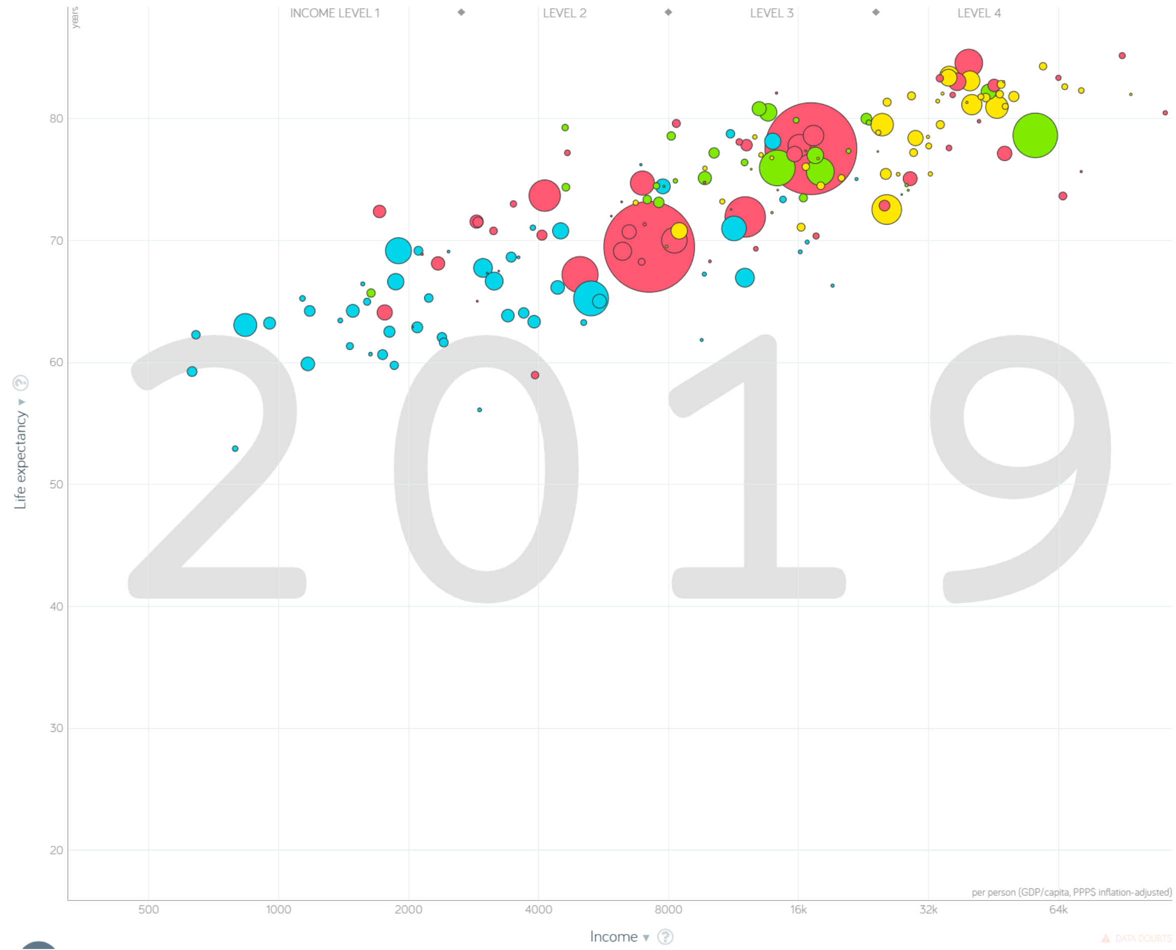

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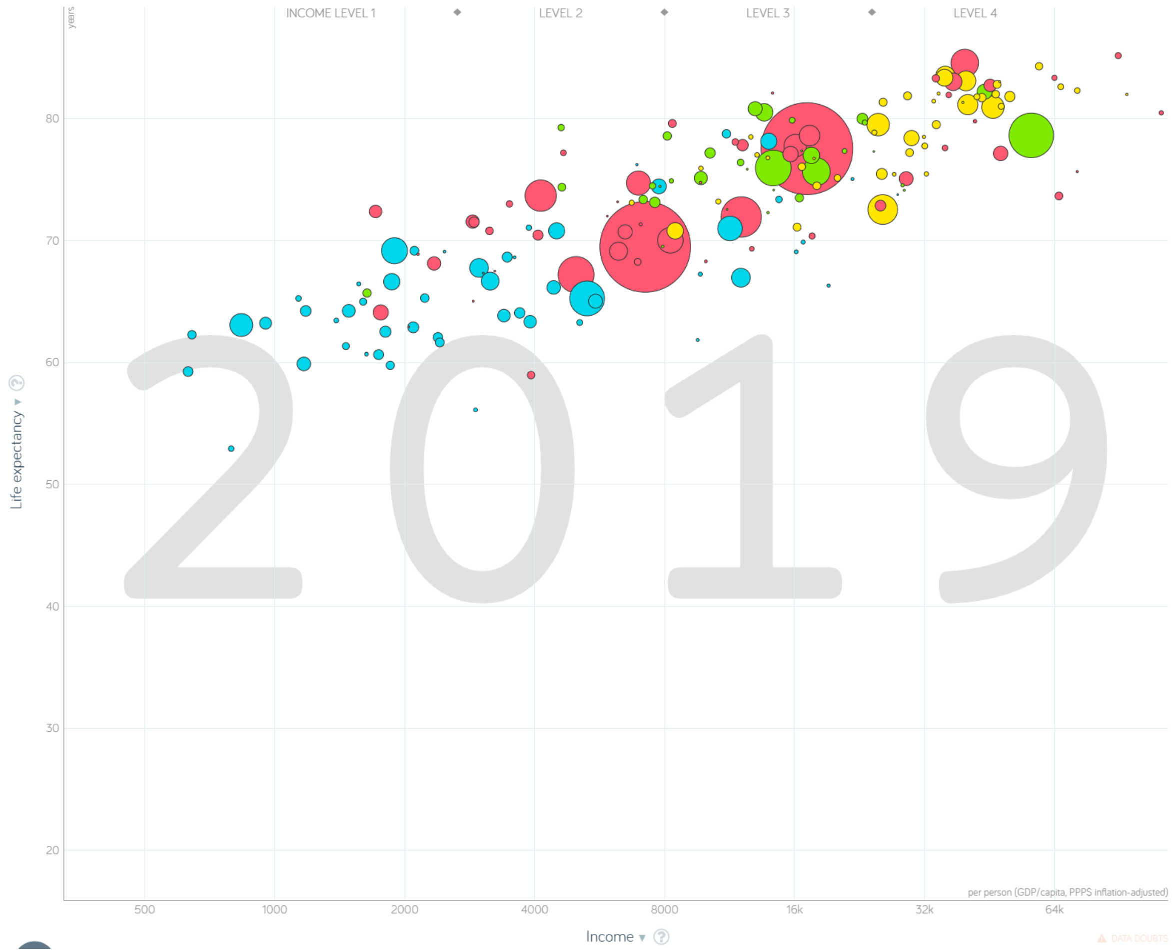
Exercise 1

https://www.gapminder.org/tools/#$chart-type=bubbles

a) Is life expectancy positively or negatively correlated with income?

POSITIVELY



Exercise 1

https://www.gapminder.org/tools/#$chart-type=bubbles

b) What variables are negatively correlated with income?

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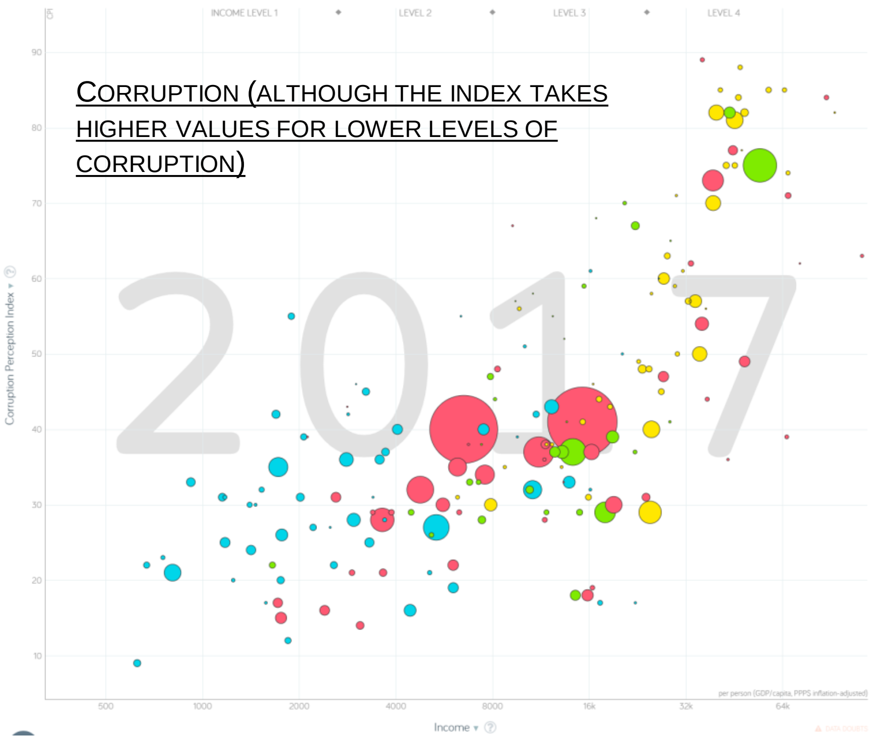

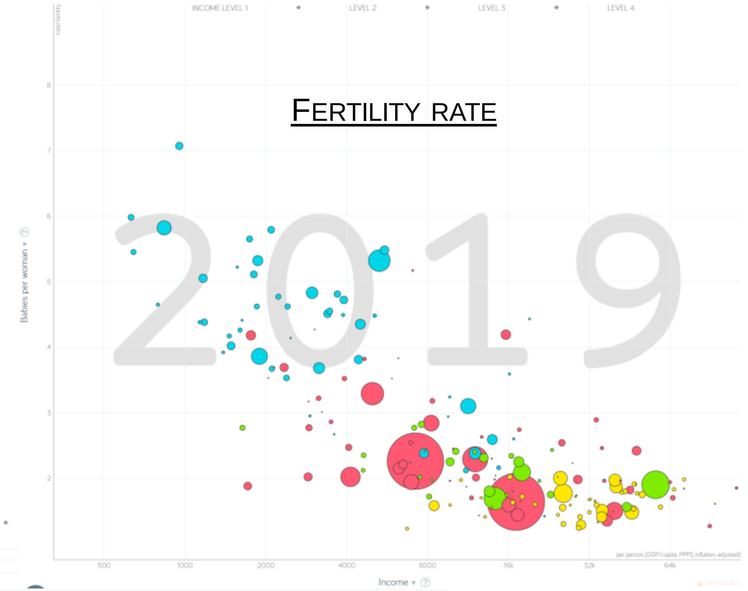

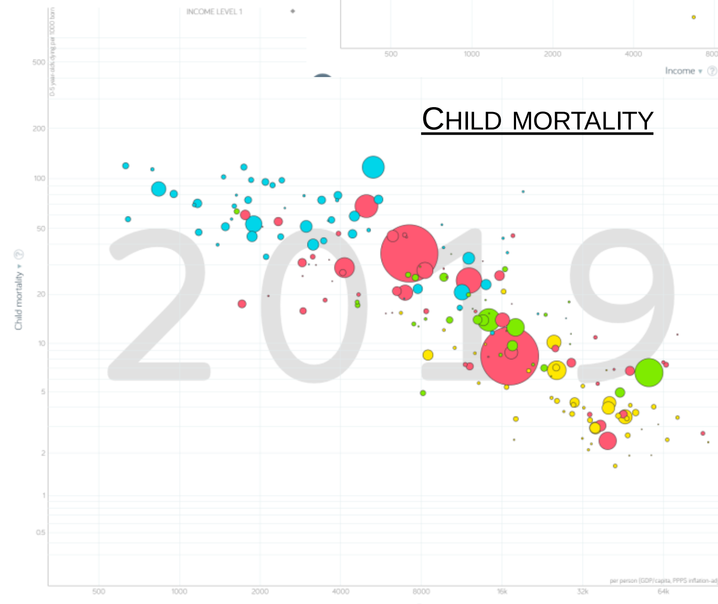

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Exercise 1

https://www.gapminder.org/tools/#$chart-type=bubbles

c) What is the relationship between population size and income across different countries?

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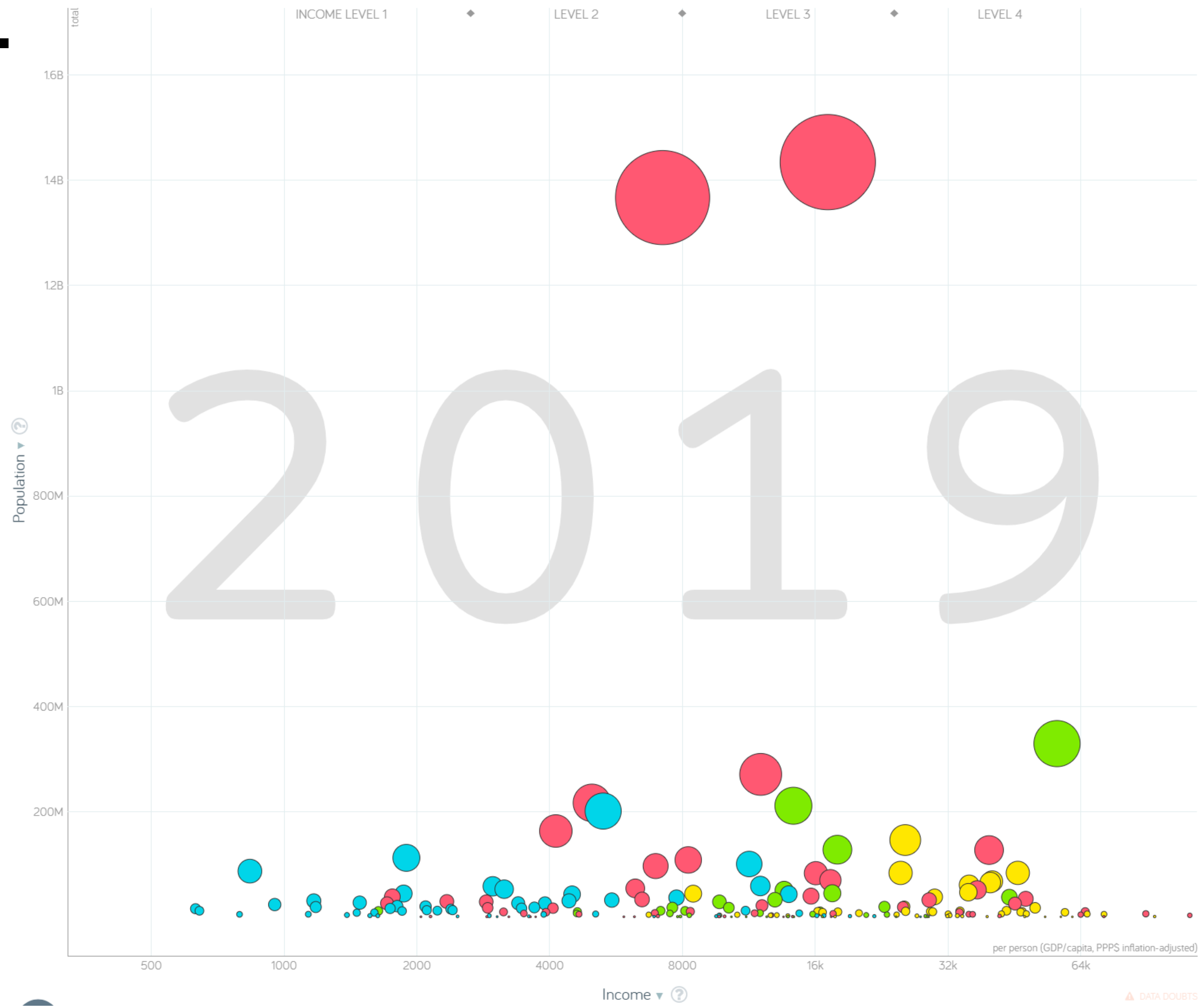

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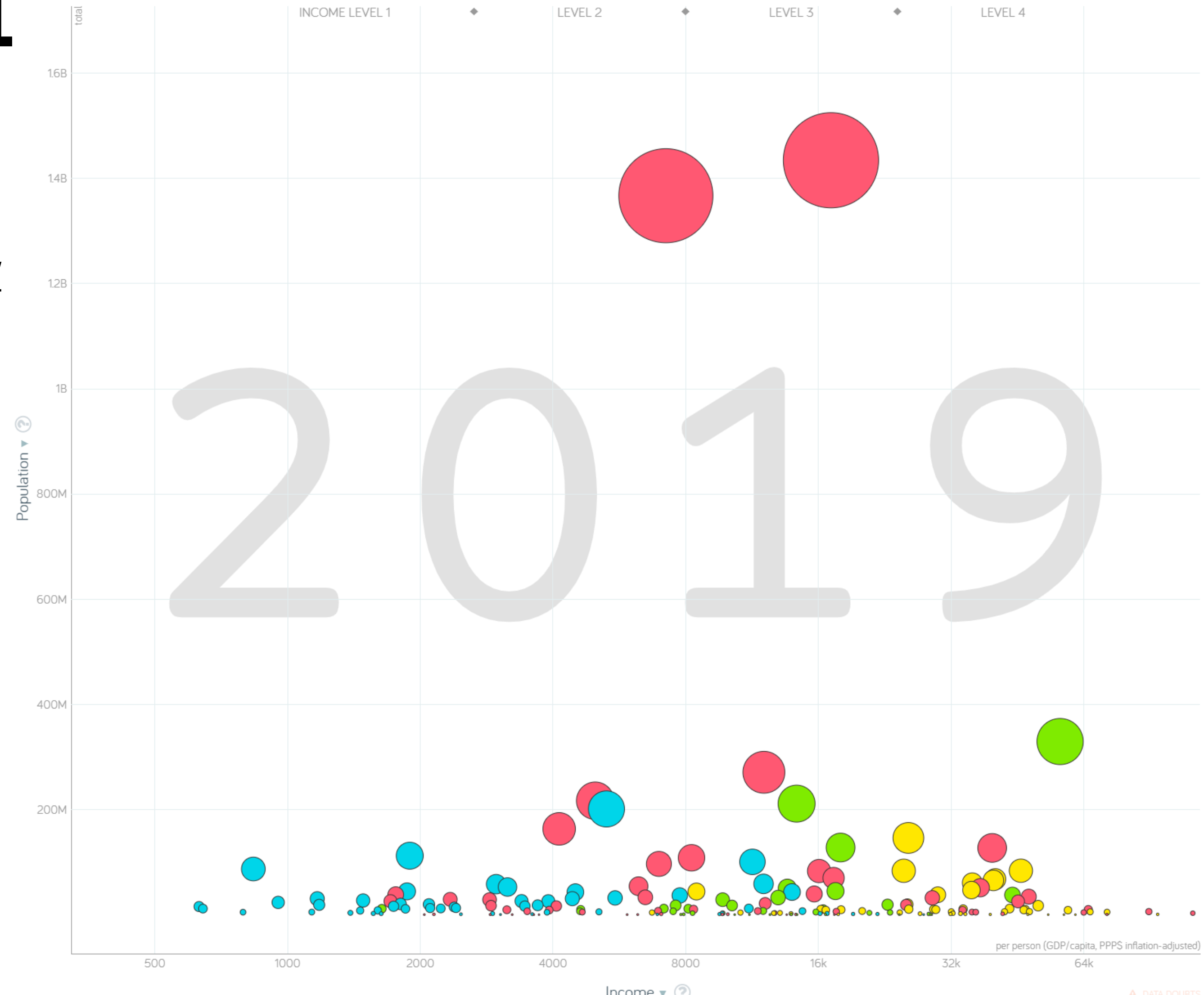
Exercise 1

https://www.gapminder.org/tools/#$chart-type=bubbles

c) What is the relationship between population size and income across different countries?

<u>NO CLEAR RELATIONSHIP / NOT CORRELATED</u>



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Exercise 1

https://www.gapminder.org/tools/#$chart-type=linechart

d) Which country among China, USA and Finland has had a close to constant growth rate over time? Can you find explanations to the periods of negative growth?

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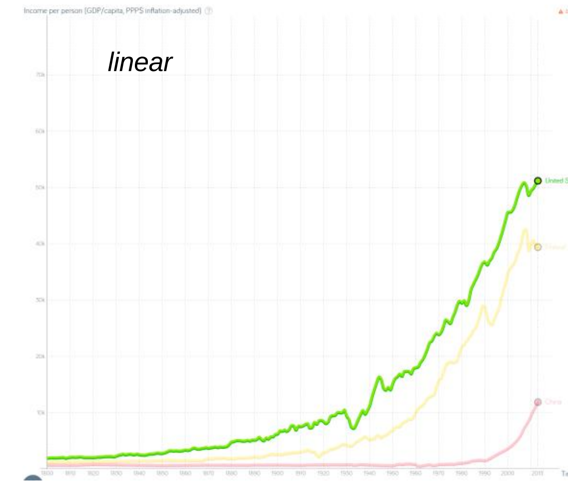
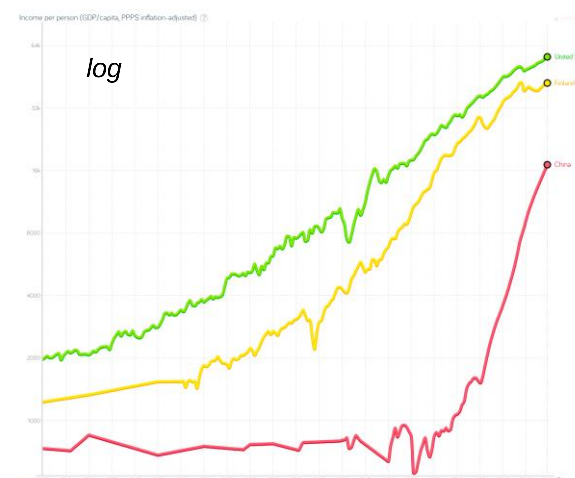
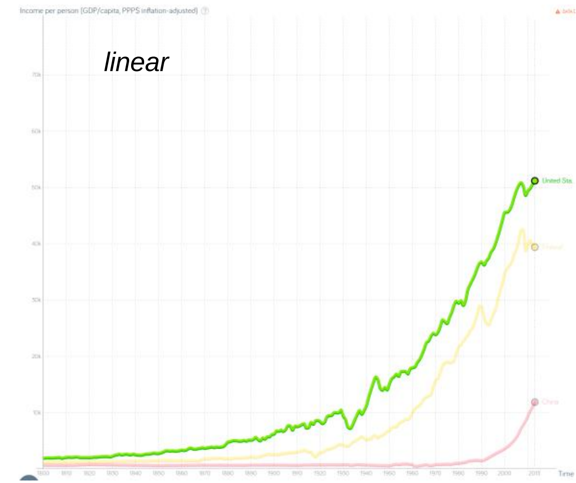


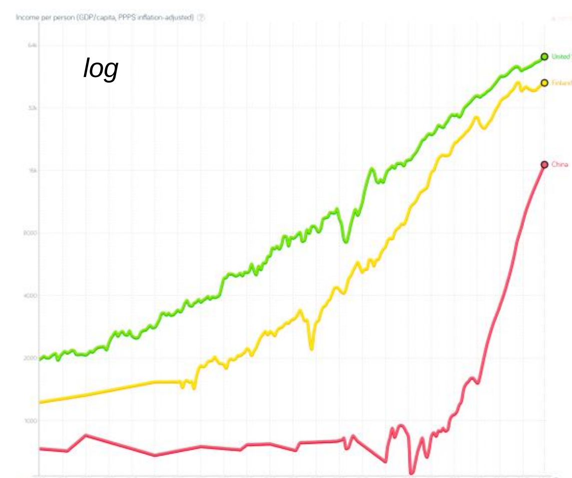


Exercise 1

https://www.gapminder.org/tools/#$chart-type=linechart

d) Which country among China, USA and Finland has had a close to constant growth rate over time? <u>USA</u>





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Exercise 1

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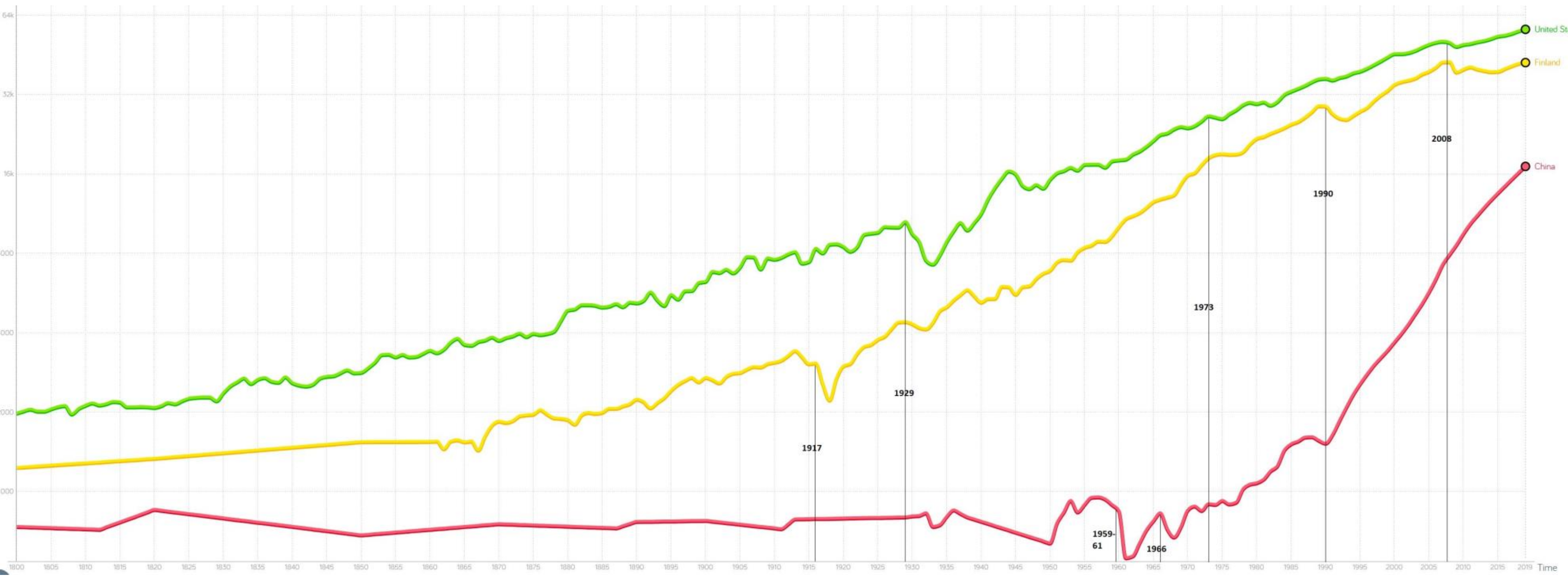
d) Can you find explanations to the periods of negative growth?



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Exercise 2

Budget Sets

Ingredients to draw a budget line:

- Available budget *w*
- Price p_x of good *x*
- Price p_y of good *y*

Equation of the budget line: $w = p_x \cdot x + p_y \cdot y$

Find the intercepts of the *x* and *y* axes, where we read amount of good *x* on the *x-axis* and amount of good *y* on the *y-axis.*

- How much of good *x* can I get if I spend all my budget *w* on it? $w = p_x \cdot x + p_y \cdot 0 \Leftrightarrow x = {}^{w}/_{p_x}$

- How much of good *y* can I get if I spend all my budget *w* on it?

The line joining the two points shows all the combinations of good *x* and good *y* (bundles) that you can afford. All the points above and on the right of the line are unaffordable bundles. All the points below the line are affordable but will leave some of the budget not spent, which is not desirable.

Equation of the budget line:

$$\boldsymbol{w = p_x \cdot x + p_y \cdot y}$$

where:

- *w:* available budget
- p_x and x: price and quantity of good *x*
- p_y and y: price and quantity of good *y*

To draw the budget line, start from the intercepts of the *x* and *y* axes (we read amount of good *x* on the *x-axis* and amount of good *y* on the *y-axis)**:**

- How much of good *x* can I get if I spend all my budget *w* on it?

$$w = p_x \cdot x + p_y \cdot 0 \Leftrightarrow x = \frac{w}{p_x}$$

- How much of good *y* can I get if I spend all my budget *w* on it?

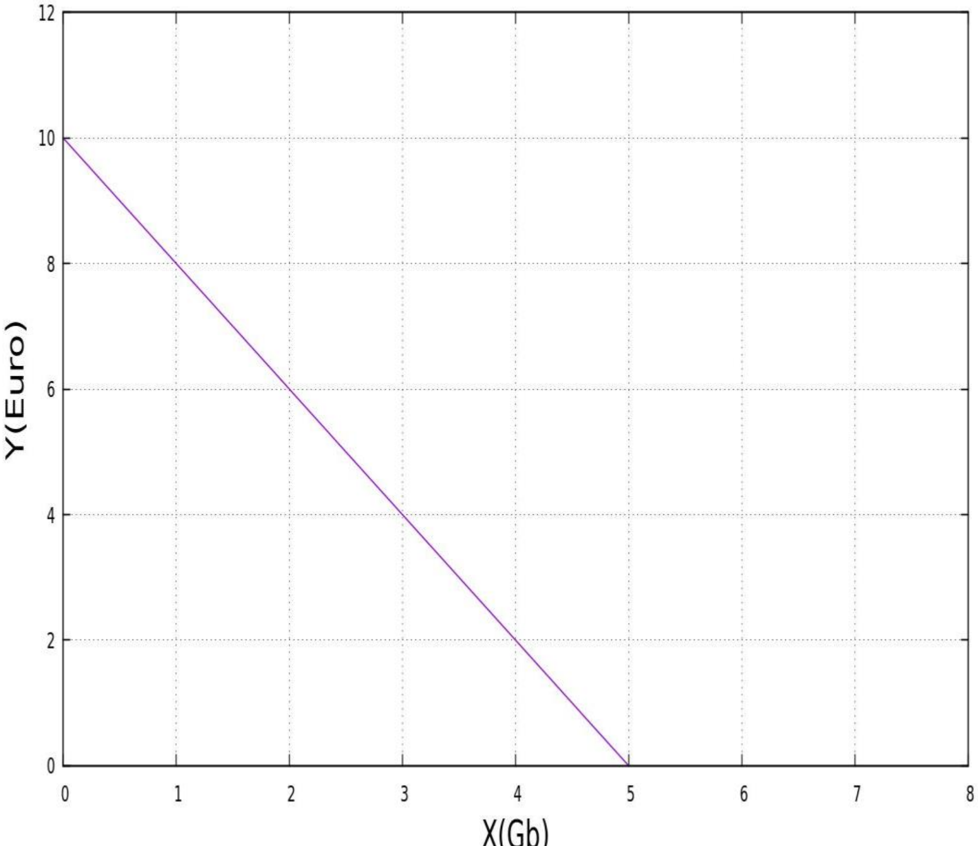
$$w = p_x \cdot 0 + p_y \cdot y \Leftrightarrow y = \frac{w}{p_y}$$

The line joining the two points shows all the combinations of good *x* and good *y* (bundles) that you can afford.

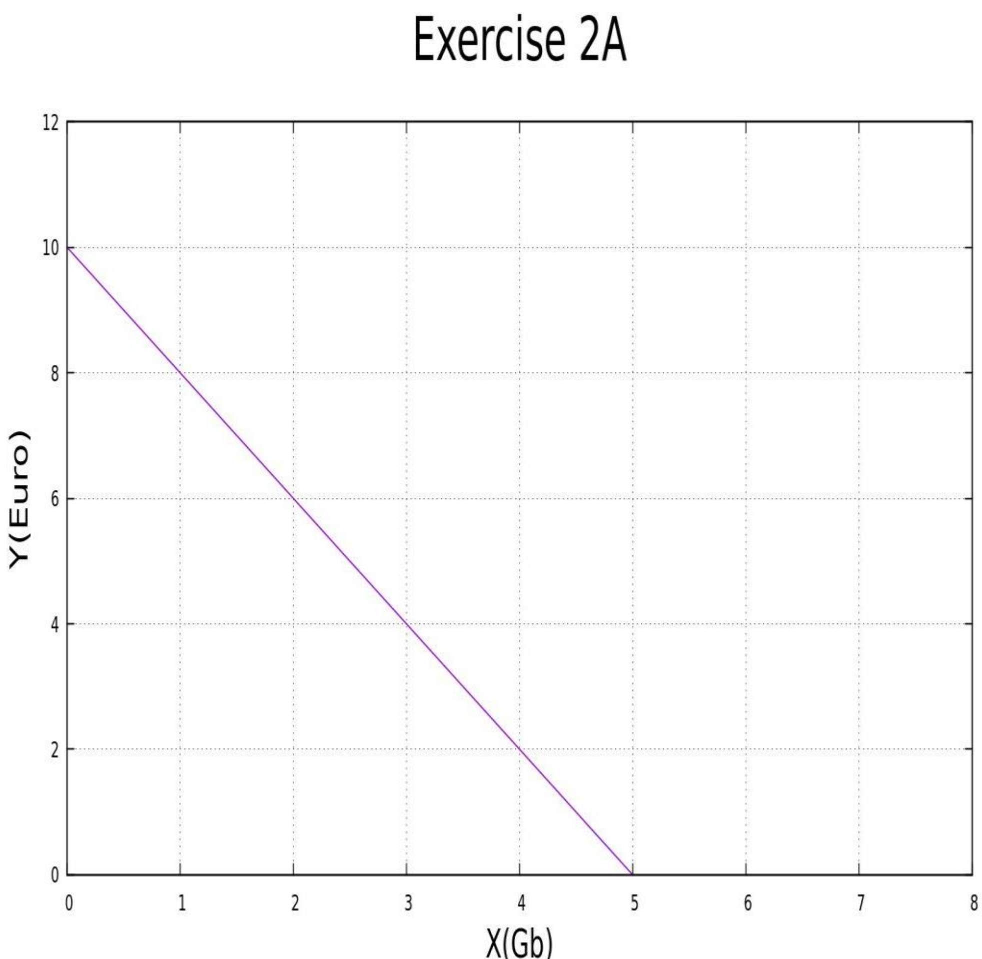
All the points above and on the right of the line are unaffordable bundles.

All the points below the line are affordable but will leave some of the budget not spent, which is not desirable.

Exercise 2A



$w = 10$
$p_x = 2$
$(p_y = 1)$



$$w = 10$$
$$p_x = 2$$
$$(p_y = 1)$$

If I spend all my money on data, the max number of GB that I can buy is *x=5.* If I buy 0 GB, I can spend 10€ on all other goods.

In the *xy-plane*, point *(0,10)* and point *(5,0)* are the extremes of the budget line. Any combination *(x,y)* on the purple budget line joining these two points (and anything below it) is feasible.

Or actually, if GB can only be bought in discrete amounts (0,1,2,…), this is not entirely correct (the line should be shaped "like stairs").

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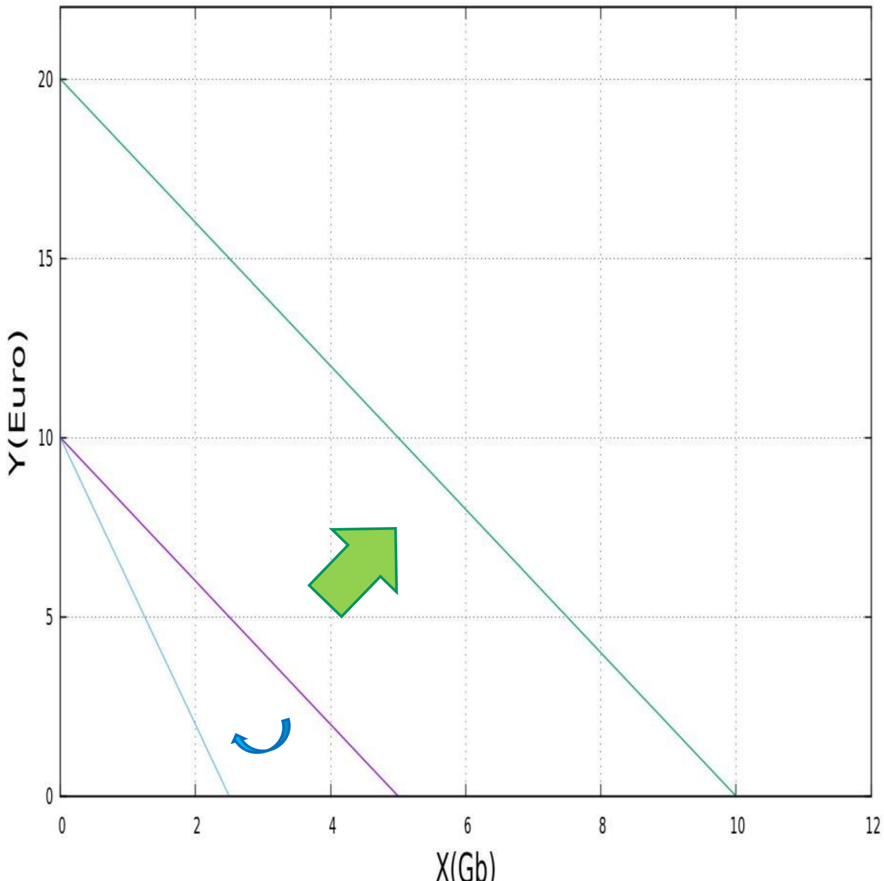
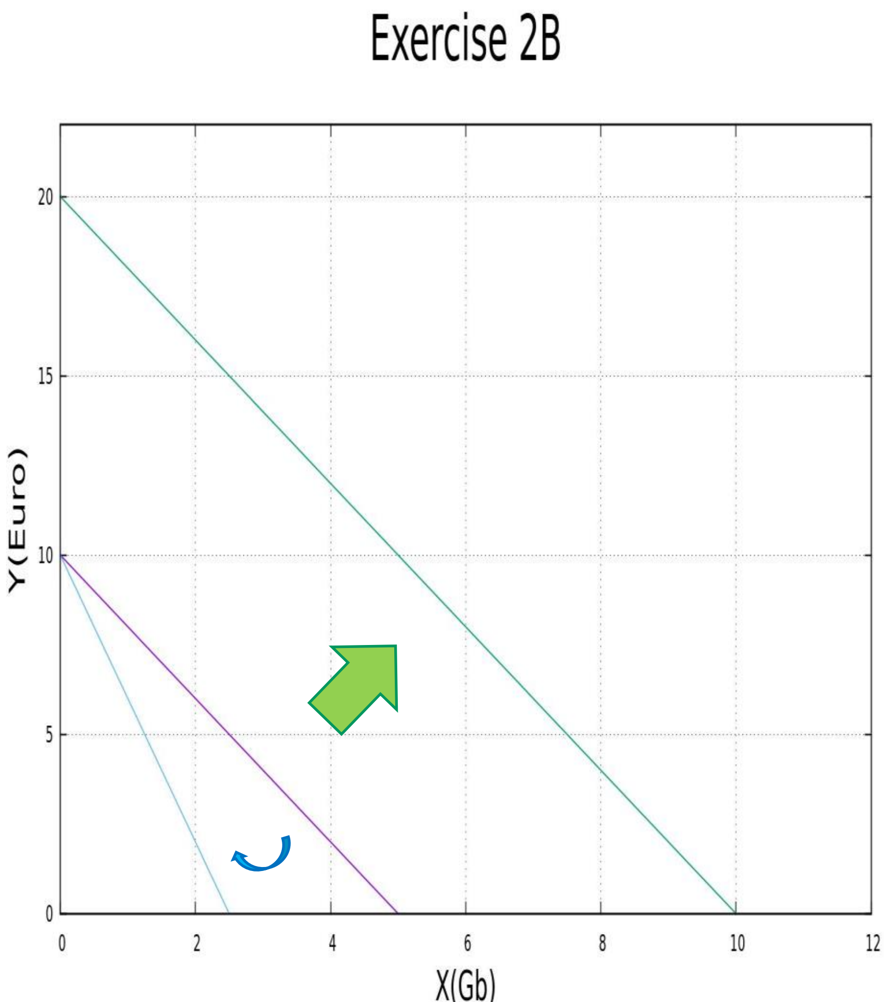
Exercise 2B



$w = 10$
$p_x = 4$
$(p_y = 1)$

$w = 20$
$p_x = 2$
$(p_y = 1)$



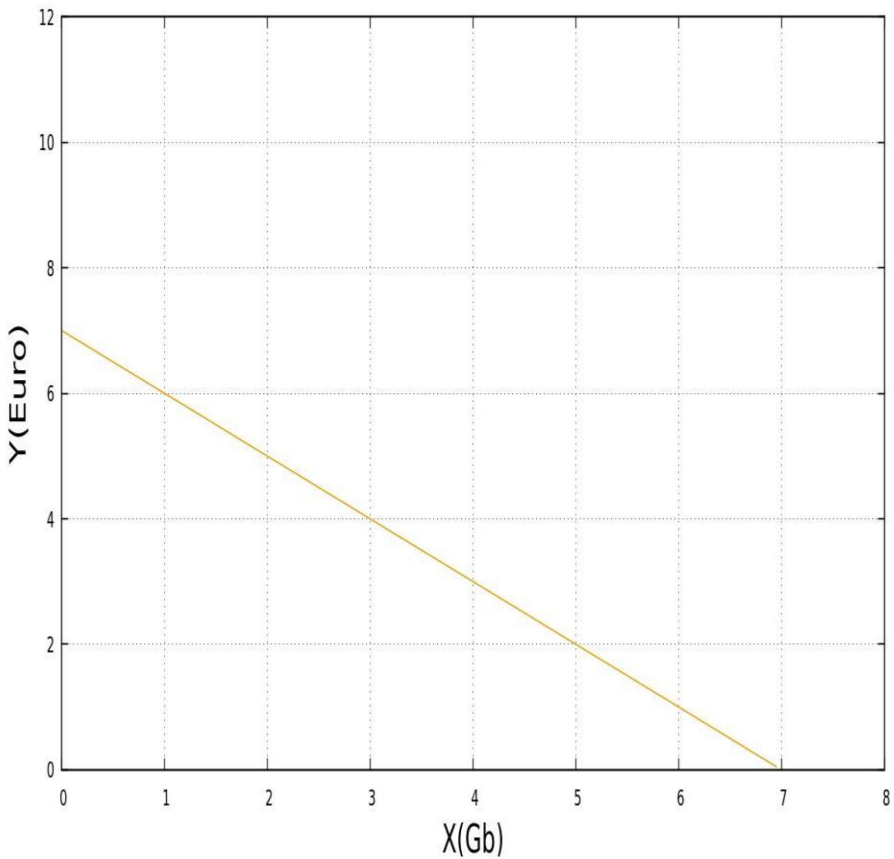
$$w = 10$$
$$p_x = 4$$
$$(p_y = 1)$$

When the price of GB doubles, I can afford only half of the original amount of GB: *10/4=2.5.* The budget line tilts inwards (blue line).

$$w = 20$$
$$p_x = 2$$
$$(p_y = 1)$$

If instead the available money *w* increases to *20*, without any change to prices, I can now afford twice as much consumption as before. The budget line simply shifts outwards to the green line.

Exercise 2C



$$w = 10$$
$$f = 3$$
$$p'_x = 1$$
$$(p_y = 1)$$

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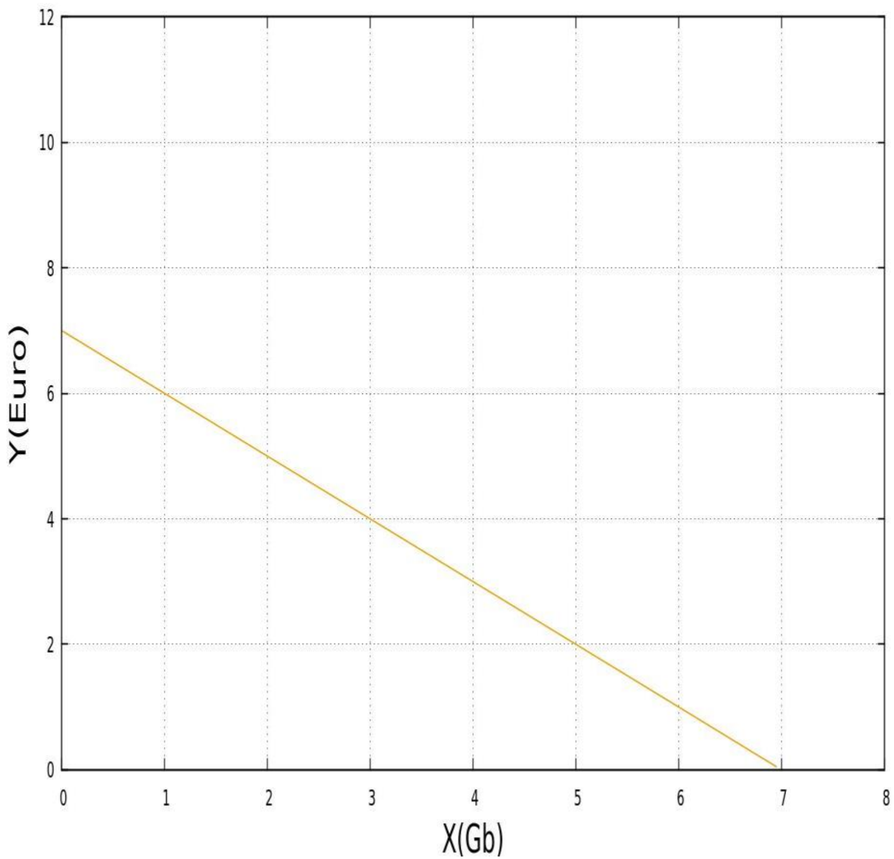
Exercise 2C



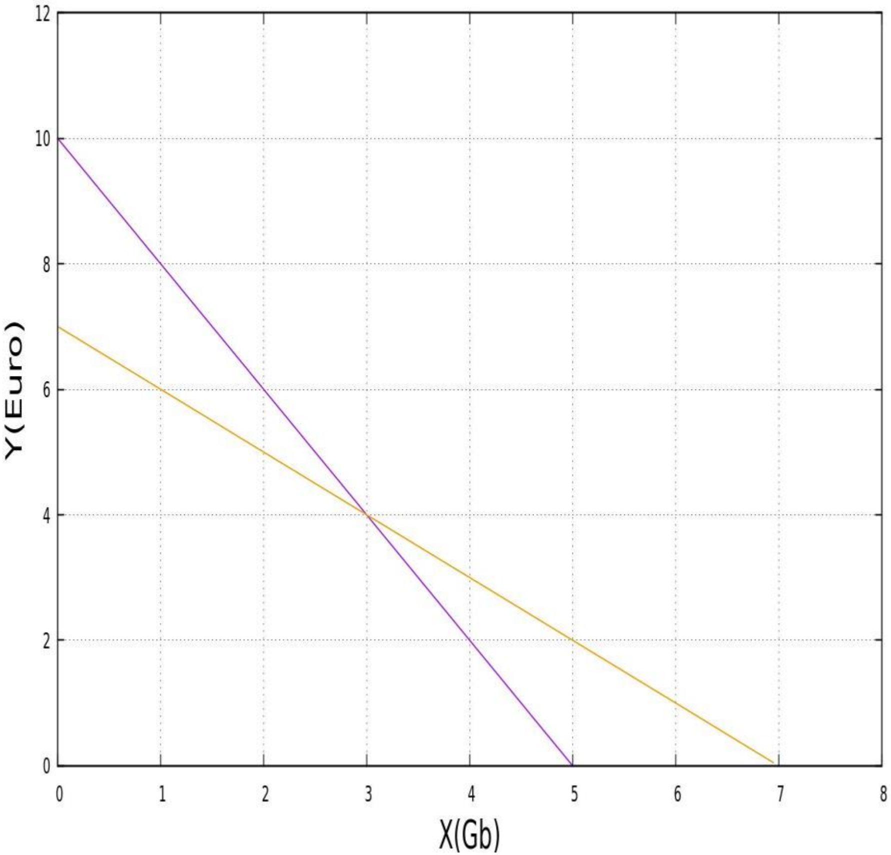
$$w = 10$$
$$f = 3$$
$$p'_x = 1$$
$$(p_y = 1)$$

The new plan requires to pay a monthly fixed fee and a lower unit price for every GB of data purchased.

With this plan, I always spend 3€, even if don't use any data, so I have 7€ left for other consumption. The max amount of data I can get is 7GB.

The yellow budget line is the new feasible consumption set.

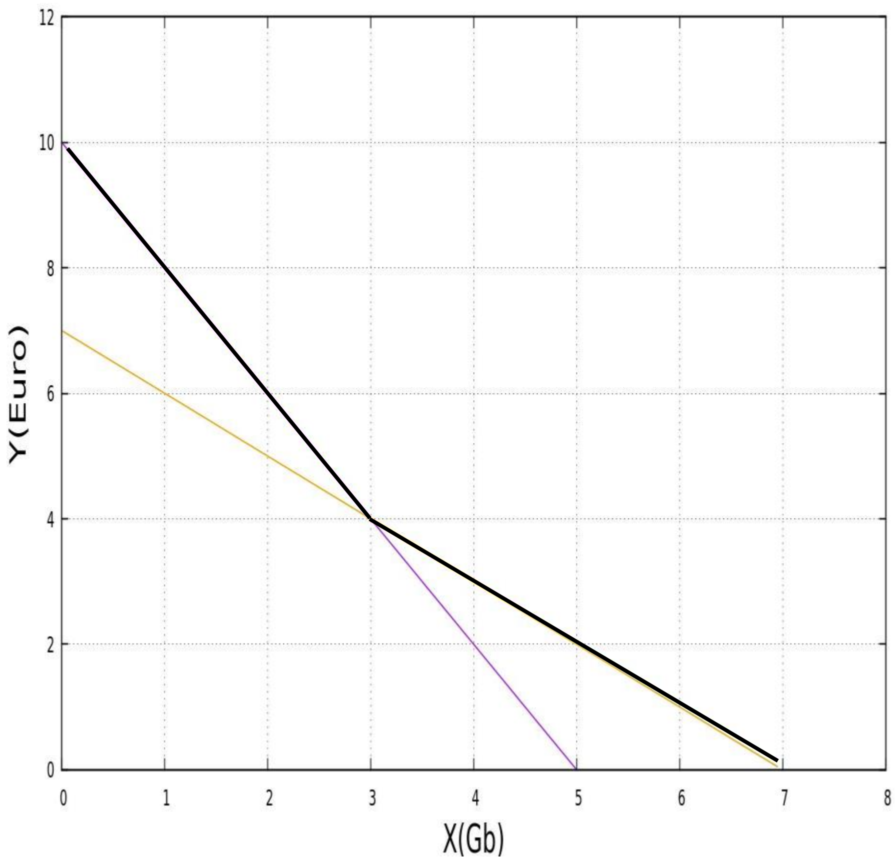
Exercise 2D



If we can choose between the two plans, we see that when *x<3*, the purple plan is cheaper (p·x < f + p´·x), when *x=3*, both plans cost the same, and when *x>3,* the yellow plan is cheaper (p·x > f + p´·x).

x (GB)	p·x	f + p´·x
0	0	3
1	2	4
2	4	5
3	6	6
4	8	7
5	10	8
6	12 (unaffordable)	9
7	14 (unaffordable)	10

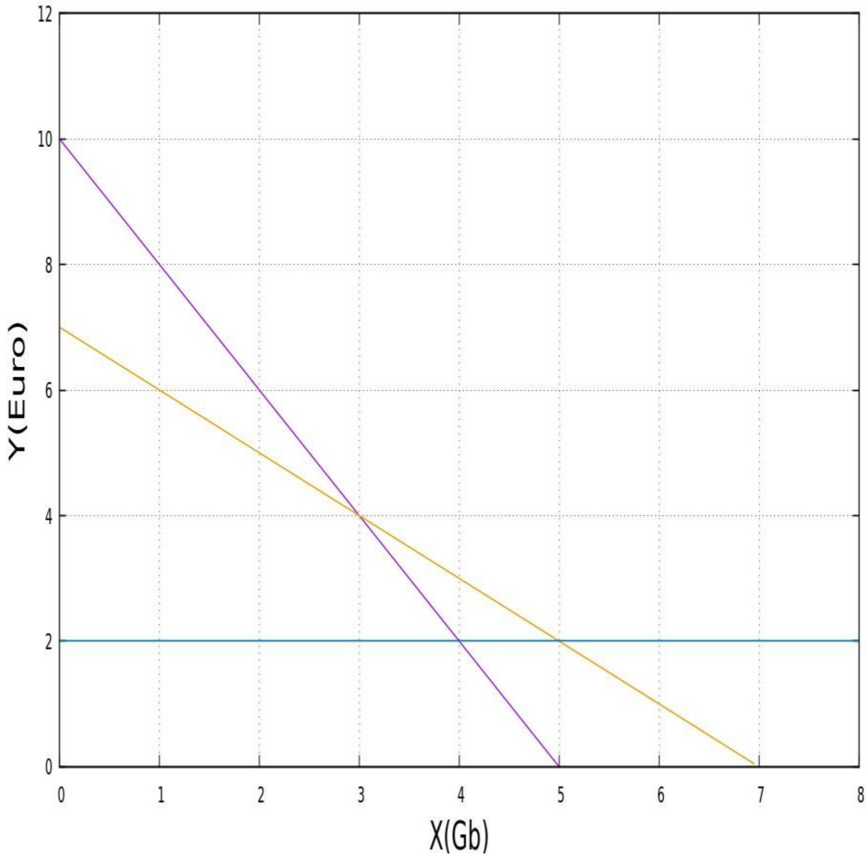
Exercise 2D



If we can choose between the two plans, we see that when *x*<*3*, the purple plan is cheaper (p·x < f + p´·x), when *x*=*3*, both plans cost the same, and when *x*>*3,* the yellow plan is cheaper (p·x > f + p´·x).

x (GB)	p·x	f + p´·x
0	0	3
1	2	4
2	4	5
3	6	6
4	8	7
5	10	8
6	12 (unaffordable)	9
7	14 (unaffordable)	10

Exercise 2E



$$w = 10$$
$$f^* = 8$$
$$p_x^* = 0$$
$$(p_y = 1)$$

Now we can choose between 3 plans. The two described so far (represented by the purple line and the yellow line), and one where you pay *f*=8*, and you can consume as many GB as you want (blue line). See from the table how we construct the actual feasible set when we can choose.

x (GB)	p·x	f + p´·x	f*
0	**0**	3	8
1	**2**	4	8
2	**4**	5	8
3	**6**	**6**	8
4	8	**7**	8
5	10	**8**	**8**
6	12 (unaffordable)	9	**8**
7	14 (unaffordable)	10	**8**

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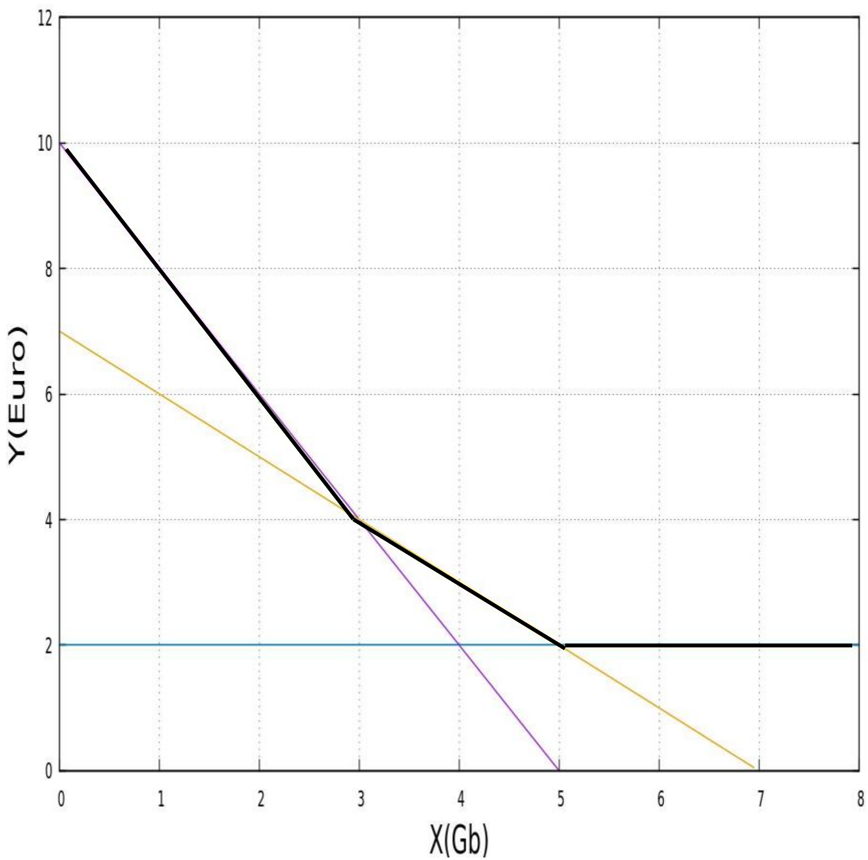

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4	8	**7**	8
5	10	**8**	**8**
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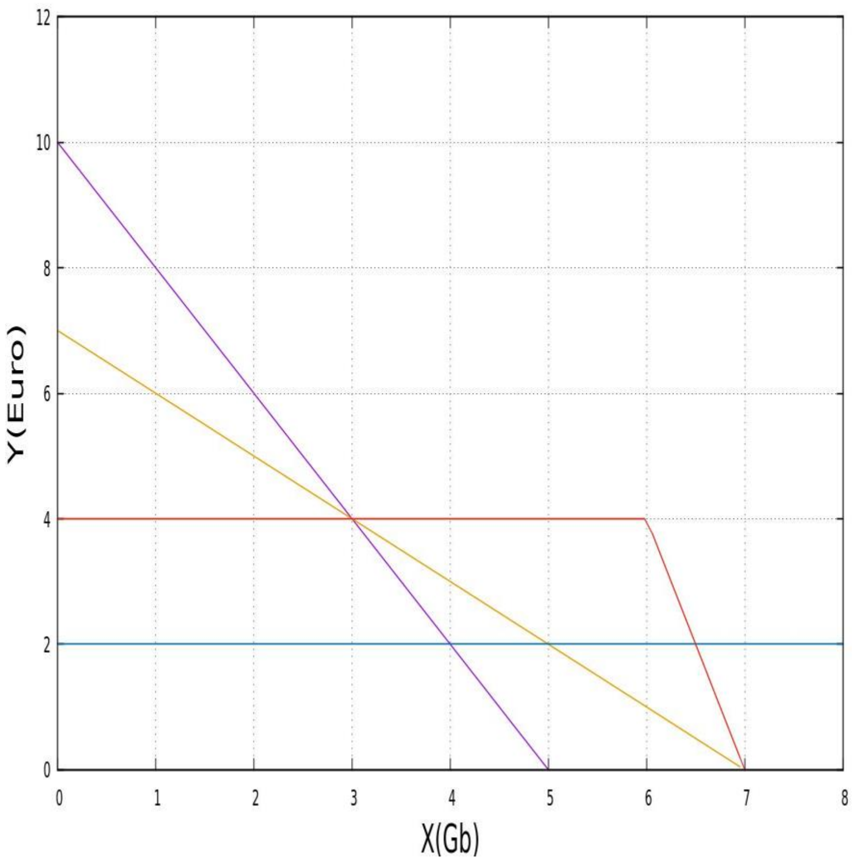
Exercise 2F



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$$w = 10$$
$$f'' = 6 \; if \; x \leq 6$$
$$p_x'' = 4 \; for \; x - 6 \; if \; x > 6$$
$$(p_y = 1)$$

Another plan: *f´´=6* up to x=*6* and then *p´´=4* for every further GB of data, see in the table and the red line on the graph.

Now we can choose between 4 plans. See from the table how we construct the actual feasible set when we can choose.

x (GB)	p·x	f + p´·x	f*	• f´´ if x≤x • f´´+p´´·(x-x) if x>x
0	**0**	3	8	6
1	**2**	4	8	6
2	**4**	5	8	6
3	**6**	**6**	8	**6**
4	8	**7**	8	**6**
5	10	**8**	**8**	**6**
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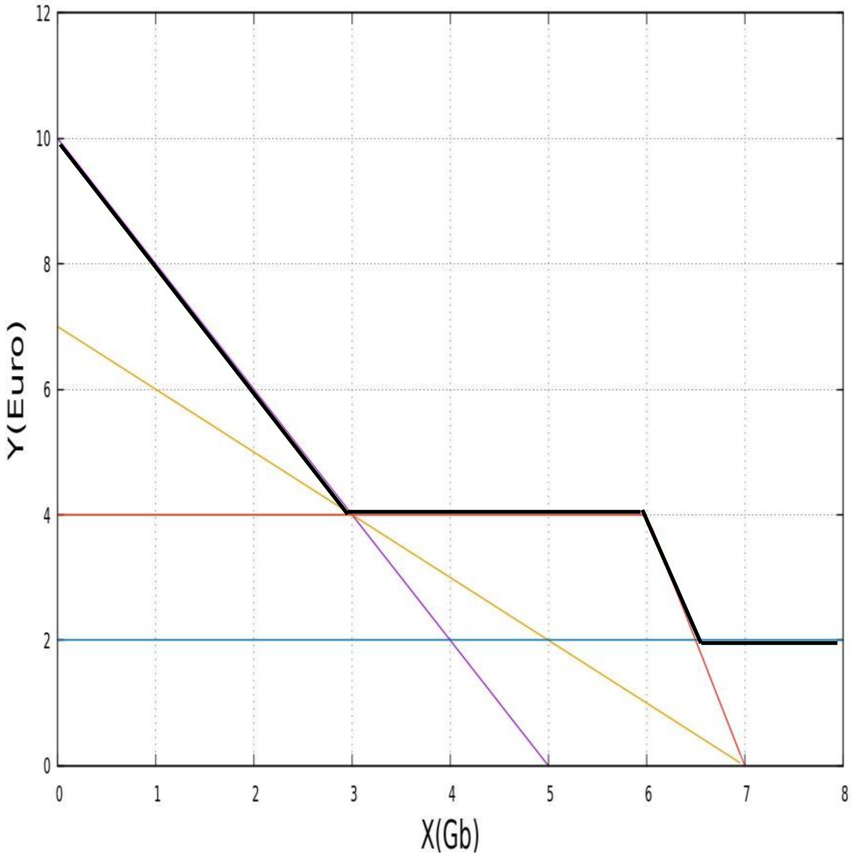
Exercise 2F



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3	**6**	**6**	8	**6**
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Exercise 3

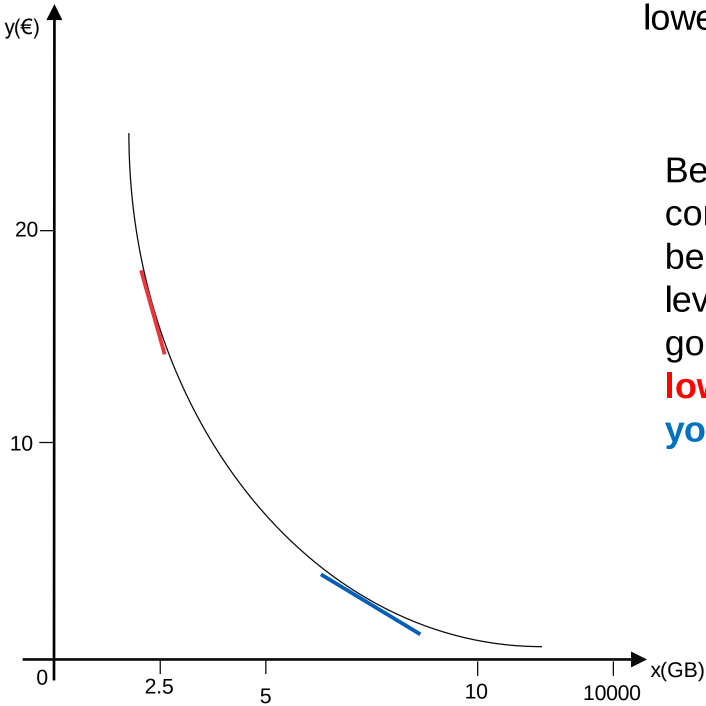
Indifference curves and Preferences

Exercise 3 a

- What is the shape of the indifference curve for GB and other consumption?
- Does the MRS between GB and other consumption become lower as you increase the number of GB?

Exercise 3 a

- What is the shape of the indifference curve?
- Does the MRS between GB and other consumption become lower as you increase the number of GB?

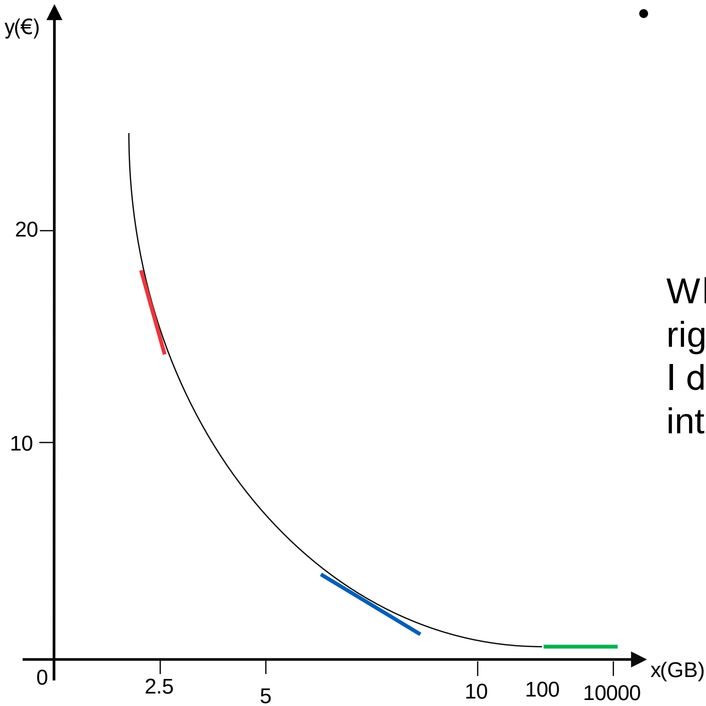


Because of **diminishing marginal utility**, at low levels of consumption of good x, we are willing to give up more of good y to be able to consume more of good x. On the opposite, at high levels of consumption of good x, we are willing to give up less of good y to be able to consume more of good x. So **MRS is large at low numbers of GB** (it tends to infinity) and **becomes smaller as you increase the number of GB** (it tends to zero).

Exercise 3 b

- Suppose that because of time constraints, you will never use more than 100GB/month. What is your MRS at any point *(x,y)* with x>100?

Exercise 3 b



- Suppose that because of time constraints, you will never use more than 100GB/month. What is your MRS at any point *(x,y)* with x>100?

When x>100, the MRS is zero (there will be a horizontal line to the right of 100).
I don't want to give up any consumption in order to consume more internet.

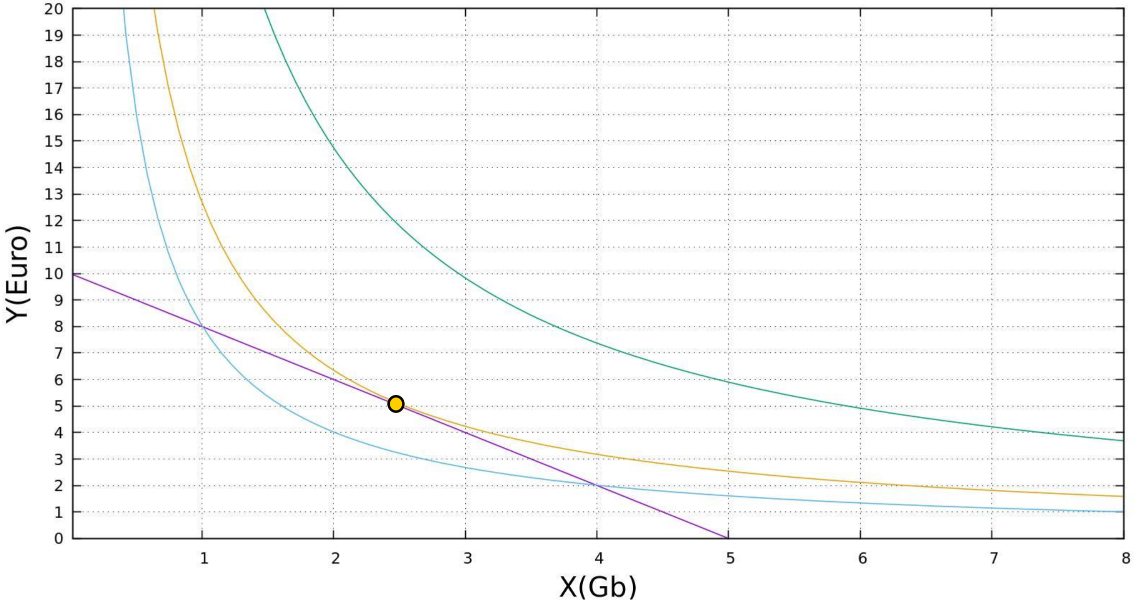
Exercise 3 c

- Determine graphically the optimal choice for the consumer with the indifference curves from Q2A.

$$w = 10$$
$$p_x = 2$$
$$(p_y = 1)$$

- How can you express the MRT in this problem?

Exercise 3C



Determine graphically the optimal choice for the consumer with the indifference curves from Q2A. How can you express the MRT in this problem?

$$y = 10 - 2x$$

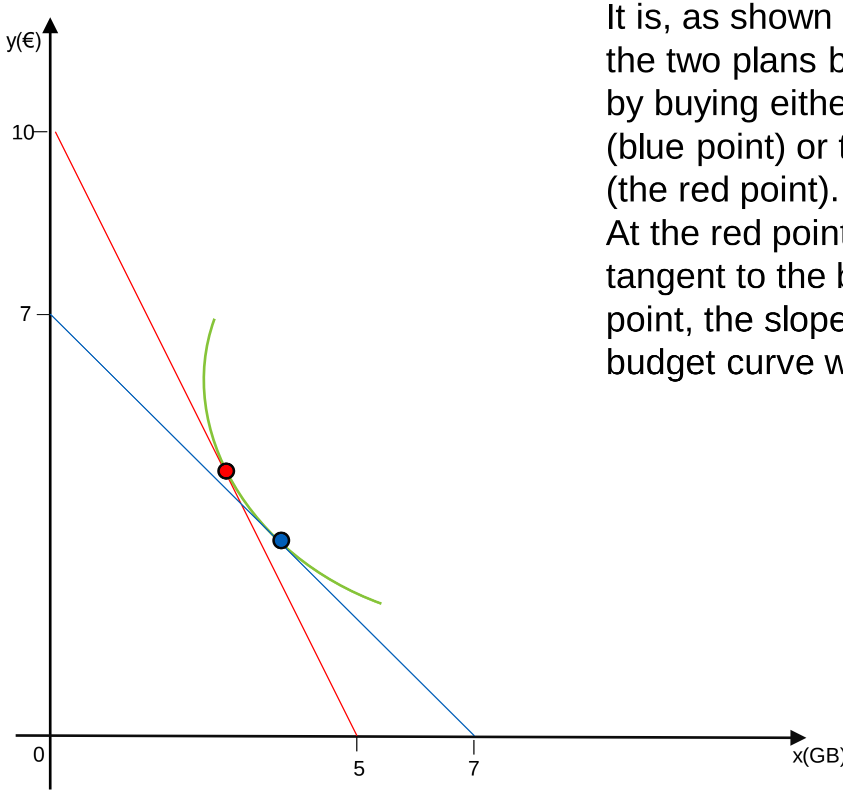
MRT is the slope of the budget constraint -2

Exercise 3 d

- Consider plans from part a and c of the previous question. Is it possible that a consumer is indifferent between the two plans?

Plan a:	**Plan c:**
$w = 10$ $p_x = 2$ $(p_y = \mathbf{\mathit{1}})$	$w = 10$ $f = 3$ $p'_x = 1$ $(p_y = \mathbf{\mathit{1}})$

Exercise 3 d



Consider plans from part a and c of the previous question. Is it possible that a consumer is indifferent between the two plans?

It is, as shown in the graph. The consumer is indifferent between the two plans because he can reach the same level of satisfaction by buying either the blue plan and consuming around 4 GB of data (blue point) or the red plan and consuming a bit more than 2.5 GB (the red point).
At the red point, the slope of the indifference curve is 2 (it is tangent to the budget curve with MRT=2), whereas at the blue point, the slope of the indifference curve is 1 (it is tangent to the budget curve with MRT=1).

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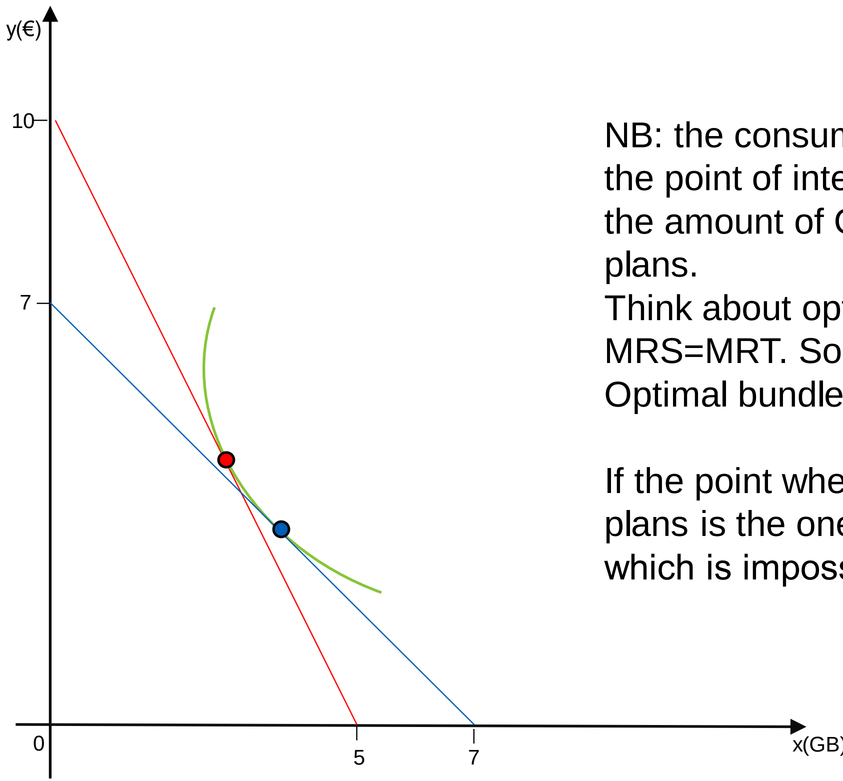
Exercise 3 d

Consider plans from part a and c of the previous question. Is it possible that a consumer is indifferent between the two plans?



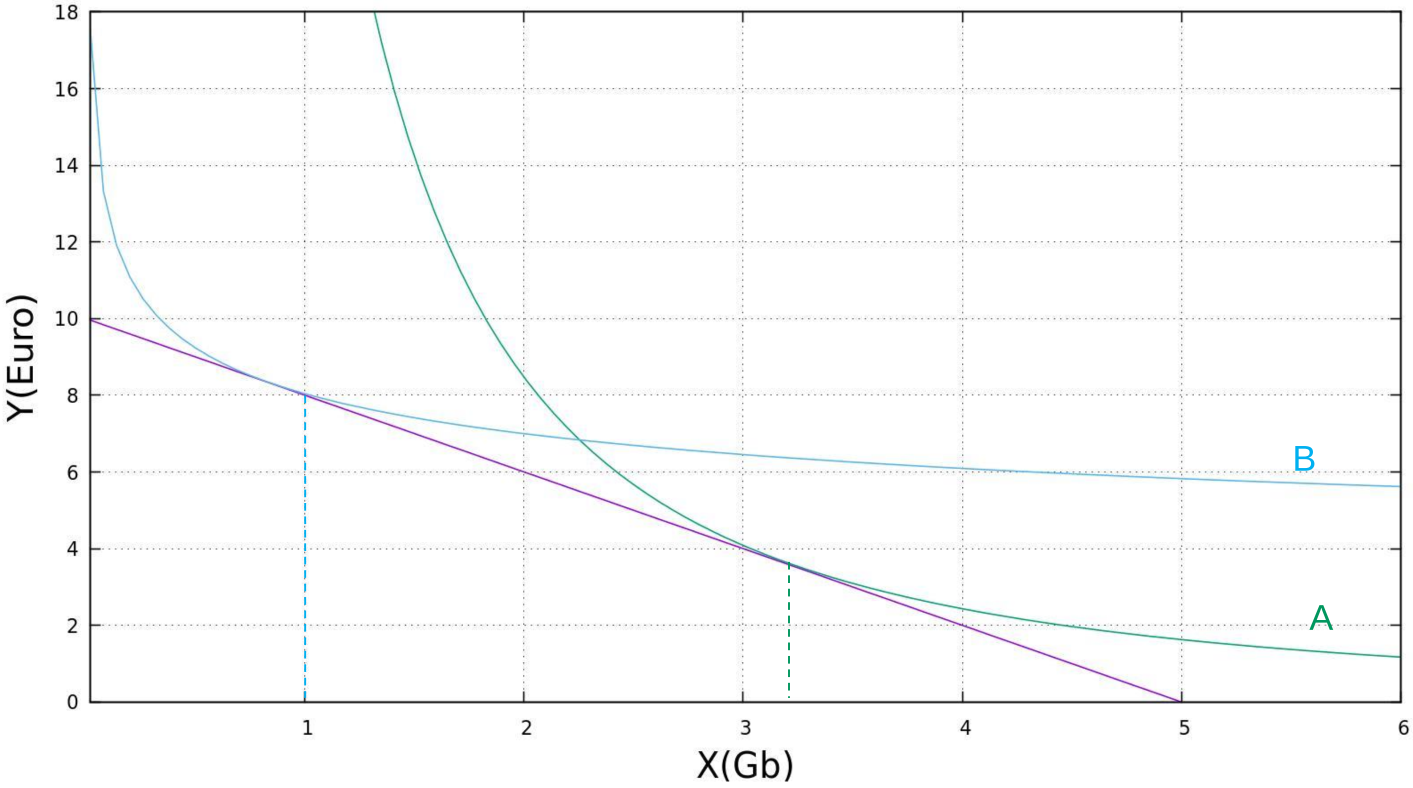
NB: the consumer cannot be indifferent between the two plans at the point of intersection, even though it is tempting to say so since the amount of GB and other consumption is the same in the two plans.
Think about optimal consumption. At the optimal bundle, MRS=MRT. So optimal bundle on blue line is MRS=MRTb=2. Optimal bundle on red line is MRS=MRTr=1.

If the point where the consumer is indifferent between the two plans is the one of intersection, it must be that MRS=MRTb=MRTr, which is impossible as 2≠1.

Exercise 3 e

- We say that Ann likes data more than Bob if the MRS of Ann (denoted by MRS_A) is higher than the MRS of Bob (MRS_B) at all $(x;\ y)$.
- Show by drawing the picture for the plan in 2.a) that at optimum, Ann chooses a higher *x* than Bob. (Hint: draw the picture for Ann's optimal choice and consider Bob's indifference curve through Ann's optimal consumption).

Exercise 3E



Ann likes data more than Bob: MRS^A is higher than MRS^B at all *(x,y)*

Exercise 3 f

- **Why would an operator want to let consumers choose from such a set rather than just offering one plan?**

Exercise 3F

Why would an operator want to let consumers choose from such a set rather than just offering one plan?

Price discrimination: offer different products to consumers with different preferences.
Some consumers might decide to not buy any internet at all, because the only available plan is not good enough for them. By offering a range of options, the phone operator can increase the customer base and revenue.

(how has the offer of telephone data plans evolved in Finland over the past few years?)

Exercise 4

Lifetime budget sets

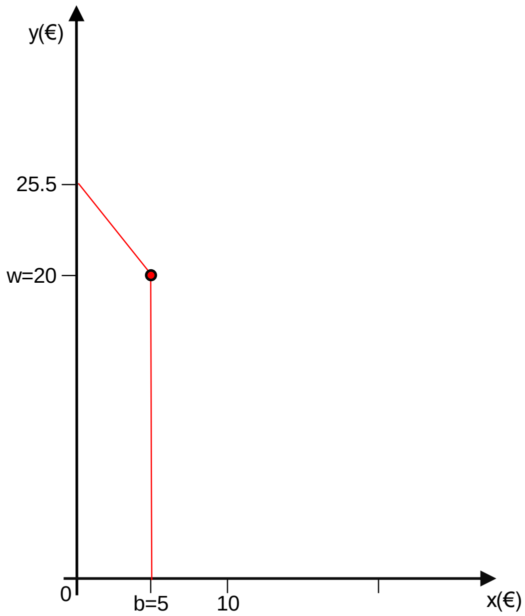
Exercise 4 a

Intertemporal budget:

- **Today, student benefit *b=5***
- **Tomorrow, wage *w=20***
- **Savings at interest rate *r=0.1***
- **Borrowing not allowed**

What is the MRT?

Exercise 4 a



$y = w + (1+r)(b-x)$ $y = 20 + 1.1(5-x)$	$x = b + \frac{w-y}{1+r}$ $x = 5 + \frac{20-y}{1.1}$

Cecilia can consume what she earns in each period: red point *(5,20)*. As she cannot borrow from future earnings, 5 is the maximum that she can consume when young, therefore we have a vertical line below the red point.
(consumption along the vertical line would mean that some money is left unspent when Cecilia is old, which is probably not going to happen: we can also leave out the vertical line altogether).

Cecilia can only save when young, so that she can consume more when old.
If $x = 0, y = 20 + 1.1 \cdot 5 = 25.5$ and that's the intercept on the y axis.

The MRT is the slope of the budget line -1.1, that is 1+r.

Exercise 4 b

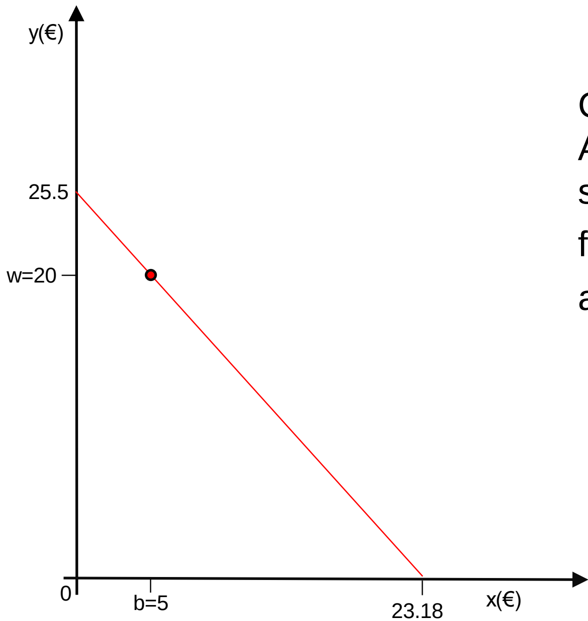
Intertemporal budget:

- **Today, student benefit *b=5***
- **Tomorrow, wage *w=20***
- **Savings at interest rate *r=0.1***
- **Borrowing at interest rate *r=0.1***

Draw the feasible set.

Exercise 4 b

$y = w + (1+r)(b-x)$ $y = 20 + 1.1(5-x)$	$x = b + \frac{w-y}{1+r}$ $x = 5 + \frac{20-y}{1.1}$



Cecilia can consume what she earns in each period: red point *(5,20)*. As she can now borrow from future earnings, she can borrow all that she is able to repay when she is old and consume nothing in the future. If $y = 0, x = 5 + \frac{20}{1.1} = 23.18$, and that's the intercept on the x axis.

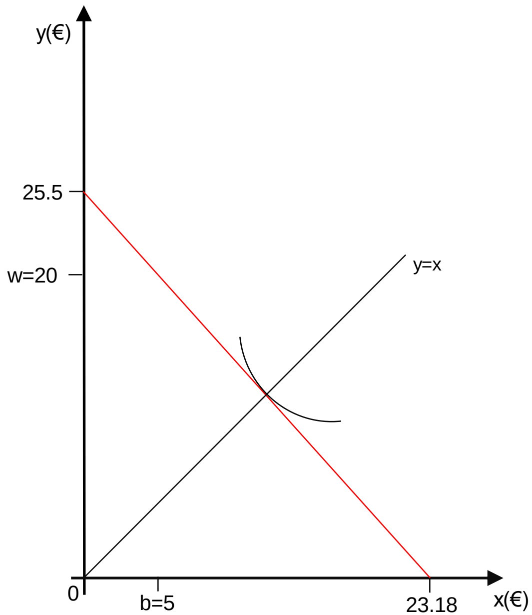
Exercise 4 c

Suppose next that has symmetric preferences over consumptions when young and when old.

This means that her MRS =1 at any point where x = y, i.e. when her consumption when student and when working are the same.

Is it optimal for Cecilia to consume the same amounts when student and when working?

Exercise 4 c



Is it optimal for Cecilia to consume the same amounts when student and when working?

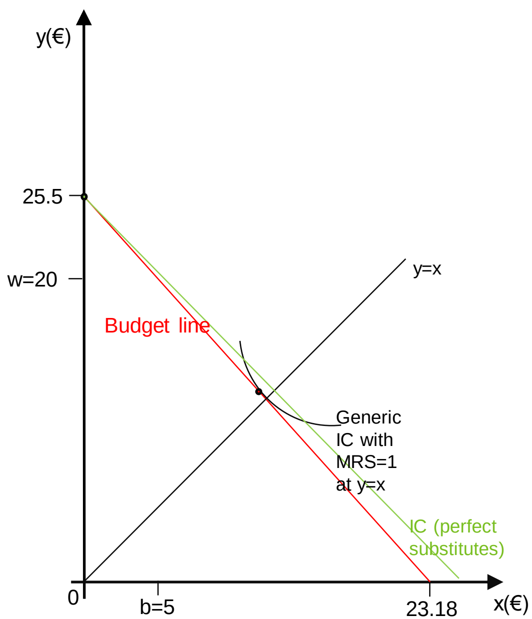
- The indifference curve shown on the graph has MRS=1 in the point where they intersect the line *y=x.*
- We already said that the slope of the budget line is *MRT=1.1*.
- Optimum consumption requires MRT=MRS, but this is never true on the *y=x* line, since MRT=1.1 ≠ 1.

It is not optimal to consume the same amount in the two periods.

Exercise 4 d

When should Cecilia consume more? Can you decide from the information thus far if Cecilia should borrow or save?

Exercise 4 c



When should Cecilia consume more? Can you decide from the information thus far if Cecilia should borrow or save?

With this information, all we know is that the optimal consumption bundle for Cecilia will be on the left of the point where y=x, meaning that we know that she **will consume more when she is a worker than when she is a student, ie. x*<y***.

Knowing that MRS=1 when x=y is not enough to determine the exact shape of the indifference curve and where the optimum will be when MRT=1.1. And therefore, **we do not know if she will borrow or save.**

In fact, there are virtually infinite indifference curves that satisfy that property.

- One is the extreme which is a "perfect substitutes" indifference curve as shown in the graph: optimum would be where x*=0 and y*=25.5.
- The more open the IC (i.e. the closer to the extreme where it is a line), the smaller x* will be.
- The less open the IC, the larger x* will be.
- But at most x* is only slightly smaller than y*, similarly to what we see in ex 5.

Exercise 5

Budget constraint: $y = w + (1 + r)(b - x)$

Slope of the budget constraint: $MRT = 1.1$

Symmetric preferences over present and future consumption: $MRS = y/x$

Find optimal consumption bundle x* and y*

Exercise 5

$$y = w + (1 + r)(b - x)$$
$$y = 20 + 1.1(5 - x)$$
$$y = 25.5 - 1.1 \cdot x$$

$$MRT = 1.1\ ;\ MRS = y/x$$

Optimal consumption requires: $MRT = MRS$

Therefore, at the optimal consumption: $1.1 = y^*/x^* \rightarrow y^* = 1.1 \cdot x^*$

Let's substitute $y^* = 1.1 \cdot x^*$ into the expression for the budget constraint $(y = 25.5 - 1.1 \cdot x)$

$$1.1 \cdot x^* = 25.5 - 1.1 \cdot x^*$$
$$2.2 \cdot x^* = 25.5 \rightarrow x^* = 25.5/2.2 = 11.591$$

Then, we substitute the value for x* into the expression for y* obtained above

$$y^* = 1.1 \cdot x^* = 1.1 \cdot 11.591 = 12.75$$

So optimal consumption when working is $\underline{y^* = 12.75}$; optimal consumption as a student is $\underline{x^* = 11.591}$

Therefore, is optimal to borrow $12.75 - 5 = 7.75$ as a student.