



Aalto University
School of Electrical
Engineering

Network Traffic Measurements and Analysis

Lecture I: Data analysis

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Version 0.2, September 20, 2017

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- ▶ Preface
- ▶ Data and exploratory data analysis
- ▶ Single variable analysis
- ▶ Relationships of variables
- ▶ Multidimensional data
- ▶ Time and measurements

“Abstract” of the lecture

- ▶ Measurements have provided a set of numbers - what can we do with those?
- ▶ Idea: Basic statistical methods even without much mathematics are sufficient to very many tasks that arise from network measurements

Reality, however,

- ▶ Measurement data available is for certain purpose . . .
- ▶ . . . while the current objective something different (cf. traffic classification from flow data)

Objective

- ▶ On the one hand, the goal is to learn to utilize basic statistical tools to distill the essential features of measurement data
- ▶ On the other hand, the goal is to learn to interpret statistical summaries of measurement data
 - ▶ Especially important is to understand the shortcomings of different types of summaries

Objective

- ▶ Computational tools
- ▶ Network measurements frequently produce **vast amounts of data**:
 - ▶ Packet traces without the payload
 - ▶ Flow data
- ▶ There are many software tools that can be utilized to
 - ▶ Manipulate data into a form that is easy to analyze (e.g. scripts)
 - ▶ Perform statistical analyses
 - ▶ Visualize the results (e.g., gnuplot)
- ▶ On this lecture we will use the R software environment for demonstration purposes
 - ▶ Downloadable freely: <http://www.r-project.org/>
 - ▶ In Ubuntu Linux: `sudo apt-get install r-recommended`

Data preprocessing

- ▶ In the network measurement context the numbers are generally obtained by **preprocessing** the raw measurement data
- ▶ Preprocessing may include
 - ▶ *Cleaning*, e.g., removing incomplete entries
 - ▶ *Integration*, e.g., multipoint measurements
 - ▶ *Transformation*, e.g., aggregation
 - ▶ *Reduction*, e.g., categorization
- ▶ Not necessarily difficult, but often more time consuming than the statistical analysis itself!

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Data

- ▶ Goal of statistics is to gain understanding from **data**
- ▶ Data are numbers with a **context**
 - ▶ Statistical tools can be utilized to organize, display and summarize the numbers
 - ▶ Understanding of the context is then utilized to draw conclusions of the data

Why network traffic measurements?

- ▶ Input for system design (e.g., WWW caches, CDNs, etc.)
- ▶ Performance evaluation afterwards
- ▶ Anomaly detection, protection against DDoS attacks
- ▶ Billing, etc.

Measurement data basics

- ▶ Measurement data consists of
 - ▶ **individuals** (packet, flow, user,...) and a set of
 - ▶ **variables** (packet length, flow size, ...)
- ▶ Variables can be
 - ▶ **categorical** (protocol type, port number) or
 - ▶ **quantitative** (packet size, delay)
- ▶ **Distribution** of a variable defines the values the variable can take and how often the variable takes these values
- ▶ **Spreadsheet** is a format where each row is an individual and each column is a variable.

Exploratory data analysis (EDA)

- ▶ Our approach to measurement data can be considered to be an exploratory one
- ▶ EDA: “Describe what you observe”
 - ▶ Uncover underlying structure
 - ▶ Extract important variables
 - ▶ Detect outliers and anomalies
 - ▶ Develop (parsimonious) models

Exploring the measurements

EDA approach

- ▶ Not to make any assumptions on the data but try to find out what kind of assumptions could be made

Problem ⇒ Data ⇒ Analysis ⇒ Model ⇒ Conclusions

Other common approaches

- ▶ “Classical” statistical approach

Problem ⇒ Data ⇒ Model ⇒ Analysis ⇒ Conclusions

- ▶ Engineering approach:
Specifications on what and how to measure (e.g., ITU-T)

Basic methodology

- ▶ Pre-process the measurements to obtain a spreadsheet
- ▶ Rules of thumb (Moore&McCabe):
 - ▶ Begin by examining each variable by itself.
 - ▶ Then move on to study the relationships of the variables
 - ▶ Begin with a graph or graphs.
 - ▶ Then add numerical summaries of specific aspects of data

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Single variable analysis

- ▶ Begin with a graph or graphs and decide which numerical summaries are the most suitable
- ▶ Available tools depend on whether the variable is categorical or continuous

Analyzing a categorical variable

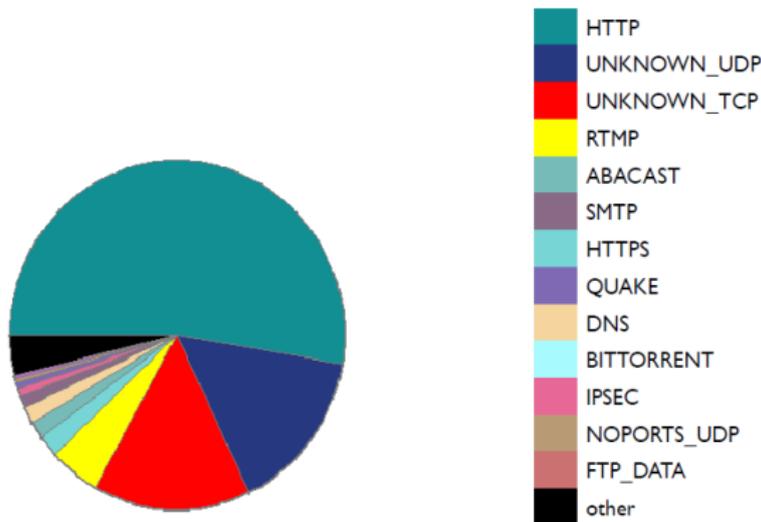
- ▶ Categorical variables have two qualities to analyze:
 - Counts:** How many instances there are in each category
 - Percents:** What are the relative shares of instances in each category
- ▶ Generally with low number of instances the counts are more interesting and with large number of instances the percents are more informative
 - ▶ Exceptions include e.g. cases where we are interested only in one category

Visualizing a categorical variable

- ▶ Categorical variables are easy to grasp from a **bar graph** or from a **pie chart**
- ▶ Rules of Thumb:
 - ▶ Use bar graphs if the actual number of instances is relevant
 - ▶ Use pie charts if the proportions are more interesting.
 - ▶ If the categories do not have a “natural order”, it is often convenient to visualize the data so that the categories are ordered according to their relative frequency

Example

- ▶ Pie chart of application packets in Chicago monitor A (www.caida.org) on March 10 - March 11 2011



Analyzing a quantitative variable

- ▶ Quantitative variable is analyzed by the distribution
- ▶ Always try to plot your data first

1. Look for overall pattern
2. Look for deviations from the pattern
3. Produce a numerical summary to briefly describe center and spread of data

- ▶ Simplest approach: Raw data plot
- ▶ Plot values “one-by-one”
 - ▶ Discrete or continuous?
 - ▶ Range, spread?
 - ▶ Special values?
- ▶ Is not a meaningful description of the distribution by itself

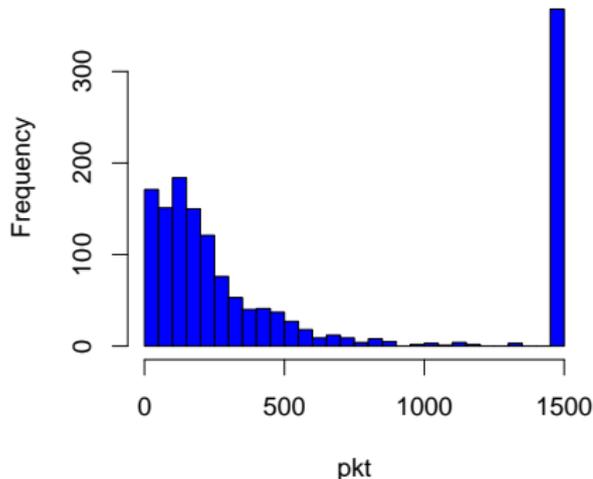
Visualization: Histogram

- ▶ Describes the distribution of the variable
- ▶ Break the range of values of a variable into classes and count the measurements that fall into each class - frequency table
- ▶ Plot the frequencies (or normalized frequencies) using bars
- ▶ Use your own judgment in choosing the classes (a.k.a. bins)
- ▶ Goal: the distribution should be well illustrated
- ▶ The appearance of the histogram may change significantly you change the classes
- ▶ Try different selections

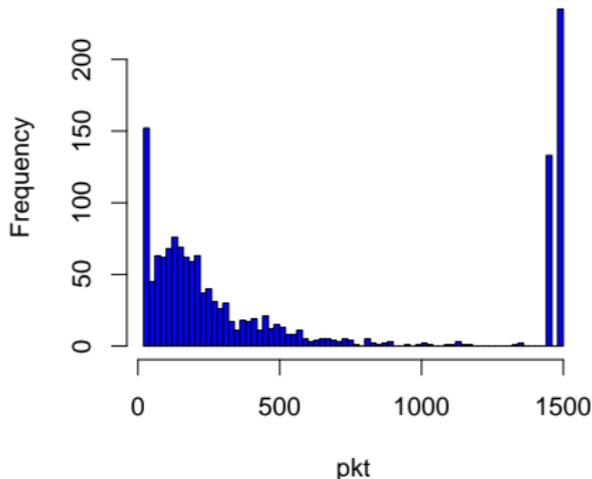
Example

```
> pkt<-scan("packetsizes.txt")  
> hist(pkt,col=4,breaks=50)
```

Histogram of pkt



Histogram of pkt

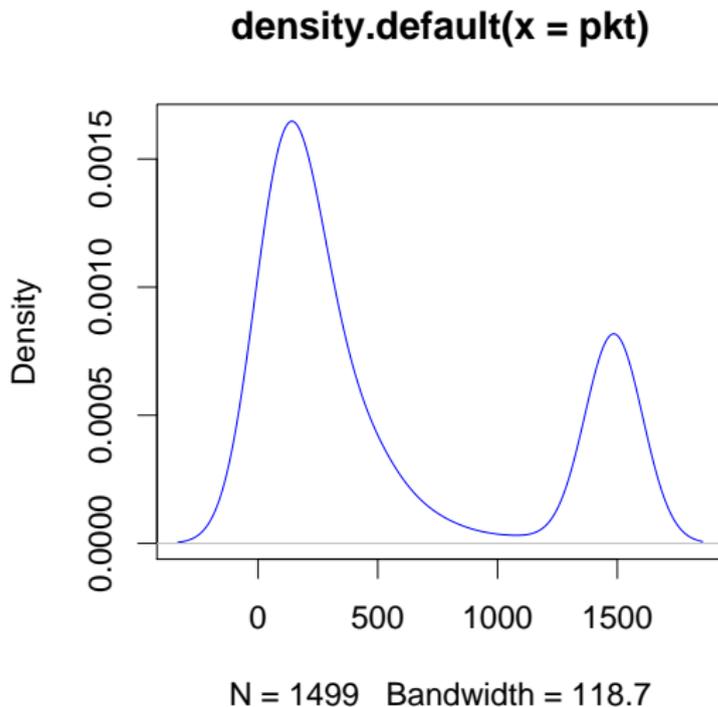


Visualization: Density curves

- ▶ Histogram depends heavily on the class selection
- ▶ Another useful tool for characterizing the data is to estimate the density curve of the underlying variable
- ▶ Good for “smooth” distributions
- ▶ Density curve has an area of 1 (pdf!)
- ▶ Always non-negative
- ▶ Determined by statistical softwares such as R

Example (of not so good density curve)

```
> plot(density(pkt), col=4)
```



Examining the distribution: Pattern

- ▶ Overall pattern
- ▶ Shape
 - ▶ One or several modes
 - ▶ Symmetricity, skewness
- ▶ Center
 - ▶ Where the distribution lies, “mean value”, “typical value”
- ▶ Spread
 - ▶ How much the values vary

Outliers, outlying observations

Outlier is an observation that is **clearly** outside the overall pattern

- ▶ Outlier can result from a measurement error ... or not
- ▶ Outliers can significantly complicate the numerical description and analysis of data
 - ▶ Screen the data and remove outliers (careful!)
 - ▶ Use robust statistics to describe the data

Describing the distribution with numbers

- ▶ Distribution shape is described by inspecting the histogram
- ▶ Numbers are generally used for the center and spread
- ▶ Remember: The numbers and graphs are aids to understanding the data not the goals themselves

Measuring center

- ▶ For measurements $\{x_i\}$, the center can be described by the **sample mean**

$$\bar{x} = \frac{1}{N} \sum_{i=1}^N x_i.$$

- ▶ Sample mean is an optimal estimator of the center of the normal distribution
- ▶ However, the mean is sensitive to the influence of outliers
 - ▶ Even one gross error can make the mean arbitrarily large
 - ▶ The **breakdown point**¹ of mean is 0%

¹ The breakdown point is the fraction of incorrect (arbitrarily large) observations an estimator can handle before going haywire.

Measuring center

- ▶ Robust methods provide alternative way of characterizing the center

- ▶ **Median** is defined as

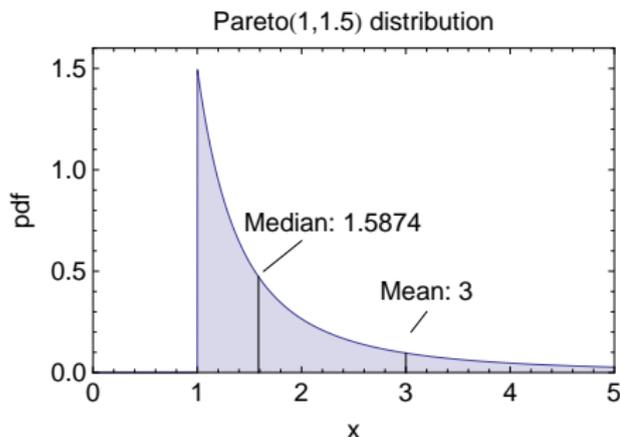
the centermost value of the ordered data.

If the number of data is even, the median is the mean of two centermost values.

- ▶ Median is more resistant measure of the center
 - ▶ It has breakdown point of 50% (it can tolerate 50% of gross errors before becoming arbitrarily large)

Mean vs. Median

- ▶ Mean is the “average” value of the variable whereas the median is the “typical” value
- ▶ If the distribution is symmetric, both are close to each other (and identical if the distribution is exactly symmetric)
- ▶ If the distribution is skewed, mean tends to be farther out in the long tail



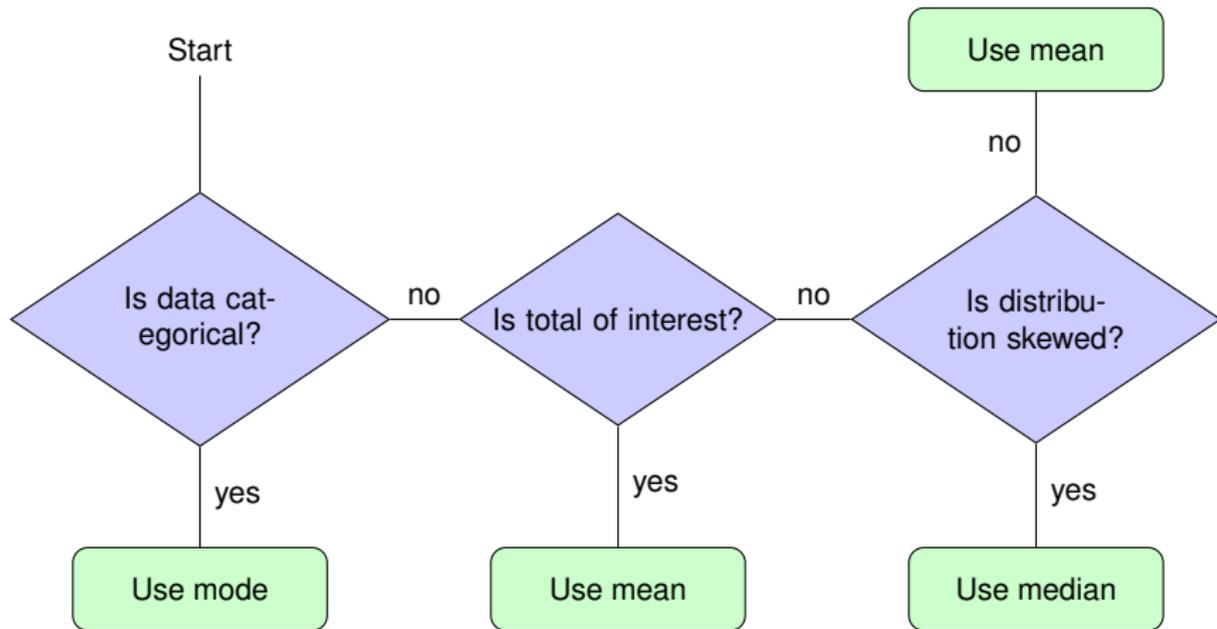


Figure: Selecting measure for center.

Note #1

Suppose you have managed to improve transfer rate in a communication system:

Wireless layer (MAC)	3%
Network layer (IP)	10%
Application layer	50%

Q: What is the average improvement?

- ▶ $(3\% + 10\% + 50\%)/3 = 21\%$?
- ▶ No, improvements are multiplicative!
- ▶ $(1.03 \cdot 1.10 \cdot 1.50)^{1/3} \approx 1.193 \Rightarrow \underline{19.3\%}$

This is the so-called **geometric mean**,

$$(a_1 \cdot a_2 \cdot \dots \cdot a_n)^{1/n}.$$

Note #2

Suppose you have measured a transfer time of a file n times:

file size	b [byte]
transfer time	t_i [sec]
mean rate	$r_i = b/t_i$

Q: What is the average transfer rate?

- ▶ Mean of sample rates?

$$\frac{r_1 + \dots + r_n}{n} = \frac{b/t_1 + \dots + b/t_n}{n}$$

- ▶ No, a better metric is the **harmonic mean**

$$\frac{n}{1/r_1 + \dots + 1/r_n} = \frac{nb}{t_1 + \dots + t_n}$$

Measuring spread

- ▶ Simplest useful numerical description of data consists of both a measure of center and a measure of spread
- ▶ Easiest way of describing spread is to give several percentiles. The p :th **percentile** is the value such that $p\%$ of the measurements fall at or below it
- ▶ Median is the 50th, first **quartile** (Q1) is the 25th and third quartile (Q3) is the 75th percentile
- ▶ **IQR** = Interquartile range, $Q3-Q1$

The five number summary and boxplots

- ▶ The **five number summary**,

(Minimum, Q1, Median, Q3, Maximum)

is a good summary of the distribution as a whole

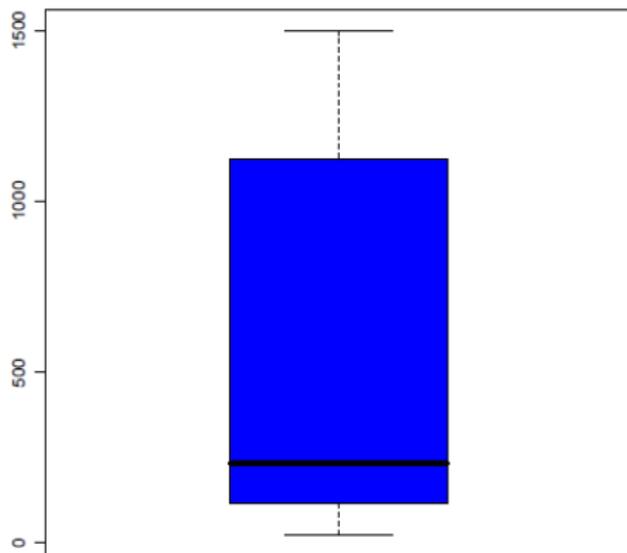
- ▶ The five number summary is generally depicted using boxplots:
 - ▶ Central box spans the quartiles Q1 and Q3
 - ▶ Line marks the median
 - ▶ Lines extend from the box to mark the maximum and minimum
 - ▶ Conventionally observations more than 1.5 times the inter-quartile range ($Q3-Q1$) from the median are plotted separately
 - ▶ Especially suitable for comparison of distributions

Example

```
➤ summary(pkt)
```

Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
22.0	115.5	233.0	536.9	1123.0	1500.0

```
➤ boxplot(pkt, col=4)
```



Measuring spread: Standard deviation

- ▶ **Sample standard deviation** is the most common measure of spread

$$s = \sqrt{\frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2}$$

- ▶ **Sample variance** is given by s^2
- ▶ Why squared distances from mean?
 - ▶ Sum of the squared deviations from mean smaller than from any other point
 - ▶ Optimal measure in normal distributions
- ▶ Why dividing by $n - 1$ and not by n ?
 - ▶ Sum of unsquared deviations is zero, so if you know $n - 1$ deviations you immediately can derive the missing one
 - ▶ There are $n - 1$ degrees of freedom
 - ▶ Important to remember when n is small

Notes on standard deviation

- ▶ Should be used only when using mean as a measure of the center
- ▶ Like mean, standard deviation is not robust against outliers
- ▶ Squared deviations make the situation even worse!
 - ▶ For the packet data distribution on page 21
 $s = 569.27$
- ▶ Note that variance σ^2 of random variable X is

$$\sigma^2 \triangleq \mathbb{E}((X - \mu)^2),$$

where μ is the mean, $\mu = \mathbb{E}(X)$. Sample variance s^2 is an **unbiased** estimator for σ^2 .

Coefficient of variation (c_v , C.O.V.)

Sample mean m and standard deviation s include a unit (e.g. bytes or seconds)

Coefficient of variation (C.O.V):

Coefficient of variation is a *dimensionless* measure of spread

$$c_v = \frac{\text{standard deviation}}{\text{mean}} = \frac{s}{m}$$

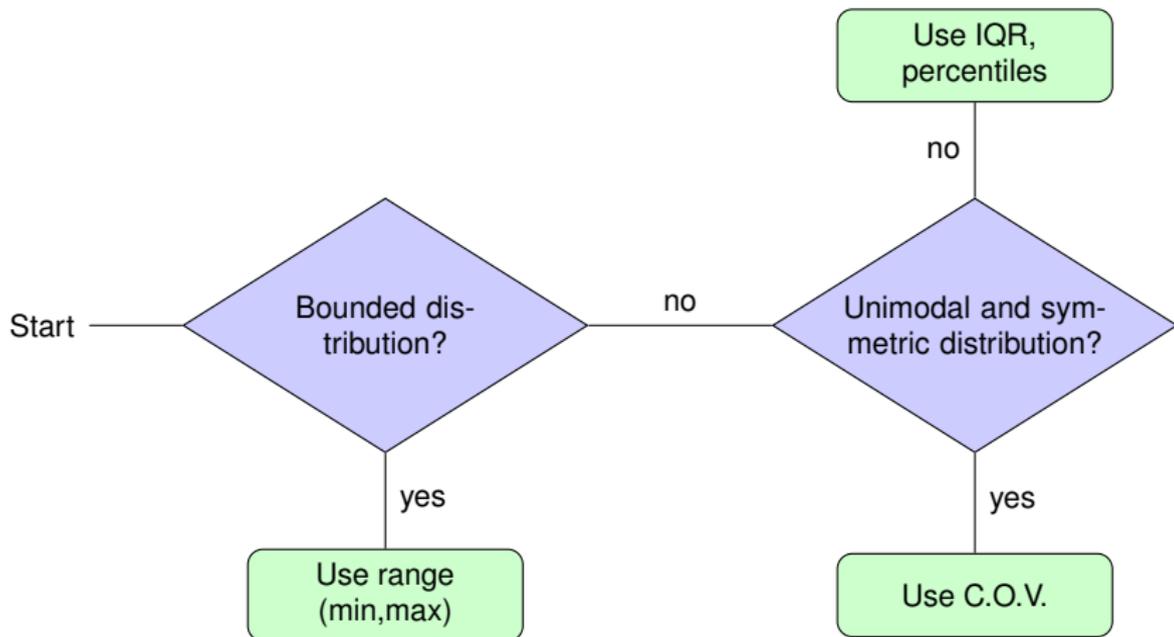


Figure: Selecting a measure for spread.

Linear transformations

- ▶ A linear transformation of type

$$x_{\text{new}} = ax + b,$$

does not change the shape of the distribution

- ▶ The transformation has the following effects on the statistics
 - ▶ Measure of spread is multiplied by a
 - ▶ Measure of center, m , becomes $am + b$
- ▶ Useful e.g. in changing the unit of measurement
 - ▶ bits vs. bytes
 - ▶ packet size vs. payload

Choosing the measures of center and spread

- ▶ Rule of thumb: Do not use mean and standard deviation for strongly skewed distributions
- ▶ Distributions with multiple peaks or gaps are ill-suited for simple numerical description in general
- ▶ Numbers report specific facts about a distribution, a graph is generally more informative than plain numbers when it comes to the overall picture

Examples of other measures

- ▶ **Skewness** and **kurtosis**, i.e., the third and fourth standardized moments of a distribution,

$$\frac{E((X - \mu)^k)}{\sigma^k}, \quad \text{where } k = 3, 4.$$

- ▶ Describe the shape of the distribution
 - ▶ Skewness < 0 , tail to the left
 - ▶ Skewness > 0 , tail to the right
 - ▶ Kurtosis low \Rightarrow flat but short tailed distribution
 - ▶ Kurtosis high \Rightarrow sharp with heavy tails

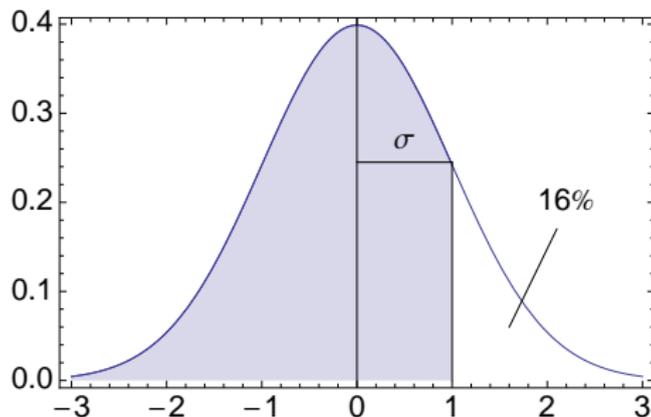
Examples of other measures

- ▶ **Trimmed mean**
 - ▶ Mean of the central $1 - 2\alpha$ part of the distribution
 - ▶ If outliers cannot be removed one-by-one
 - ▶ Uses more information than median
- ▶ **Median absolute deviation (MAD)**

$$\text{MAD} = \text{Median}_i(|X_i - \text{Median}_j(X_j)|).$$

Intuition on center and spread for density curves

- ▶ Mean is the “balance point” of the density curve
- ▶ Median is the point that divides the area of density curve into two equal parts
- ▶ Standard deviation for normal curves: the “turning point”



Empirical cumulative distribution function

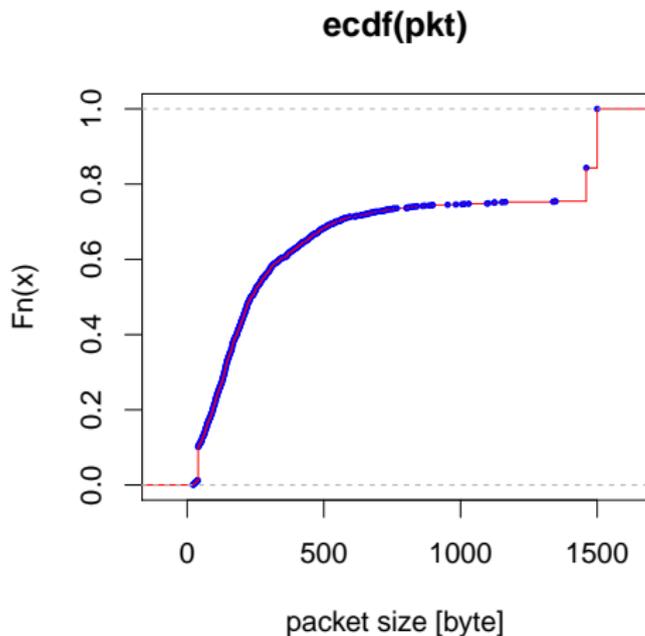
- ▶ Another important way of characterizing the distribution of a variable is the empirical cumulative distribution (cdf)

$$F_n(x) = \frac{1}{n} \sum_{i=1}^n I(x_i \leq x).$$

- ▶ Here the function $I(x_i \leq x) = 1$ if $x_i \leq x$, otherwise 0

Example

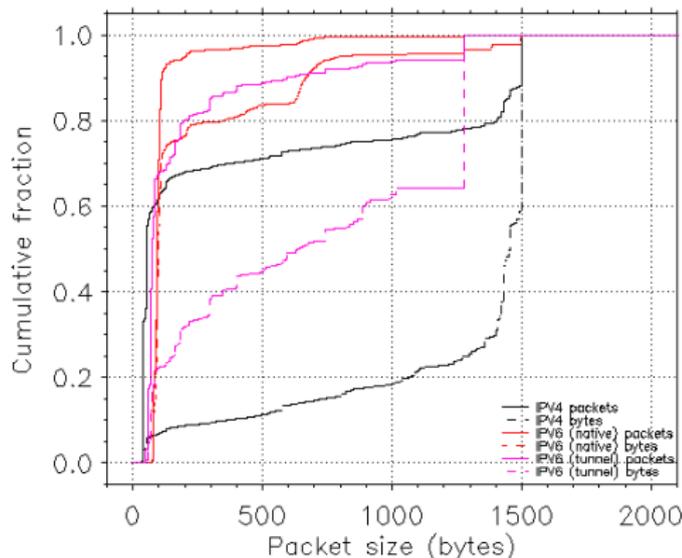
```
> pkt<-scan("packetsizes.txt")  
> fn<-ecdf(pkt)  
> plot(fn,verticals=TRUE,col.points=4,col.hor=2,col.vert=2,cex=.4)
```



Example (www.caida.org)

- ▶ Packet size distribution on a backbone link
- ▶ Feb 17 2011
- ▶ 12:59:04 - 14:01:04

2011-02.dirA



147986351749 bytes

2221625 bytes (1.5×10^{-3} %)

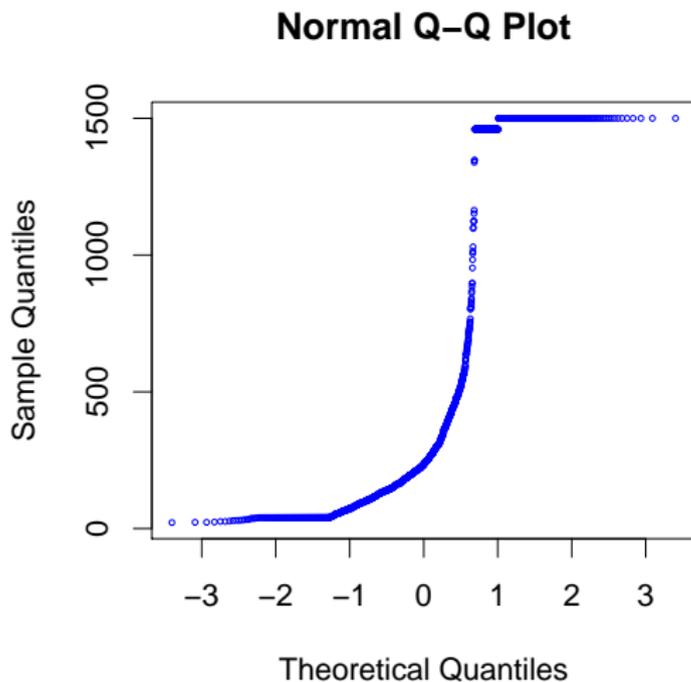
852714 bytes (5.8×10^{-4} %)

Quantile plots

- ▶ **Quantile plots** (aka Q-Q plots) are a useful tool in comparing whether your measurements can be described by a certain statistical distribution (or by another data set)
- ▶ More about distributions on the next lecture!
- ▶ For data of n measurements plot (x, y) , where:
 - ▶ $x: k/(n + 1)$, $k = 1, \dots, n$ quantile of the comparison distribution
 - ▶ y : order statistics of data, i.e., k :th smallest measurement
- ▶ If the points constitute a straight line, the distributions are “similar” (and if the line is close to the 45 degree line, the distributions are identical)

Example: Normal Q-Q plot

```
> pkt<-scan("packetsizes.txt")  
> qqnorm(pkt, col=4,cex=0.4)
```



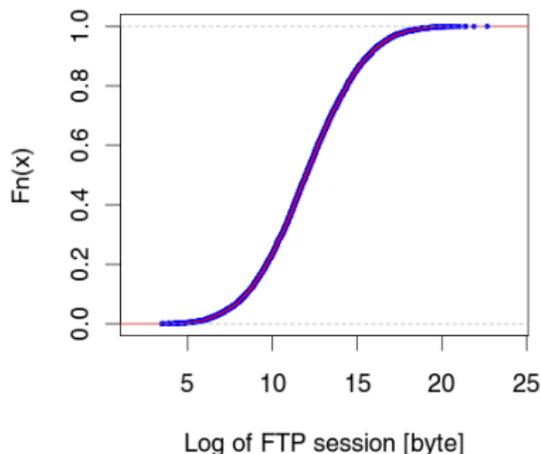
Challenges with measurement data

- ▶ Data volume
 - ▶ Storage, handling, overplotting, ...
 - ▶ Some problems can be avoided by
 - ▶ Preprocessing steps (reduction, aggregation, ...)
 - ▶ Sampling
- ▶ High variability
 - ▶ Causes instability for many metrics
 - ▶ Use robust statistics
 - ▶ Makes distribution plots less illustrative
 - ▶ Use logarithms in plots

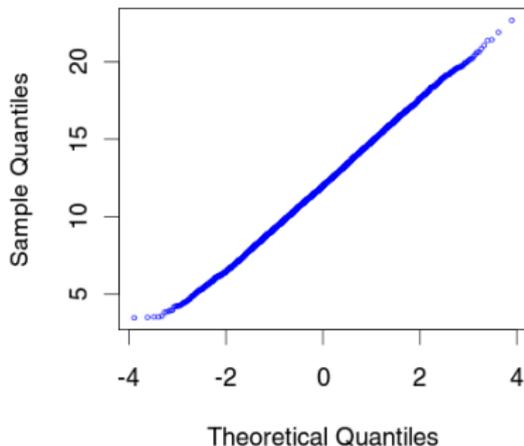
Example: Q-Q log-normal

```
> ftp<-scan("ftpsessions.txt")  
> ftplog<-log(ftp)  
> fn<-ecdf(ftplog)  
> plot(fn,verticals=TRUE,col.points=4,col.hor=2,col.vert=2,xlab="...")
```

ecdf(ftplog)



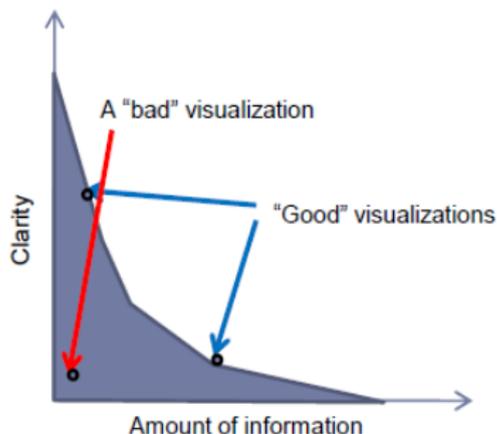
Normal Q-Q Plot



⇒ Log-normal distribution!

In reporting the results ...

- ▶ Suitable numerical summaries support understanding but may oversimplify some aspects of the data
- ▶ Graphs are efficient in reporting measurement results
- ▶ When visualizing the data
 - ▶ Make sure that the graph is as informative as possible
 - ▶ “Informativeness” sets a appropriate balance between amount of information and clarity



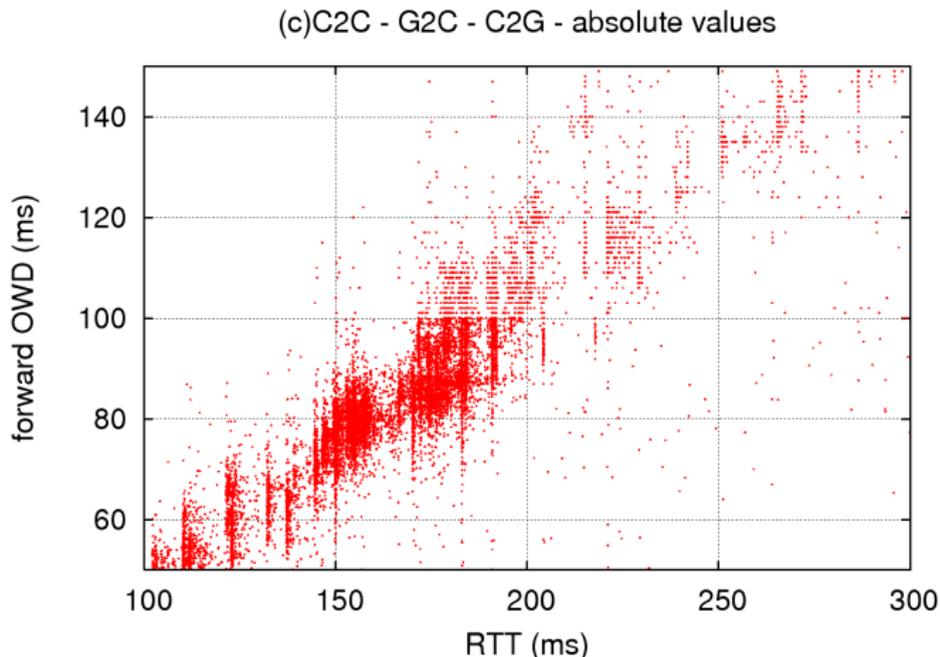
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Relationships between variables

- ▶ The most important tool for studying relationships between variables is the **scatterplot**
 - ▶ Values of one variable are plotted on the x -axis and values of the other variable are on the y -axis
 - ▶ Conventionally x is the *explanatory variable* and y is the *response variable*
- ▶ Categorical variables can be added using different markers or colors
 - ▶ Sometimes referred to as *Youden plot*

Example scatterplot



Source: A. Pathak, et al., *A Measurement Study of Internet Delay Asymmetry*, PAM 2008. [use `plot()` in R or `gnuplot` to produce scatterplots]

Analysing a scatterplot

- ▶ Form, direction and strength of the relationship
- ▶ Form
 - ▶ Linear, curved,...
 - ▶ Clusters
- ▶ Direction
 - ▶ Positive or negative association?
- ▶ Strength, outliers
 - ▶ How close the points are to the form?

Correlation

- ▶ Correlation between two variables x and y ,

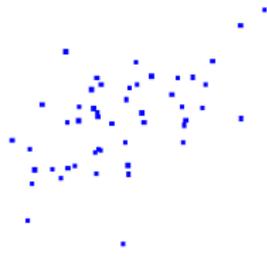
$$r = \frac{1}{n-1} \sum_{i=1}^n \left(\frac{x_i - \bar{x}}{s_x} \right) \left(\frac{y_i - \bar{y}}{s_y} \right).$$

- ▶ Gets values between $[-1, 1]$
 - ▶ Values near 0 indicate weak linear relationship
- ▶ Describes only linear relationship
- ▶ Not resistant
- ▶ r^2 is the fraction of variation in data that is explained by least-squares regression of y on x , r is the slope

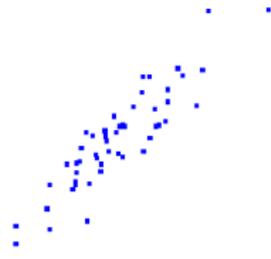
Example: correlation



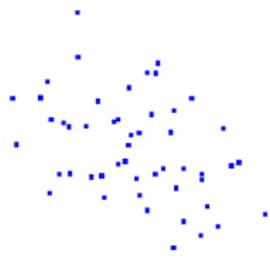
$r = 0$



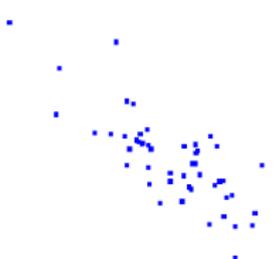
$r = 0.5$



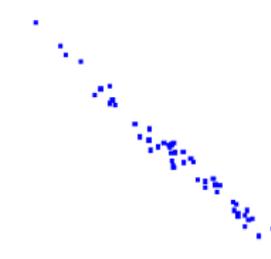
$r = 0.9$



$r = -0.3$



$r = -0.7$



$r = -0.99$

Caution on relationship analysis

- ▶ Outliers and influential observations
- ▶ Correlations on averages usually higher than on individuals
- ▶ Lurking variables
- ▶ High correlation does not imply **causation**. Possible associations
 - ▶ Causation
 - ▶ Common response
 - ▶ Confounding
- ▶ Establishing causation
 - ▶ Conduct an **experiment**, where the effects of lurking variables are controlled

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Multi-dimensional data

- ▶ Measurement data usually contains many variables for each individual
 - ▶ Try to reduce the number of variables in preprocessing
- ▶ In certain cases we need to preserve the information
→ multi-dimensional data
 - ▶ No prior knowledge what to look for
 - ▶ The studied phenomena span over several variables, e.g., intrusion detection

Exploring multi-dimensional data

- ▶ The general rule applies: **Plot your data!**
- ▶ The plots are typically not as intuitive as with one and two variables, visualization of multi-dimensional data is a challenge
- ▶ Methods:
 - ▶ Multi-dimensional plots
 - ▶ Projection methods

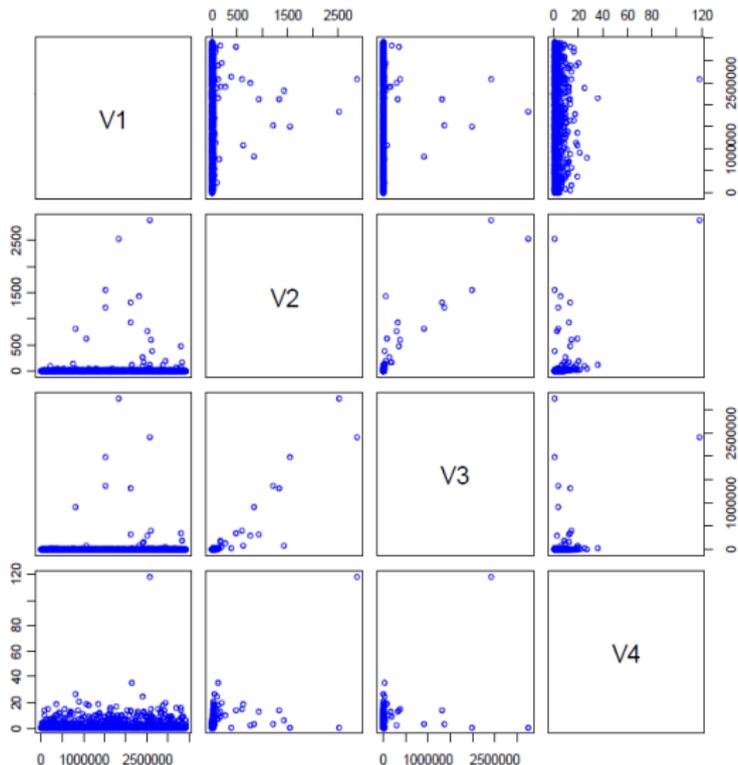
Example: Pairs plot

Matrix of
scatterplots

Variables:

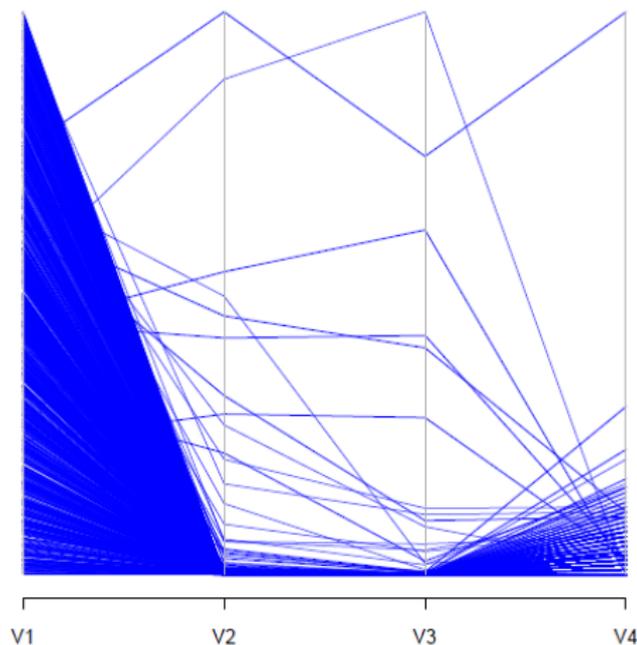
1. Source ID
2. Packets
3. Bytes
4. Flows

```
> pairs(flowdata, col=4)
```

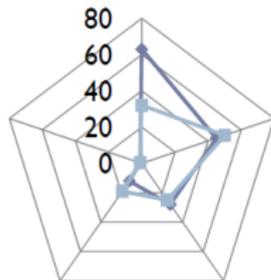


Parallel plots

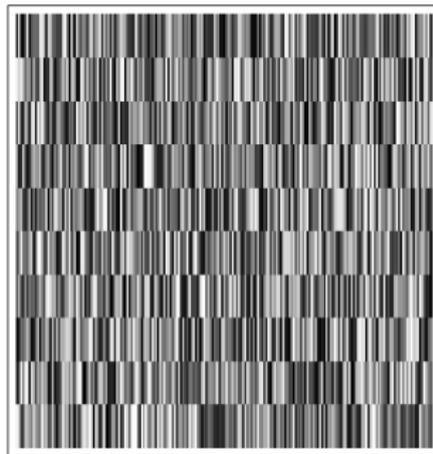
- ▶ Each measurement is represented by a horizontal path



Other plots



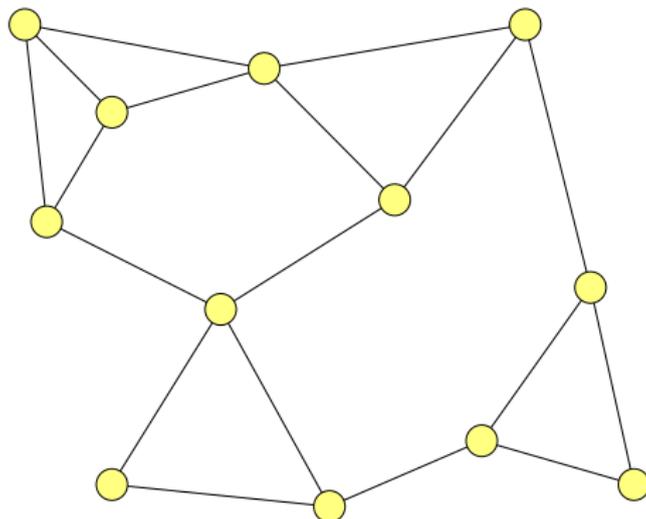
Radar / star plots



Color histograms

Multi-dimensional scaling

- ▶ A method to visualize multi-dimensional proximity data in low dimension
- ▶ E.g., *ad hoc* network MAC neighbors topology



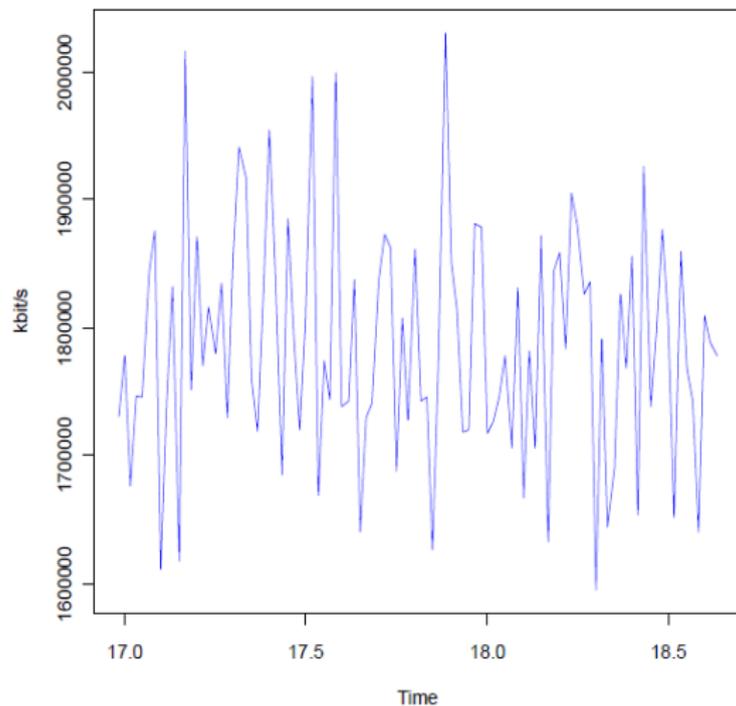
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Time plots

- ▶ Whenever data are collected over time it is a good idea to plot the measurements in time order
- ▶ Distribution studies ignore the time order, which may be misleading when there is a systematic change in time
- ▶ For example, if traffic load is high at a certain moment, it is likely that it is still high a second afterwards

Example: Time plot



Time series analysis

- ▶ Analysis of data ordered by the time the data were collected
- ▶ Usually equally spaced time instants (discrete time)
- ▶ Goals:
 - ▶ Modeling: To determine the process that has produced the data
 - ▶ (Forecasting: Point estimates and confidence intervals)
- ▶ Exploratory aspects
 - ▶ **Memory** and **stability** of the data

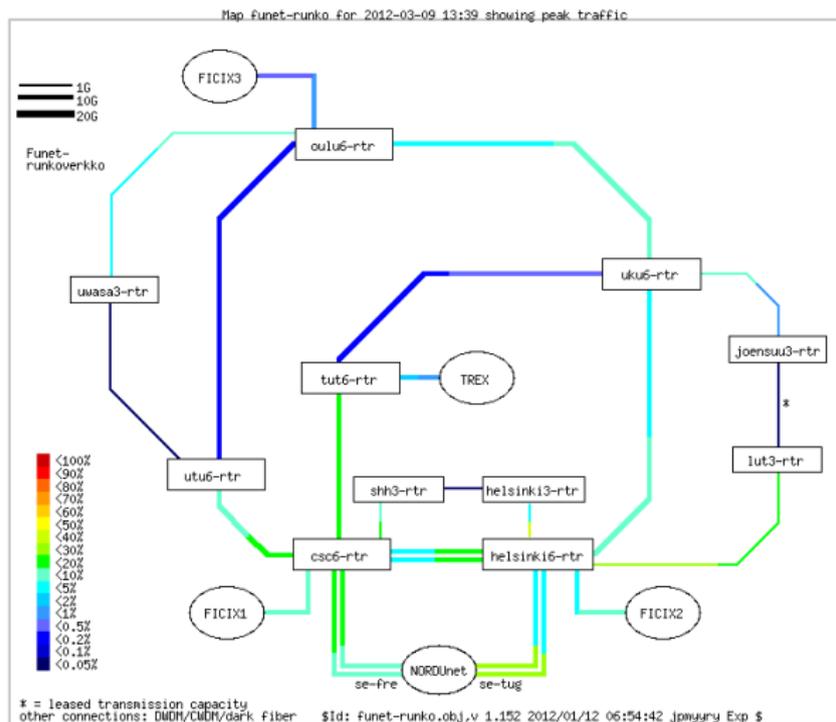
Stability of data

- ▶ Stability refers to the traffic consistency over time
 - ▶ Mean, variance, etc. do not change over time
- ▶ Distinct from **stationary**, which is a formal property of a stochastic process
 - ▶ If the data are stable, a stationary model may be applicable
- ▶ Subjective concept
 - ▶ Can be tested roughly by dividing the data into successive batches and analyzing whether some parameter estimates remain roughly constant in all the batches

Traffic at long time scales

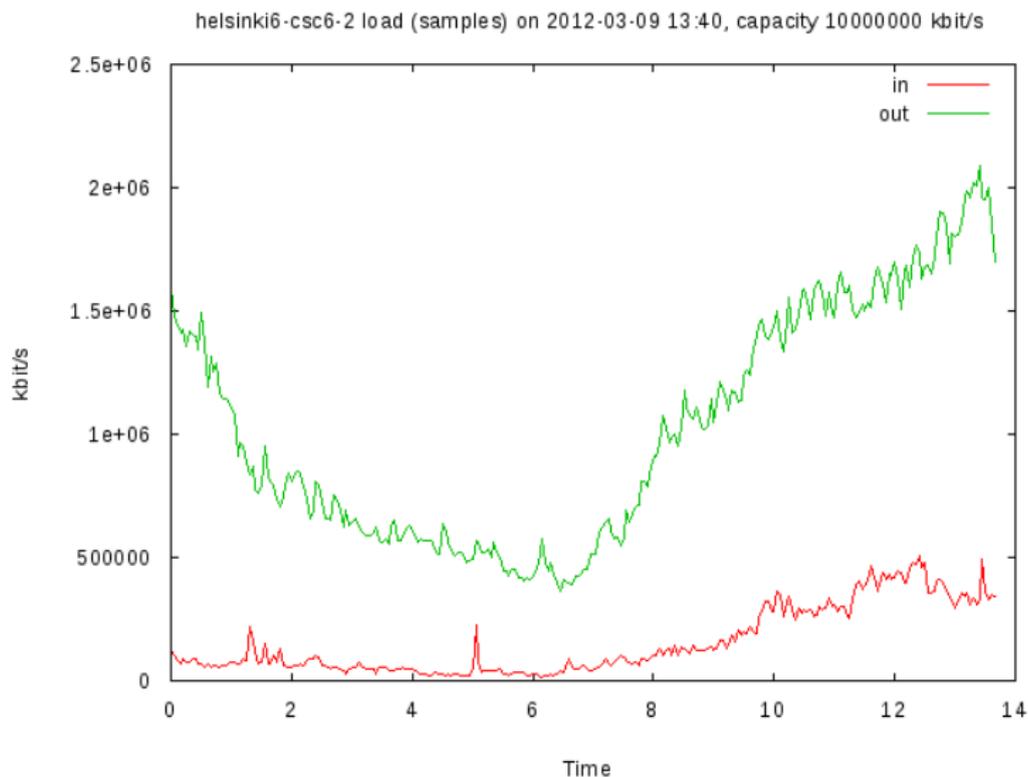
- ▶ Network traffic is not usually stable at long time scales
 - ▶ Long time scale trend of “ever increasing traffic”
 - ▶ Predictable components
 - ▶ Daily cycle
 - ▶ Weekly cycle
 - ▶ Yearly cycle (summer holidays etc.)
 - ▶ Special events (football world championships etc.)
- ▶ Unpredictable **external** effects
 - ▶ Accidents
 - ▶ Network failures

Example: Funet backbone network



<http://www.csc.fi/funet/status/tools/wm>

Time plot example: csc6rtr – helsinki6rtr



Decomposition of time series

- ▶ Statistical tools can be utilized to decompose the series into components
- ▶ Trend
- ▶ Seasonal variation
- ▶ ...
- ▶ Irregular influences

Traffic at short time scales

- ▶ At short time scales, comparable to minutes, traffic can be usually described as a stationary stochastic process
- ▶ However, networks contain buffers and control algorithms that maintain past history in a way it affects the current behavior
- ▶ Short-range memory and long-range memory are often both present at network measurements

Memory in system behavior

- ▶ Memory has both good and bad effects
 - ▶ Good: Near future more predictable
 - ▶ Bad: The amount of information in each measurement decreases, high variability
 - ▶ ... the Ugly?

Analyzing memory: Lag plots

- ▶ The easiest way to observe short-range memory is to consider **lag plots**
- ▶ Plot X_k against, e.g., X_{k+1}
- ▶ Check randomness, outliers, deterministic models

With R:

```
> lag.plot(x1)
```

Analyzing memory: ACF

- ▶ **Empirical autocorrelation function (ACF)** is defined as

$$\hat{r}(k) \triangleq \frac{\frac{1}{N-k} \sum_{i=1}^{N-k} (x_i - \bar{x})(x_{i+k} - \bar{x})}{\frac{1}{N-1} \sum_{i=1}^N (x_i - \bar{x})^2}.$$

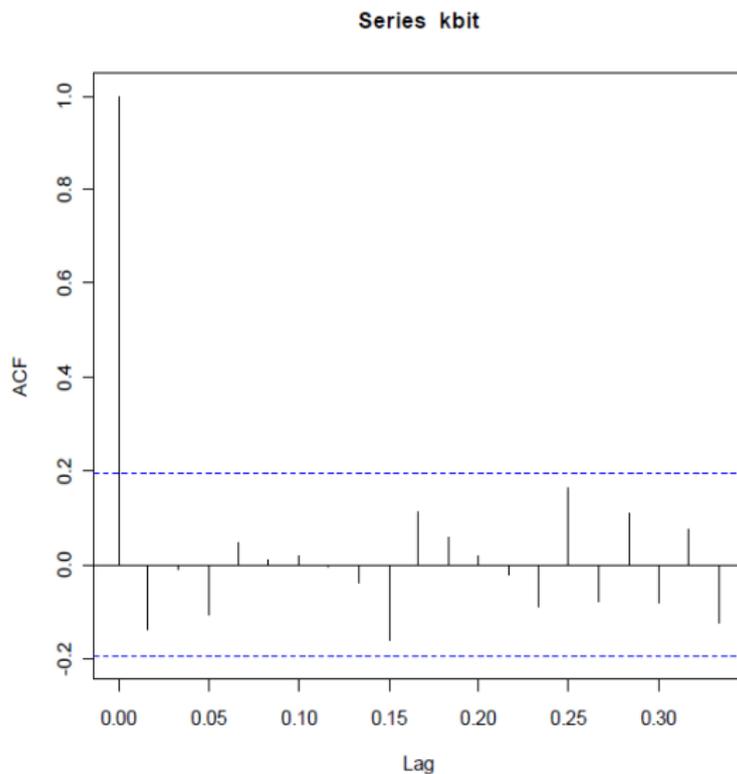
- ▶ Estimate for the normalized autocovariance function,

$$r(k) \triangleq E\left(\frac{(X_i - \mu)(X_{i+k} - \mu)}{\sigma^2}\right).$$

With R:

```
> plot(acf(x1))
```

Example: lag



Examining the ACF plot

ACF plot can be used to assess

- ▶ Are the data random?
- ▶ Are the adjacent measurements related?
- ▶ What model could be appropriate?
- ▶ Are the data self-similar?
 - ▶ Is ACF similar at different time scales?

Self-similarity

- ▶ Network traffic is often self-similar
 - ▶ Its statistical properties remain same under “zooming”
 - ▶ Cf. Koch curves, ferns, coast lines etc.
- ▶ Results essentially from long-range dependence
- ▶ We will return to self-similarity in more detail later as stochastic processes are discussed



Koch curve (Source: Wikipedia)

Literature

- ▶ David S. Moore and George P. McCabe, Introduction to the practice of statistics, 5th Edition, W.H. Freeman & Co., 2006
 - ▶ Chapters 1-2
- ▶ NIST/SEMATECH, Engineering Statistics Handbook, Chapter 1,
<http://www.itl.nist.gov/div898/handbook/index.htm>