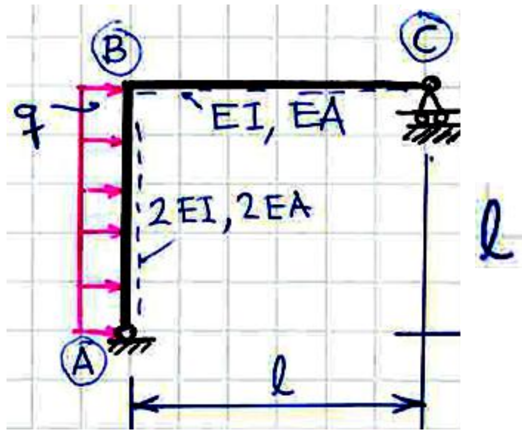
Note that remote exam will be for 4 hours.

CIV-E1020 - Mechanics of Beam and Frame Structures – Examination 6.2.2020 duration: 3h

It is compulsory to solve THREE EXERCISES. There is no extra exercises: #1, #2 and #3 or #1, #2 and #4. If someone solves all the four exercises, then the fourth exercise - #3 or #4 - with fewest points obtained will not be graded.

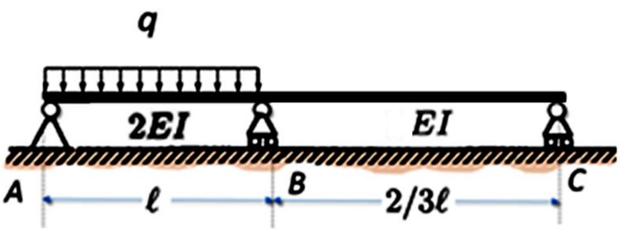
- Formulary are enclosed with the questions
- The material is linear elastic in all the structures



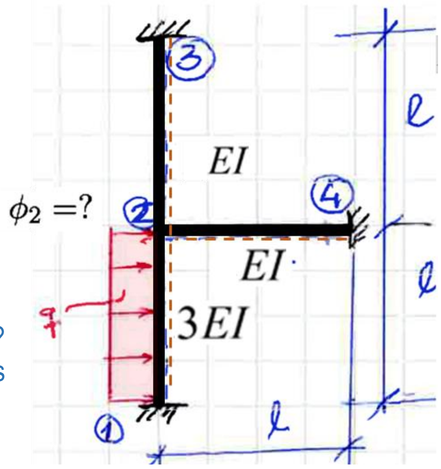
1. Use the dummy unit-load theorem (or method) and determine the horizontal displacement of the roller at support **C** [5 pnts]. Support **A** is hinged.

Determine the support reactions and draw accurately the needed internal force diagrams *N* and *M*. Account for both bending and stretching/compression in determining the displacement.

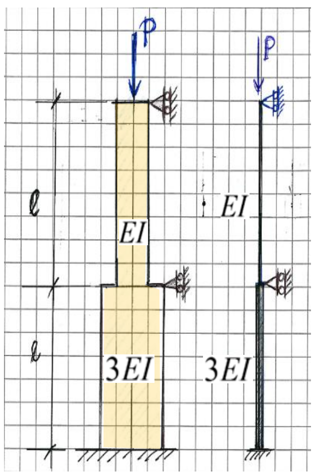
Grading 3 obligatory exercises)		GRADE
Points in this exam		
14.25	15	5
12.75	13.5	4
9.75	12	3
8.25	9	2
6	7.5	1
<6	fail	0



2. What is the degree of statical indeterminacy?
Use the general force method and **determine the bending moment at the mid-support B for the continuous beam**. Draw accurately the all needed bending moment diagrams. [5 p]
Ignore the effect of shear deformation in the flexibility coefficients (account only for bending effects)



3. Use Slope-Deflection Method **and determine the bending moment** at the rigid support *3*. All the supports are rigid (jäykkä kiinnitys)
a) Rotation at node 2? [4 pnts] $\phi_2 =?$
b) Bending moment at node 3? [1 pnt]
Hint: Is it a sway or non-sway frame? What is the degree of kinematical indeterminacy? (=number of independent translation kinematical degrees of freedom). The brown dot-lines shows the positive sides of the beams (or columns)



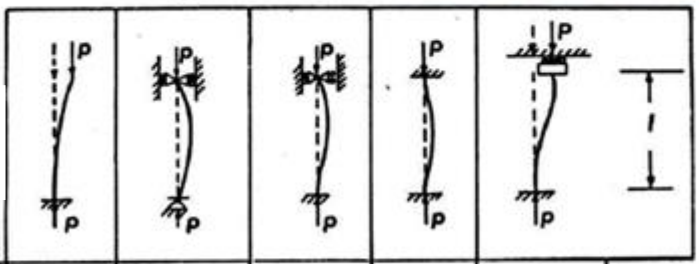
4. Use Slope-Deflection Method and derive the expression of **the *criticality condition*** to determine the critical buckling load for this continuous column [4p]. No need to solve numerically for the buckling load. The force *P* is centrically applied to the column.

Sketch the corresponding buckling mode.
Use **Euler** basic buckling cases & give a bracket (=haarukoi, in Finnish) for the value of this critical load? [1 pnt]

The slope-deflection method:

Euler's basic buckling cases Eulerin perusnurjahdus

$$P_{cr} = \mu \frac{\pi^2 EI}{\ell^2}$$



Tapaus	1	2	3	4	5
μ	0.25	1	2.046	4	1

$$M_{ij} = A_{ij}\phi_{ij} + B_{ij}\phi_{ji} - C_{ij}\psi_{ij} + \bar{M}_{ij} \qquad M^0_{ij} = A^0_{ij}\varphi_{ij} - C^0_{ij}\psi_{ij} + \bar{M}^0_{ij}$$

Beam-column with constant flexural rigidity:

$$A_{ij} = A_{ji} = \frac{2\psi(kL)}{4\psi^2(kL) - \phi^2(kL)}\frac{6EI}{L}, \qquad B_{ij} = B_{ji} = \frac{\phi(kL)}{4\psi^2(kL) - \phi^2(kL)}\frac{6EI}{L}$$

$$C_{ij} = A_{ij} + B_{ij} \qquad A^0_{ij} = C^0_{ij} = \frac{1}{\psi(kL)}\frac{3EI}{L},$$

$$kL \equiv L\sqrt{\frac{P}{EI}}$$

Berry's functions:

Olkoon $\lambda \equiv kL,$

$$\lambda \equiv kL$$

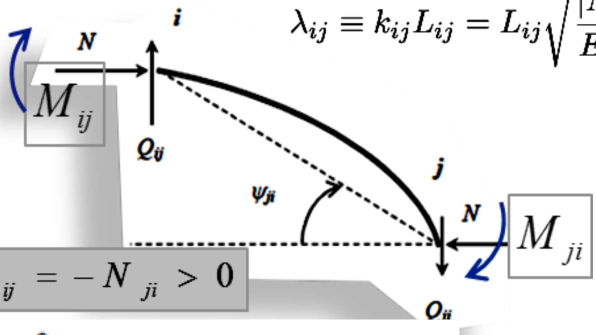
Puristettu sauva:

Compression: $\phi(\lambda) = \frac{6}{\lambda}\left(\frac{1}{\sin\lambda} - \frac{1}{\lambda}\right), \ \psi(\lambda) = \frac{3}{\lambda}\left(\frac{1}{\lambda} - \frac{1}{\tan\lambda}\right), \ \text{ja} \ \chi(\lambda) = \frac{24}{\lambda^3}\left(\tan\frac{\lambda}{2} - \frac{\lambda}{2}\right),$

Vedetty sauva:

Extension: $\phi(\lambda) = \frac{6}{\lambda}\left(-\frac{1}{\sinh\lambda} + \frac{1}{\lambda}\right), \ \psi(\lambda) = \frac{3}{\lambda}\left(-\frac{1}{\lambda} + \frac{1}{\tanh\lambda}\right), \ \text{ja} \ \chi(\lambda) = \frac{24}{\lambda^3}\left(-\tanh\frac{\lambda}{2} + \frac{\lambda}{2}\right),$

$$\lambda_{ij} \equiv k_{ij}L_{ij} = L_{ij}\sqrt{\frac{|N_{ij}|}{EI_{ij}}} \rightarrow \lambda \equiv kL = L\sqrt{\frac{|N|}{EI}}$$



$$N \equiv -N_{ij} = -N_{ji} > 0$$

$$Q_{ij} = Q^0_{ij} - (M_{ij} + M_{ji})/L - N\psi_{ij}$$

$\overline{M}_{12} \equiv MK_1$

$\overline{M}_{ij}, \overline{M}_{ji}$

N:o	Kuormitus	Kiinnitysmomentit: MK_1, MK_2
1	q, L	$MK_1 = -\frac{qL^2}{12}$, $MK_2 = \frac{qL^2}{12}$
2		

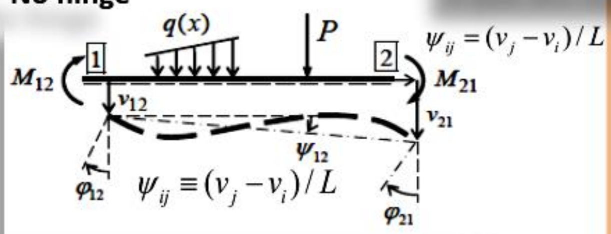
$\overline{M}_{ij}$ $\overline{M}_{ji}$

N:o	Kuormitus	Nivel oikeassa päässä: MK_1^0	Nivel vasemmassa päässä: MK_2^0
1	q, L	$MK_1^0 = -\frac{qL^2}{8}$	$MK_2^0 = \frac{qL^2}{8}$

The stiffness equation relating the end-moments to the end-displacements

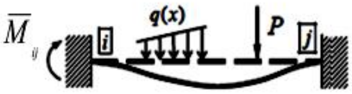
If you are using lecture's notations

No hinge



$$M_{ij} = a_{ij}\varphi_{ij} + b_{ij}\varphi_{ji} - c_{ij}\psi_{ij} + \overline{M}_{ij},\ \ i \neq j$$

$$a_{ij} = \frac{4EI}{L},\ b_{ij} = \frac{2EI}{L},\ c_{ij} = \frac{6EI}{L} \quad (EI\text{-constant})$$



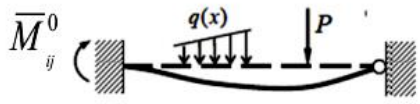
Fixed end-moment resulting from external mechanical loading, look from tables

One node is hinged

The is a superscript "0" means that the support at end *j* is hinged

$$M_{ij}^0 = a_{ij}^0\varphi_{ij} - c_{ij}^0\psi_{ij} + \overline{M}_{ij}^0$$

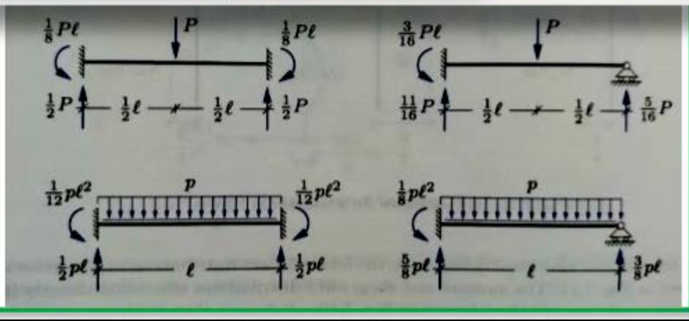
$$a_{12}^0 = c_{12}^0 = \frac{3EI}{L} \quad \psi_{ij} = (v_j - v_i)/L$$



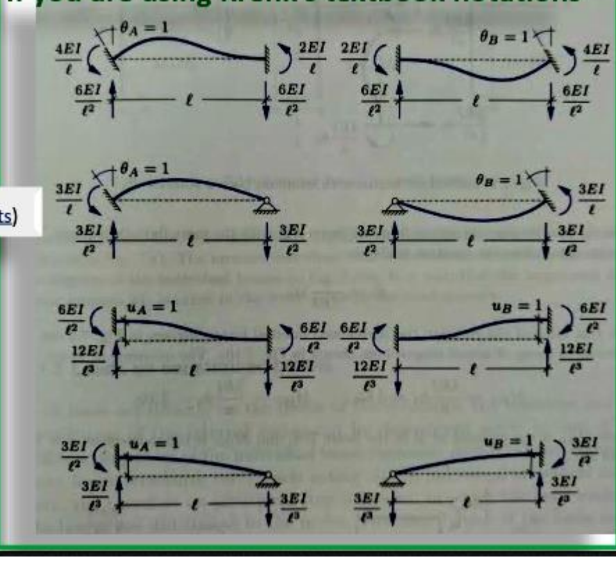
Fixed end-moment resulting from external mechanical loading, look from tables

If you are using Krenk's textbook notations

Bending Moments (pay attention to the sign convention to convert to Fixed-End-Moments)

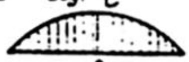
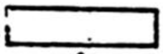
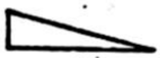
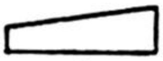
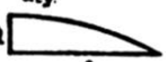


If you are using Krenk's textbook notations



Maxwell-Mohr integrals table

TABLEAU DES INTEGRALES $\int_0^{\ell} {}^iM\,{}^kM\,dx$

${}^kM$ \ ${}^iM$					
	$ac\ell$	$\frac{1}{2}ac\ell$	$\frac{1}{2}ad\ell$	$\frac{1}{2}a\ell(c+d)$	$\frac{2}{3}ac\ell$
	$\frac{1}{2}ac\ell$	$\frac{1}{3}ac\ell$	$\frac{1}{6}ad\ell$	$\frac{1}{6}a\ell(2c+d)$	$\frac{1}{3}ac\ell$
	$\frac{1}{2}bc\ell$	$\frac{1}{6}bc\ell$	$\frac{1}{3}bd\ell$	$\frac{1}{6}b\ell(c+2d)$	$\frac{1}{3}bc\ell$
	$\frac{1}{2}(a+b)c\ell$	$\frac{1}{6}(2a+b)c\ell$	$\frac{1}{6}(a+2b)d\ell$	$\frac{\ell}{6}[a(2c+d) + b(c+2d)]$	$\frac{1}{3}(a+b)c\ell$
	$\frac{1}{3}ac\ell$	$\frac{1}{4}ac\ell$	$\frac{1}{12}ad\ell$	$\frac{1}{12}a\ell(3c+d)$	$\frac{1}{5}ac\ell$
	$\frac{2}{3}ac\ell$	$\frac{5}{12}ac\ell$	$\frac{1}{4}ad\ell$	$\frac{1}{12}a\ell(5c+3d)$	$\frac{7}{15}ac\ell$