

Problem Set 0: Solutions

1. Solution

$$f(x) = \begin{cases} 2 & \text{if } x = 0, 1 \\ 0 & \text{otherwise} \end{cases}$$

2. Solution

1. $f(x) = x^a, f'(x) = ax^{a-1}$
2. $f(x) = e^{ax}, f'(x) = ae^{ax}$
3. $f(x) = (3x + 2)^3, f'(x) = 3(3x + 2)^2 \times 3 = 9(3x + 2)^2$
4. $f(x) = \frac{3x}{x^2+1}, f'(x) = 3 \times \frac{1}{x^2+1} + 3x \times \frac{-1}{(x^2+1)^2} \times 2x = \frac{3}{x^2+1} - \frac{6x^2}{(x^2+1)^2} = \frac{-3(x^2-1)}{(x^2+1)^2}$
5. $f(x) = 4e^{-3x}, f'(x) = -12e^{-3x}$
6. $f(x) = x \ln x, f'(x) = \ln x + 1$

3. Solution

1. Local maximum: $f(x) = \frac{2}{3\sqrt{3}}$ at $x = \frac{1}{\sqrt{3}}$
2. Local minimum: $f(x) = -\frac{2}{3\sqrt{3}}$ at $x = -\frac{1}{\sqrt{3}}$.
3. No global maximum
4. No global minimum

4. Solution

1. Convex
2. Convex
3. Both concave and convex
4. Concave. If $-f(x)$ is convex, then $f(x)$ is concave
5. Concave
6. Concave for $x \leq 0$, and convex for $x \geq 0$