Aalto University

Shapes in Action
Sept 22nd

Spherical patterns

Program schedule for Sept 22nd

15:15 Weekly exercise

Magic theorem for spherical symmetries

Instructions for a folding activity

16:00 Break

16:15 Spherical symmetry classes

17:00 Working in groups/individually

Possible orbifolds for planar patterns

Orientable

Sphere (632 442 333 2222)

Torus o

Annulus **

Disk (*632 *442 *333 *2222
2*22 4*2 3*3 22*)

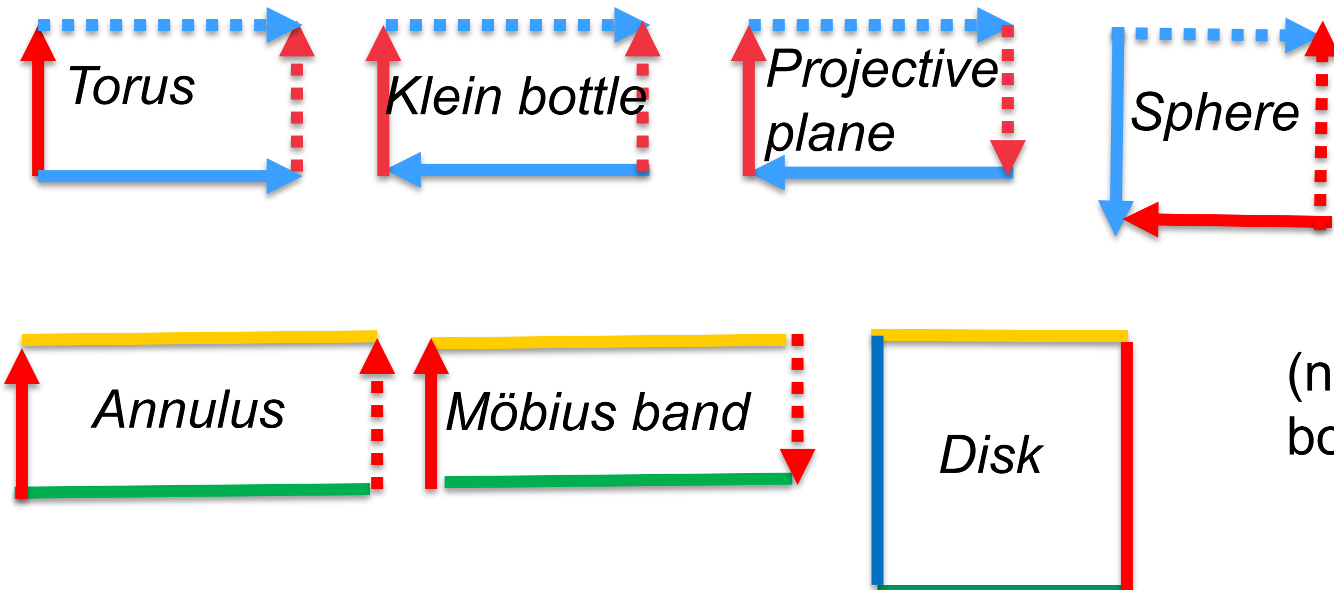
Non-orientable

Projective plane 22x

Klein bottle xx

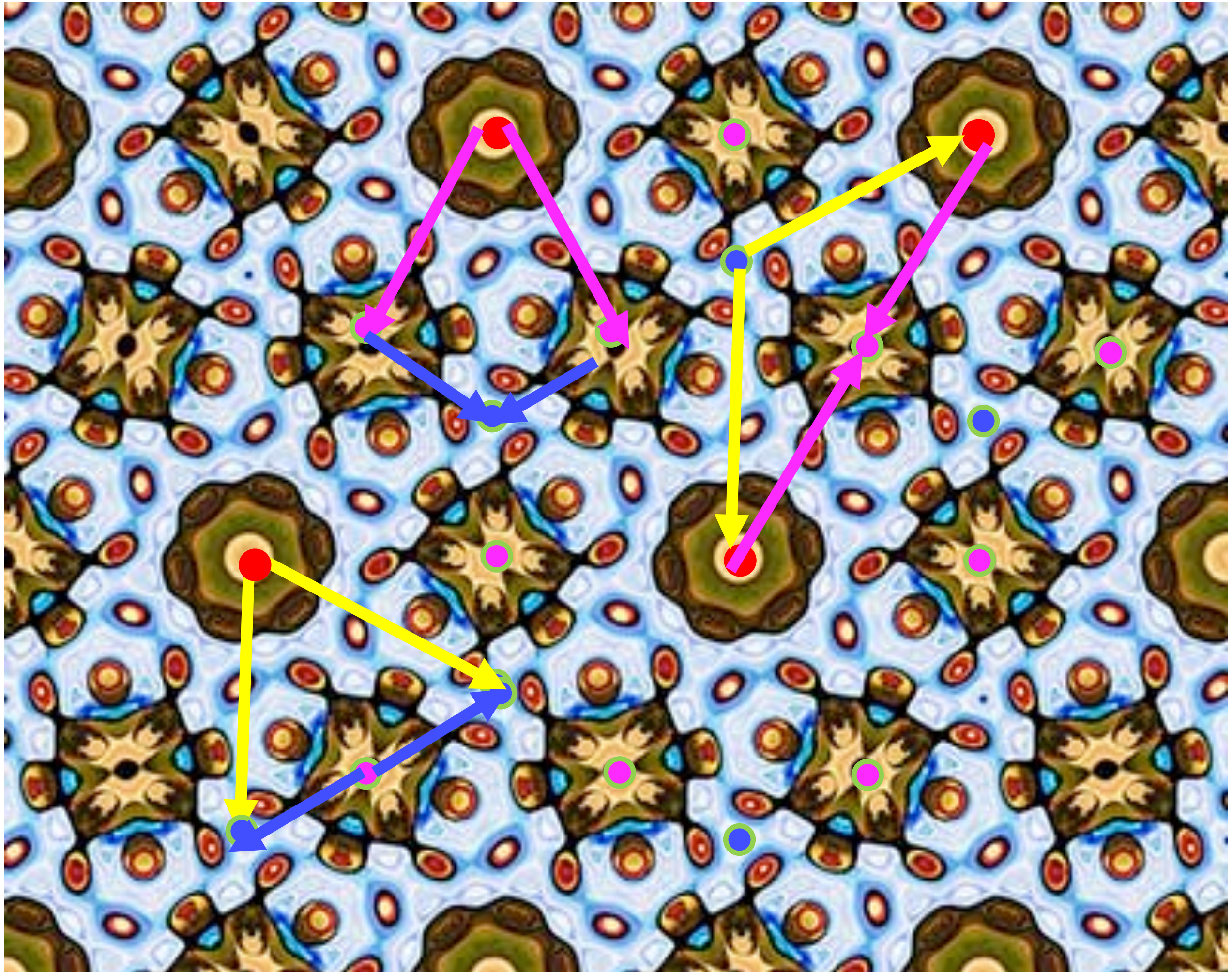
Möbius band *x

Orbifolds (of planar patterns) through boundary identifications



(no boundary after gluing)

(non-empty boundary)



632

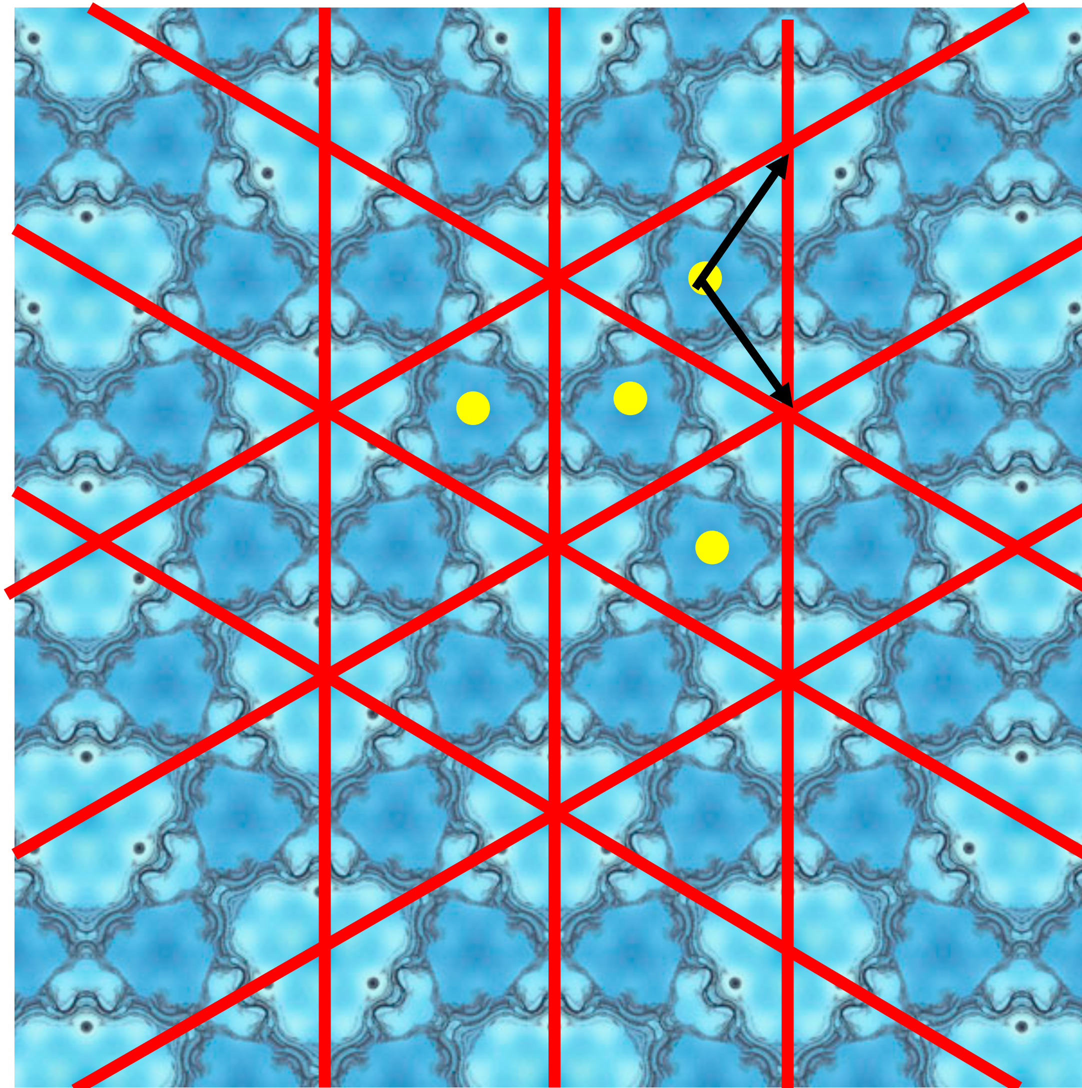
- Three rotation points
- Three possibilities for a fundamental domain

Order 6 rotation

Order 3 rotation

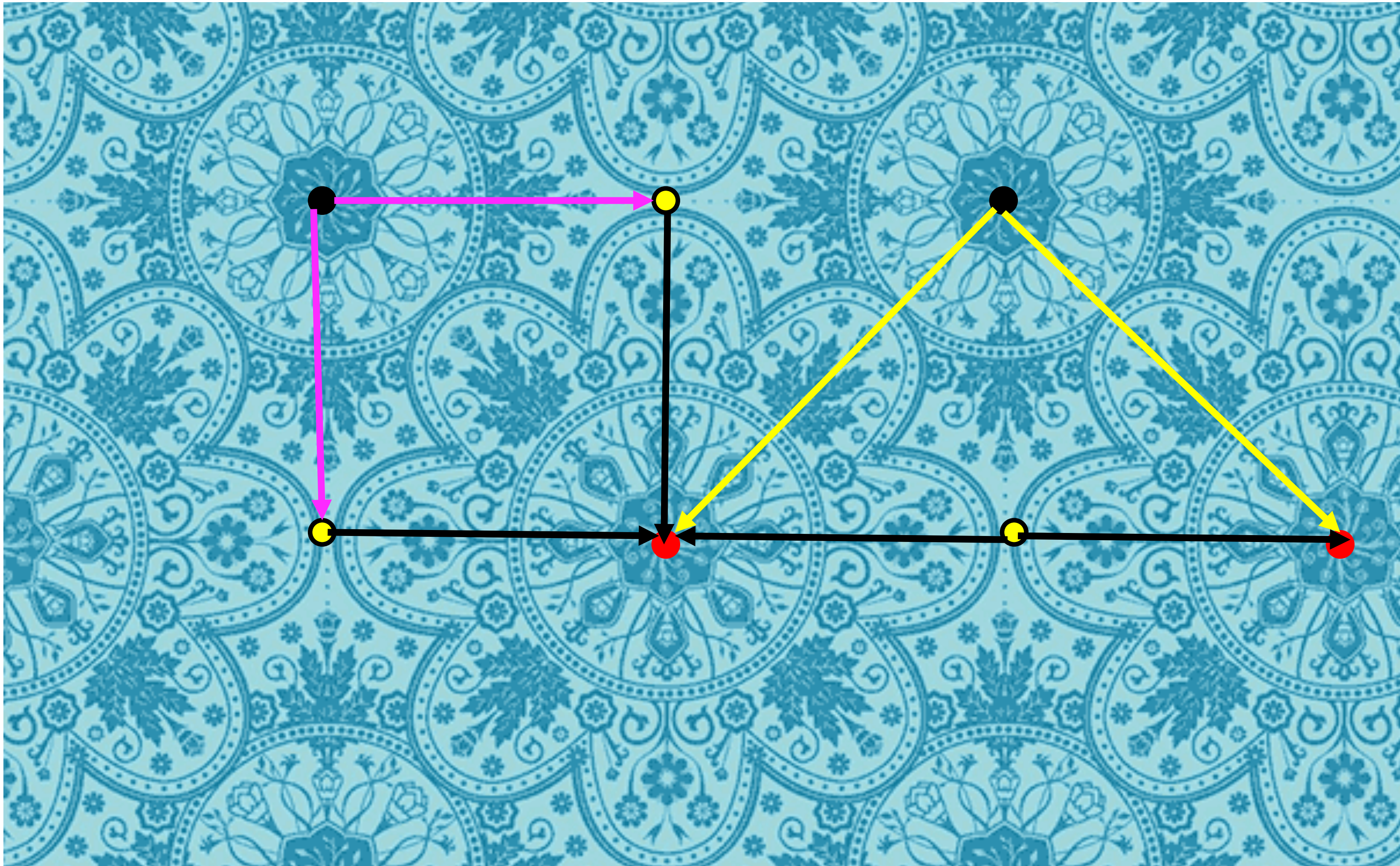
Order 2 rotation

1.



3*3

2.

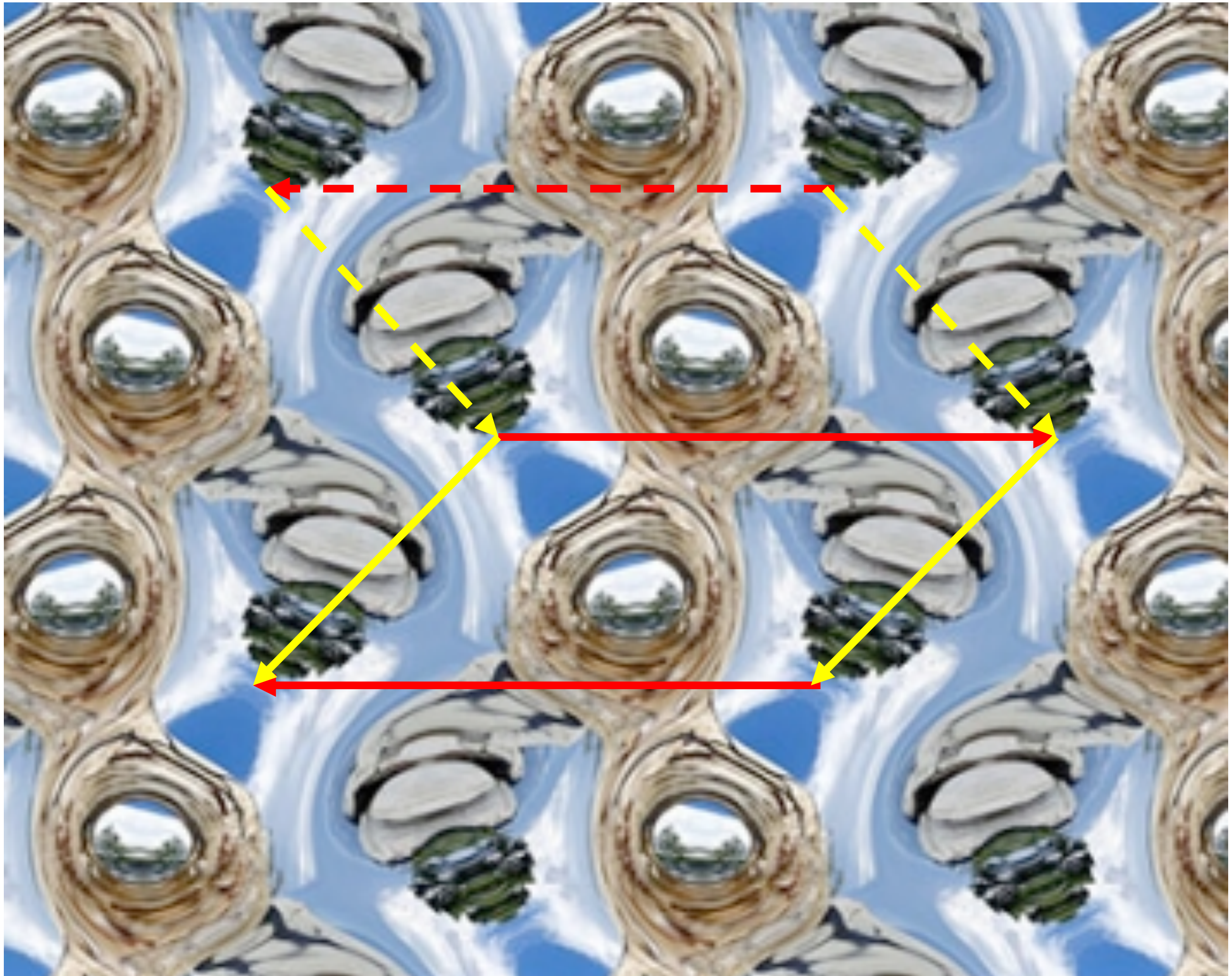


442

3.

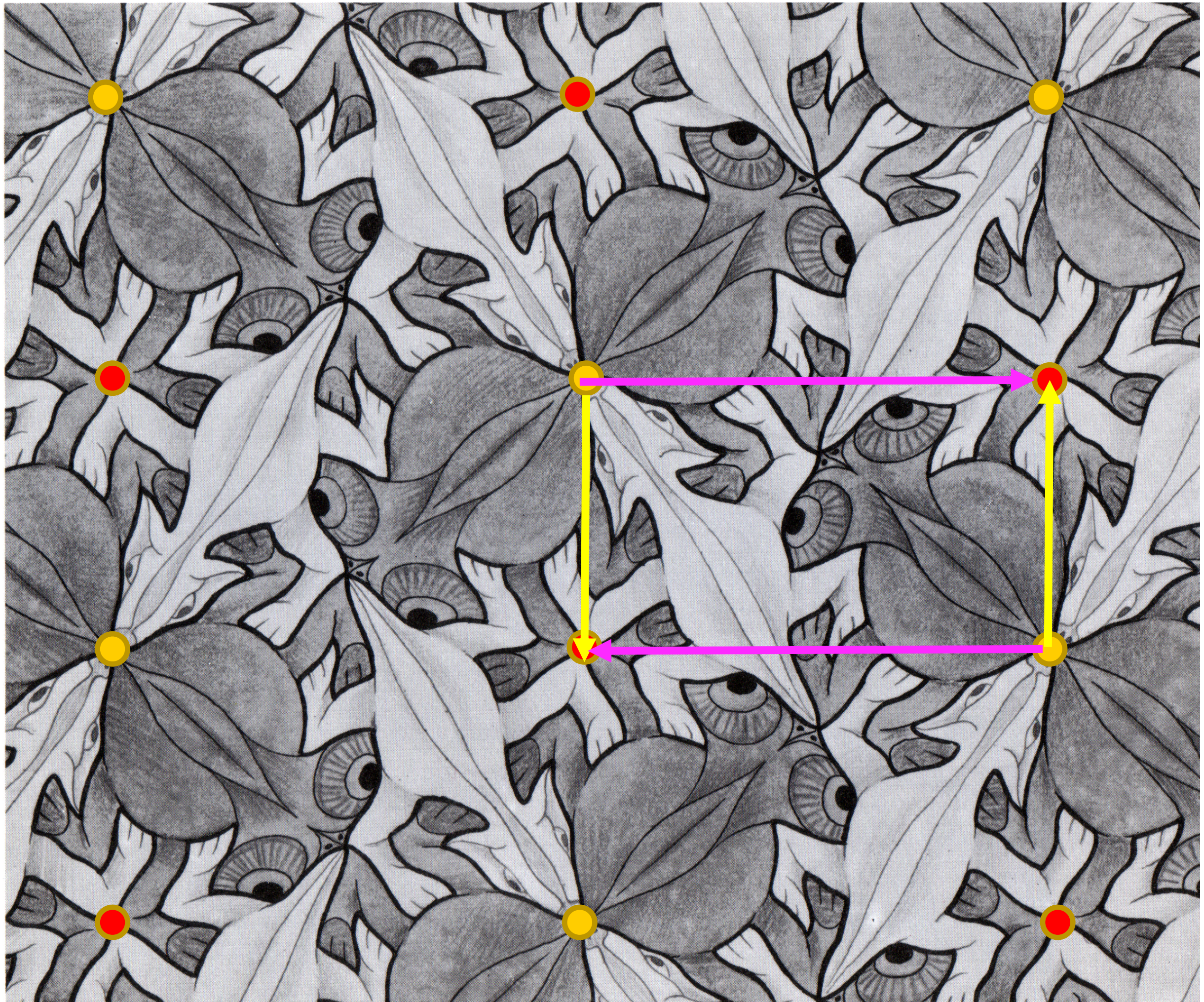
22*

4.



xx

5.



22x

6.

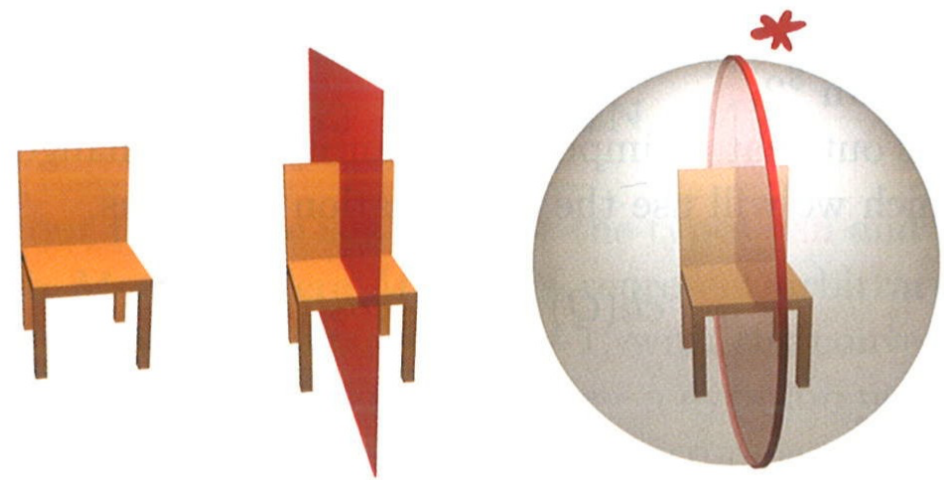
What about spherical symmetries?

Rotation lines (vs points) and reflection planes (vs lines)

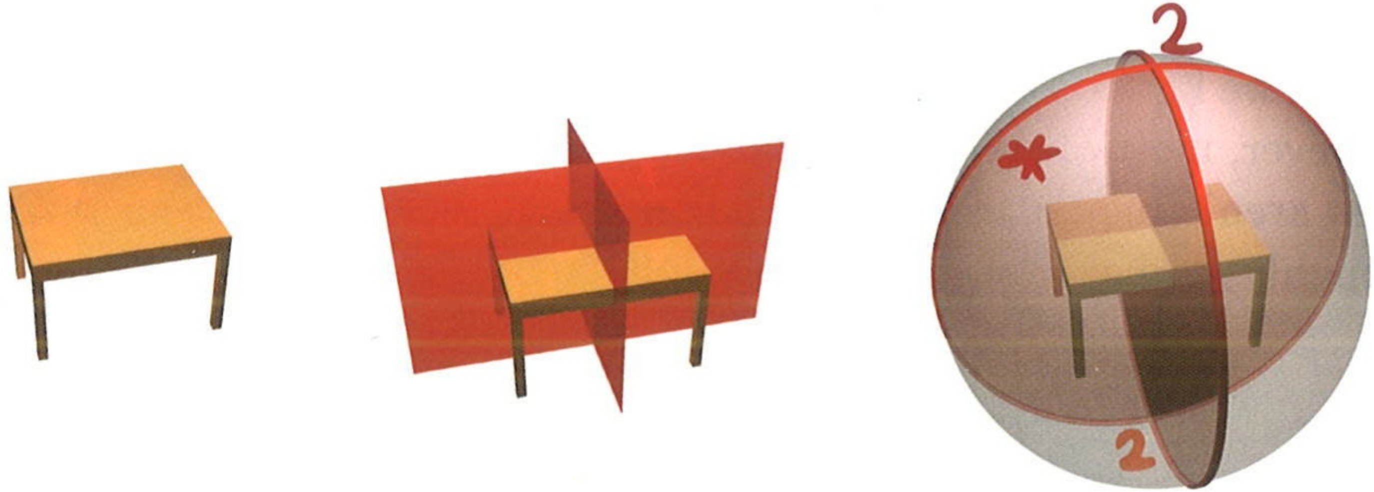
Temari balls, Bathsheba sculptures, ….

Spherical patterns are *cheaper* than planar patterns. (Will see….)

Ex: Bilateral symmetry = * interpreted as a reflection wrt to plane cost only 1 euro



Price of a rectangular table



Two intersecting reflection planes give signature *22, which cost **1+1/4+1/4=3/2** euro **=>** spherical patterns can have **different total prices.**

An important quantity ch=change (in euros)

Change from signature Q: **ch**(Q) = 2-**cost**(Q) euro

Above:

- **For the chair: ch(*)=2-cost(*)=2-1=1 euro**
- **For the table: ch(*22)=2-cost(*22)=2-3/2=1/2 euro**

The Magic Theorem for spherical patterns

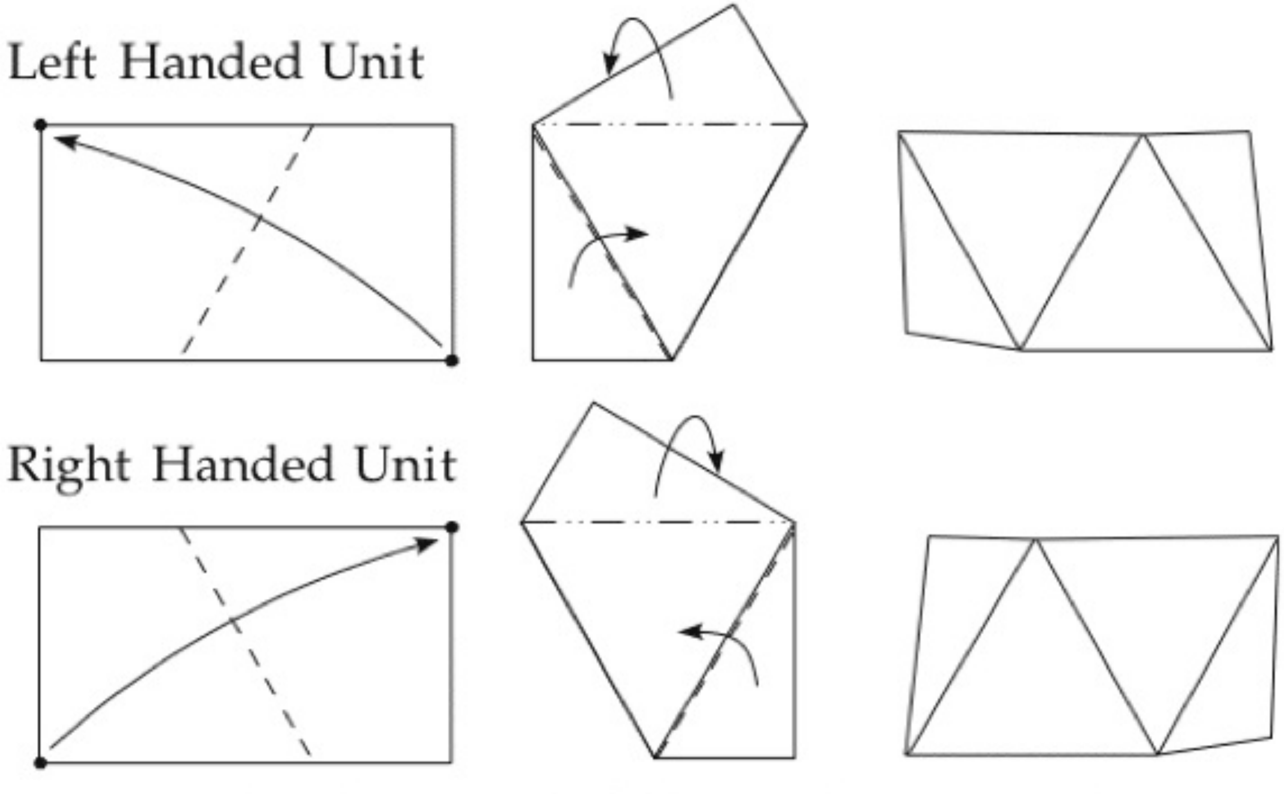
The signature of a spherical pattern costs exactly 2-2/d euros, where d is the total number of symmetries of the pattern.

Note:

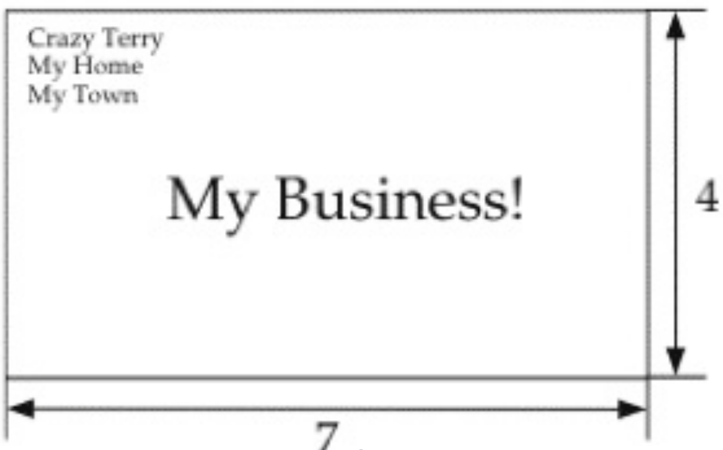
- **ch = 2/d**
- **for the chair d=2, for the table d=4**
- **In the plane case: d=∞ => only *one* Magic Theorem**

Lets produce some objects for analysis via folding …

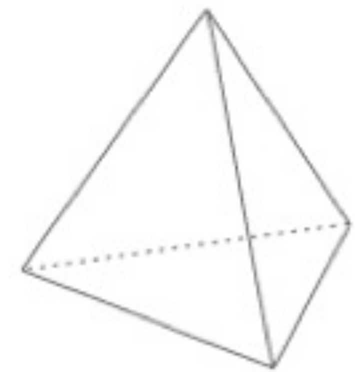
Business card modules (T. Hull, J. Mosely, K. Kawamura)



Are triangles equilateral ? Why ?

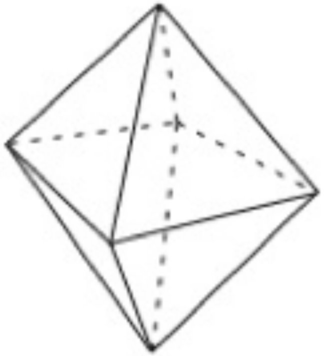


1) Make one left handed and one right handed module and try to **lock** them to a tetrahedron



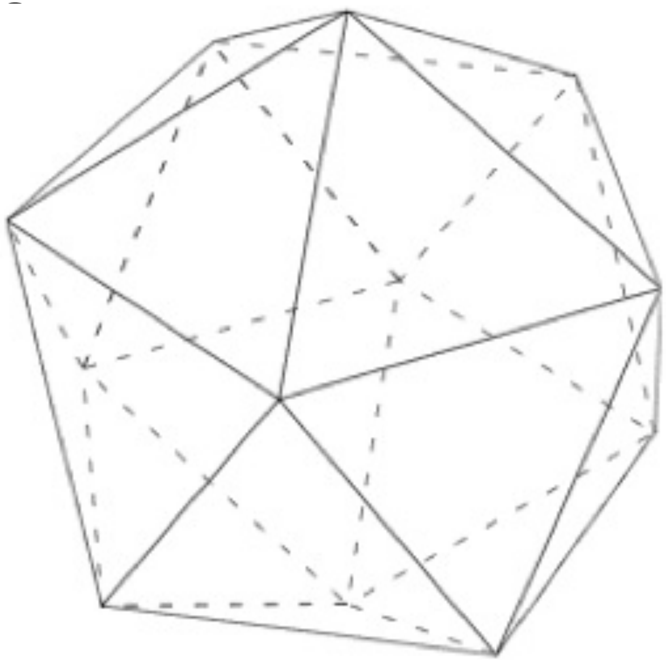
- Mark the reflection lines on your module
- What is the fundamental domain/orbifold ?
- How many reflection lines (=reflection plane intersection with the module) meet on the vertices of the fundamental domain?
- What is the number of symmetries ?
- Check that the Magic theorem holds

2) Construct an octahedron from 4 units



- Same questions as for the tetrahedron above
- Calculate V-E+F, V=number of vertices, E=number of edges, F= number of faces (also for the tetrahedron)

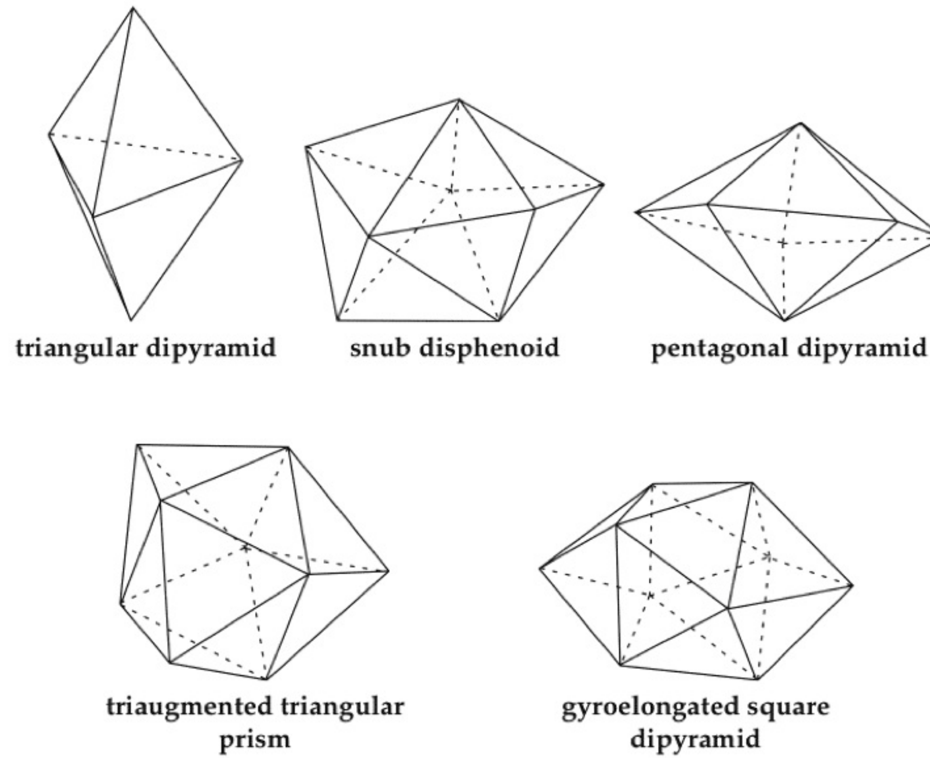
Possible to construct also an icosahedron from these modules



Hint: Use tape in construction
What other polyhedrons can be constructed from these modules ?

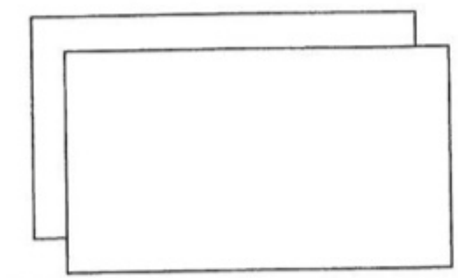
Same questions as for previous polyhedrons

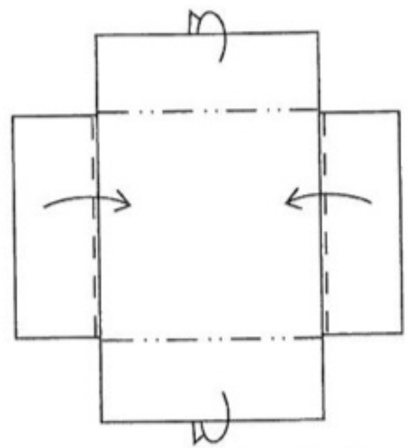
Johnson solids with triangular faces

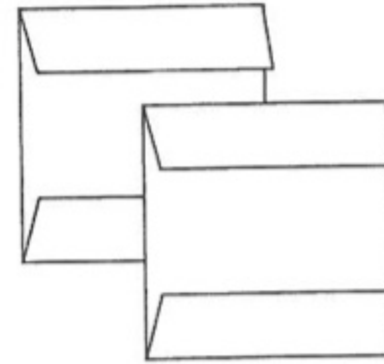


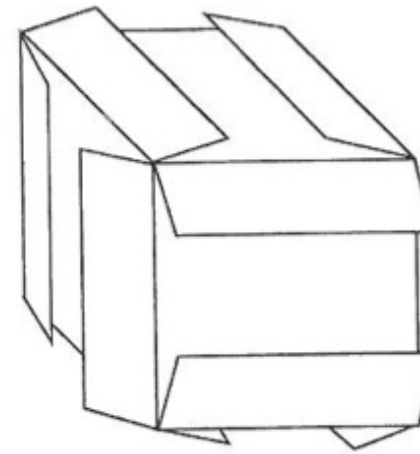
Business card cube

6 modules (one/face) constitute a ('unpaneled') cube, that can be joined together with flaps that remain outside.





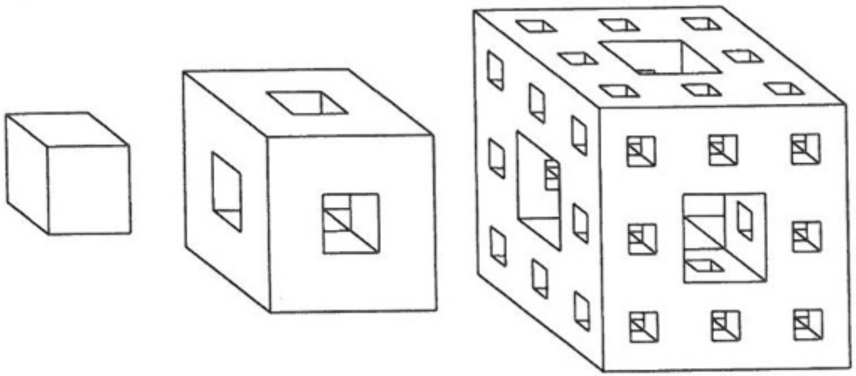




How do you 'panel' a cube **?**

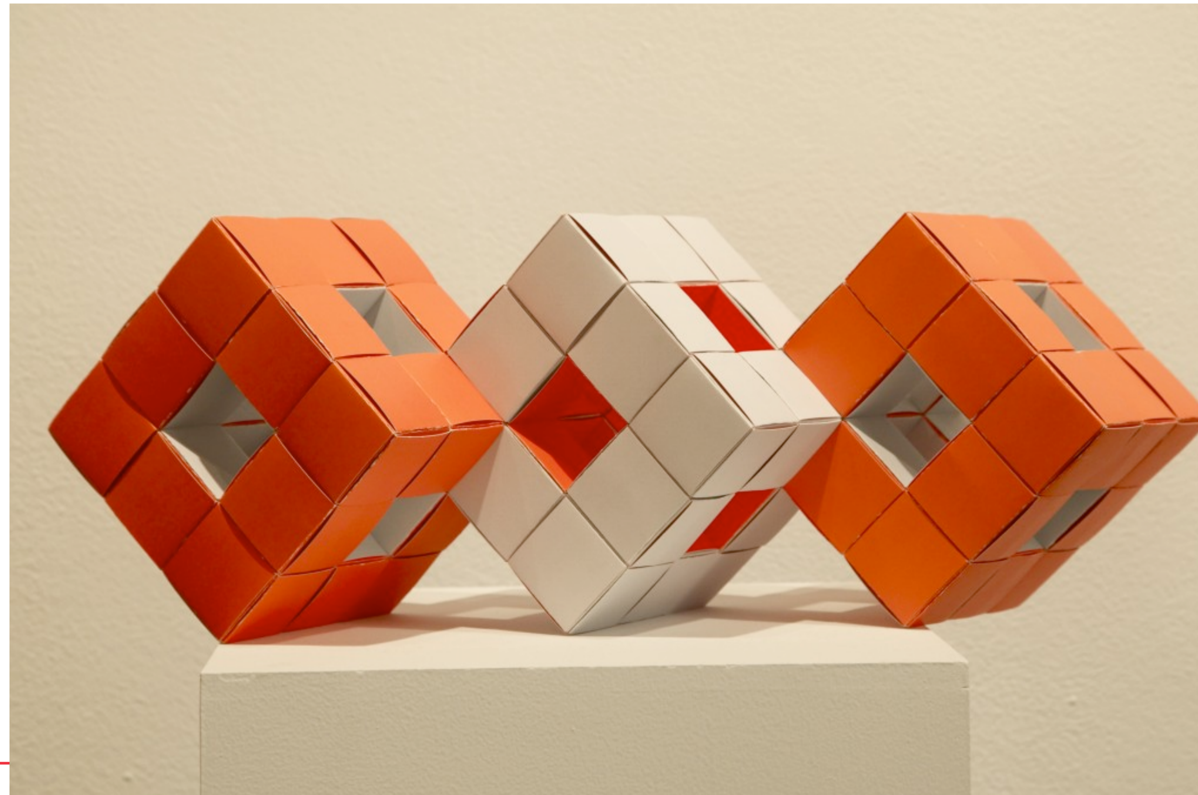
Same questions as for earlier polyhedrons !

Building idea: Menger's Sponge



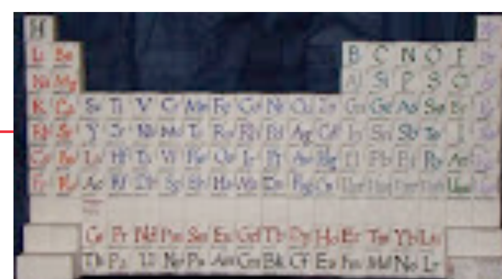
Jeannine Mosely
66048 business cards

Three interlinked Level One Menger Sponges, by Margaret Wertheim.



Mosely snowflake sponge 2012
49 000 business cards

Union Station 2014, Worcester
more thand 60 000 business cards,
500 assistants



James Lucas 2011,
periodic table
1414 business cards

14 different spherical symmetry classes

*532	*432	*332	*22N	*NN
				N*
		3*2	2*N	
				Nx
532	432	332	22N	NN

Note:

- **N=** 1,2,3… **but** digits 1 are omitted
- 1*=*11=*
- **However:** For example 11 11 = two rotation points of order 11

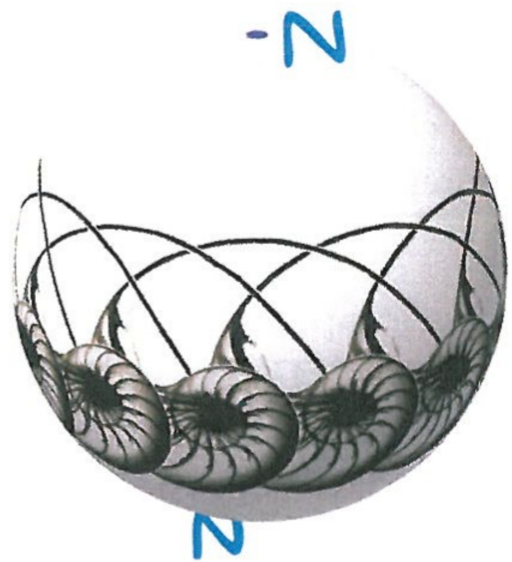
The five ‘true blue’ types (first one)

Total cost = 2-2/d <2 for every **d=** 1, 2, 3, …. =>

- no wonder rings ○
- no more than 3 digits (distinct to 1): (N-1)/N ≥ ½ for all, N=2,3,…
- if three digits, then **at least one** must be 2 (⅔+⅔+⅔=2, (N-1)/N≥⅔ for all N≥3)

Two digit case: MN

(In fact only case M = N occurs)



Case two 2's: 22N (second)

$1+(N-1)/N<2$ for all $N=2,3,4,5,\ldots$

Last 3 of the five 'true blue' types

Three digits, one 2:

- one digit must be 3 ($\frac{1}{2}+\frac{3}{4}+\frac{3}{4}=2$)
- the remaining digit must be 3, 4 or 5 ($\frac{1}{2}+\frac{2}{3}+\frac{5}{6}=2$)

$\Rightarrow$ 332, 432, 532

Note: ch(332)=2-($\frac{2}{3}+\frac{2}{3}+\frac{1}{2}$)=$\frac{1}{6}$**=2/12**

ch(432)= 2/24

ch(532)=2/60

The five ‘true red’ types

No **, *x, xx signatures, all of type *AB…N

$ch(*AB\ldots N)=2-1-((A-1)/2A+\ldots+(N-1)/2N)$,

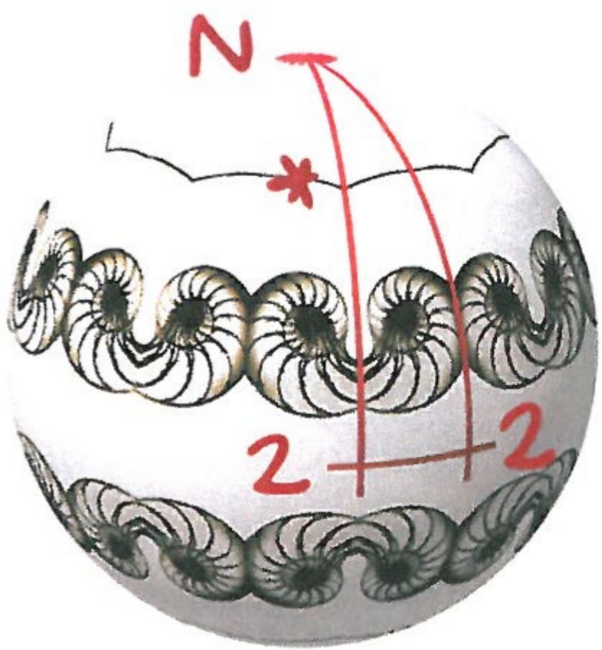
$ch(AB\ldots N)=2-((A-1)/A+\ldots+(N-1)/N)$,

=>

$ch(*AB\ldots N)=\frac{1}{2}ch(AB\ldots N)$

Note: only *NN is possible with two digits !

*22N



*MN2

*432, *532, *332

ch(*332) = $2-(1+\frac{1}{4}+\frac{1}{3}+\frac{1}{3})= 1/12$

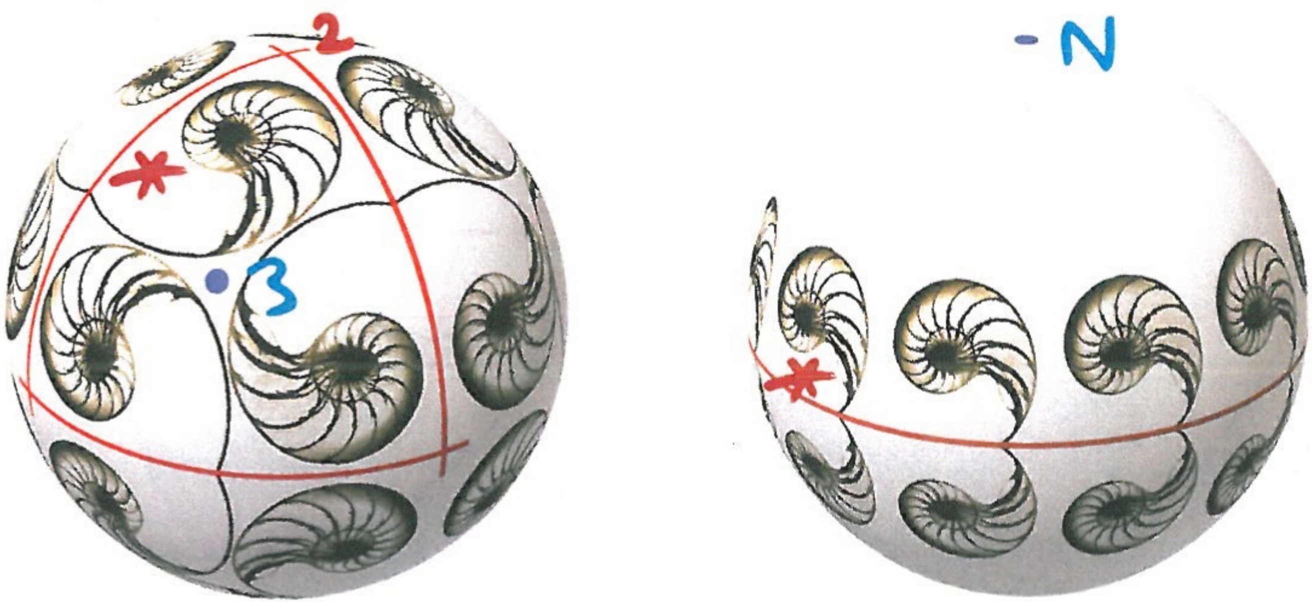
Compare with orientation reversing symmetries of five platonic solids.

The four Hybrid types

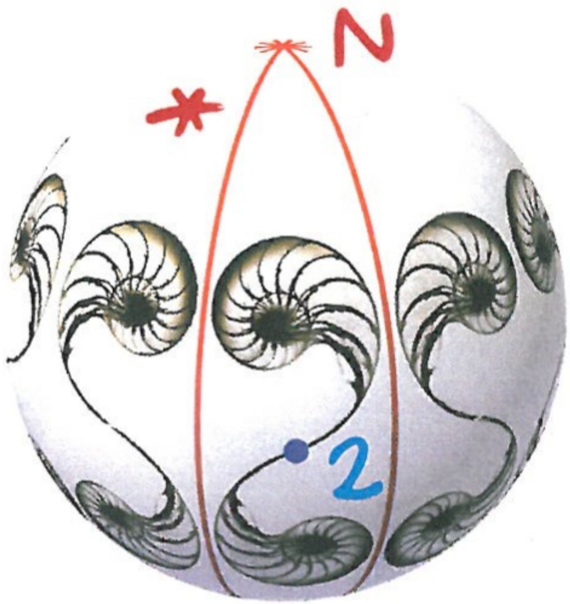
All possible variants (as in the plane case)

- *532
- *432
- *332 -> 3*2
- *22N -> 2*N
- *NN -> N* -> Nx

3*2 and N*



2*N and Nx



Some examples

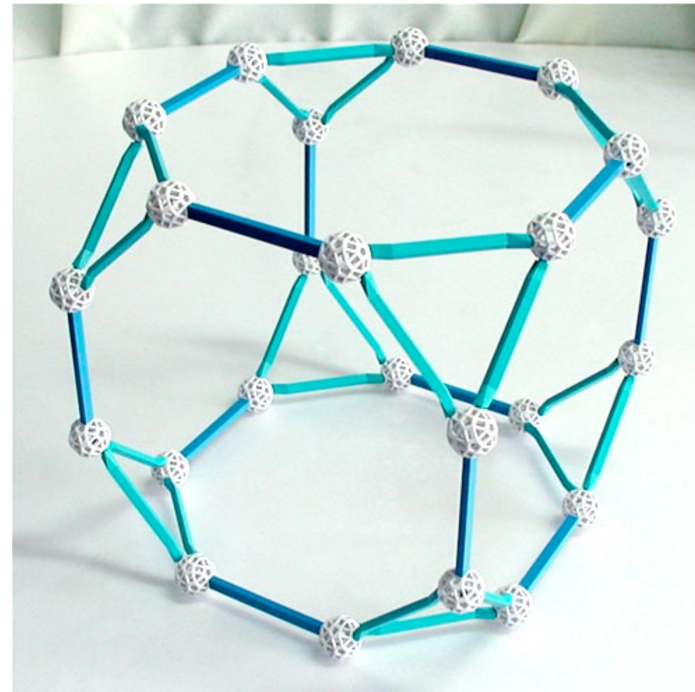
Archimedean solids (13):

- Regular polygonal faces
- Identical vertex arrangement

EX: Truncated cube

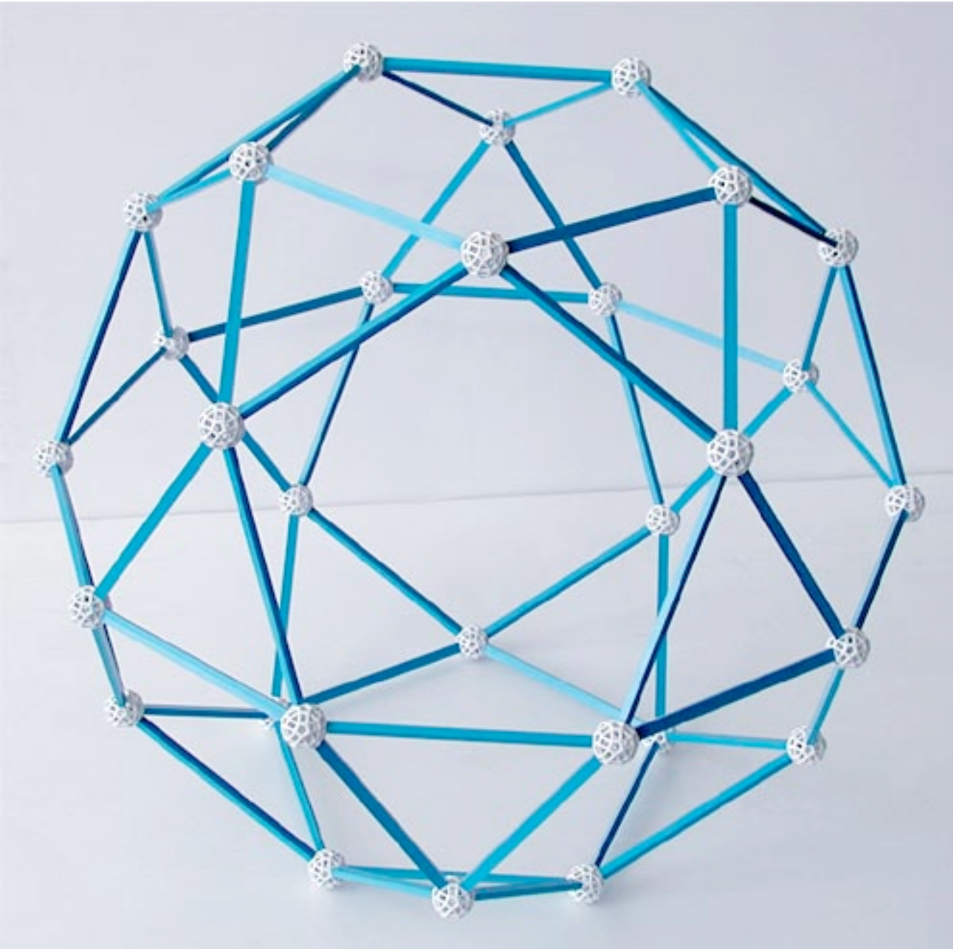
- 8 triangles
- 6 octagons

Dual Catalan solid: triakis octahedron

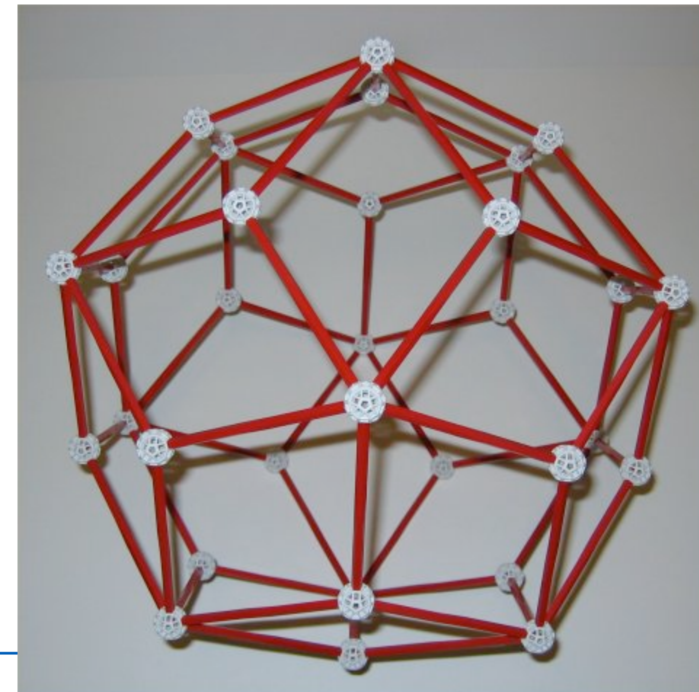


Ex: http://kovacsv.github.io/JSModeler/documentation/examples/solids.html

Icosidodecahedron (20 triangles,12 pentagons)



Dual Catalan solid: Rhombic triacontahedron



Truncated Icosidodecahedron

- 30 squares
- 20 hexagons
- 12 decagons

Dual Catalan solid: Disdyakis triacontahedron

Exercise to be returned on 29th Sept

1) Find fundamental domain and signature of Platonic solids and check the validity of Magic theorem:

Prize(symmetry)=2-2/d, d=number of symmetries (*Make use of the models you built)*

2) Find signature of at least four different spherical shapes (Archimedean/Catalan solids or other spherical shape you can find)

3) What is the value of V-E+F in each case?

4) Take photos of the pieces you folded and upload to MyCourses