Exercise session 2

Aalto University
School of Business

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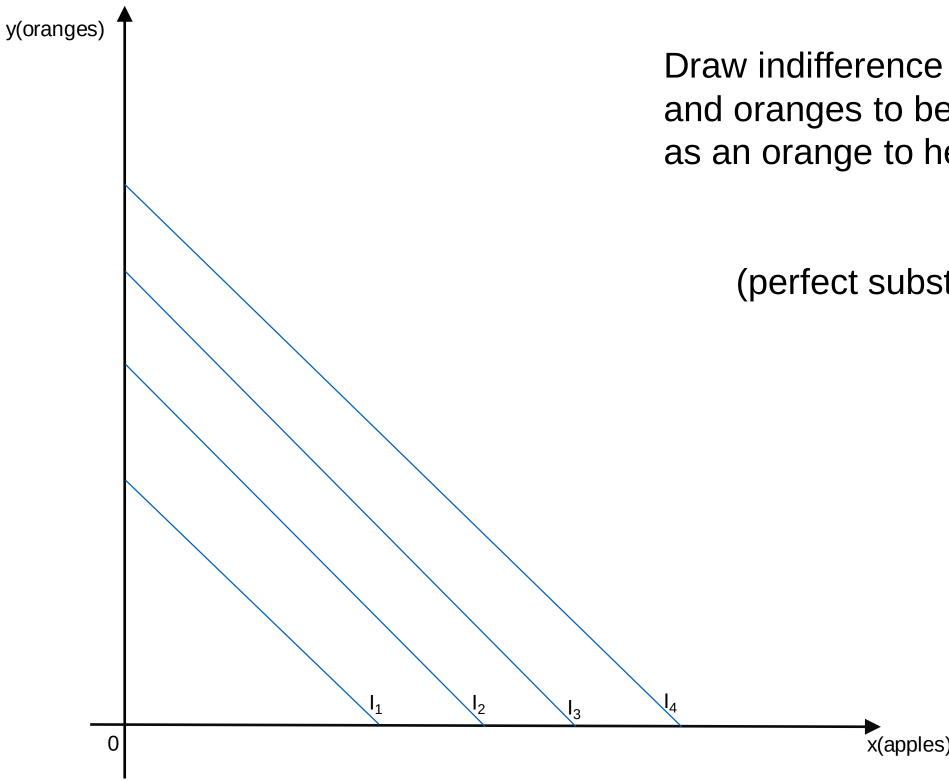
Exercise 1

Budget Sets and Indifference Curves

Exercise 1 a

Draw indifference curves for a consumer that considers apples and oranges to be interchangeable, i.e. an apple is equally good as an orange to her.

Exercise 1 a



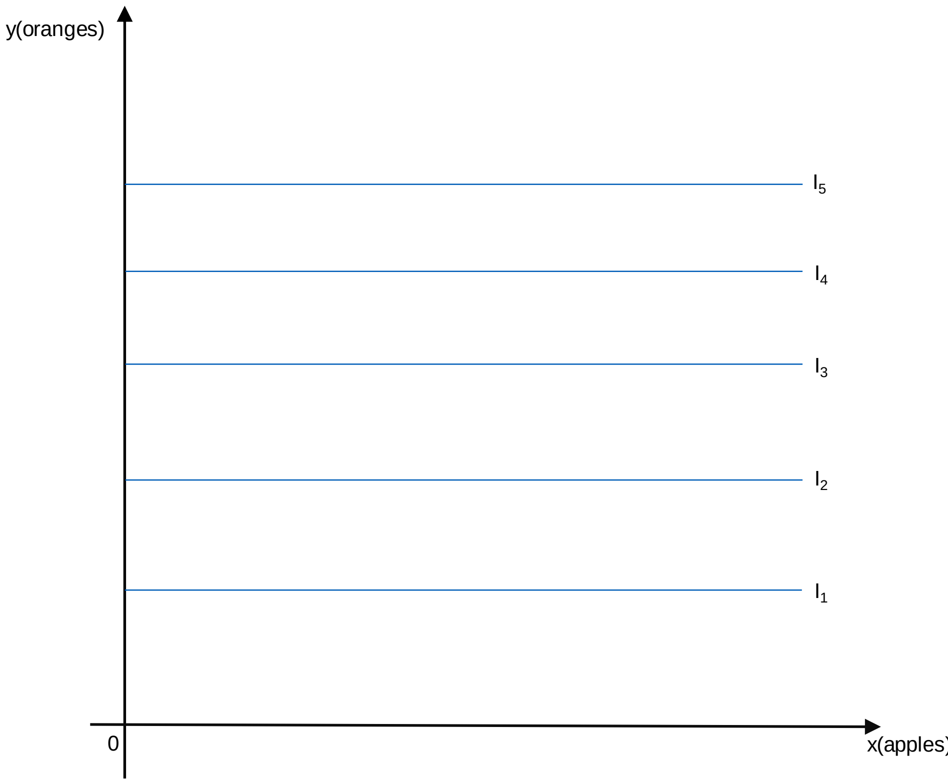
Draw indifference curves for a consumer that considers apples and oranges to be interchangeable, i.e. an apple is equally good as an orange to her.

(perfect substitutes)

Exercise 1 b

Draw indifference curves for a consumer that is allergic to apples but likes oranges.

Exercise 1 b



Draw indifference curves for a consumer that is allergic to apples but likes oranges.

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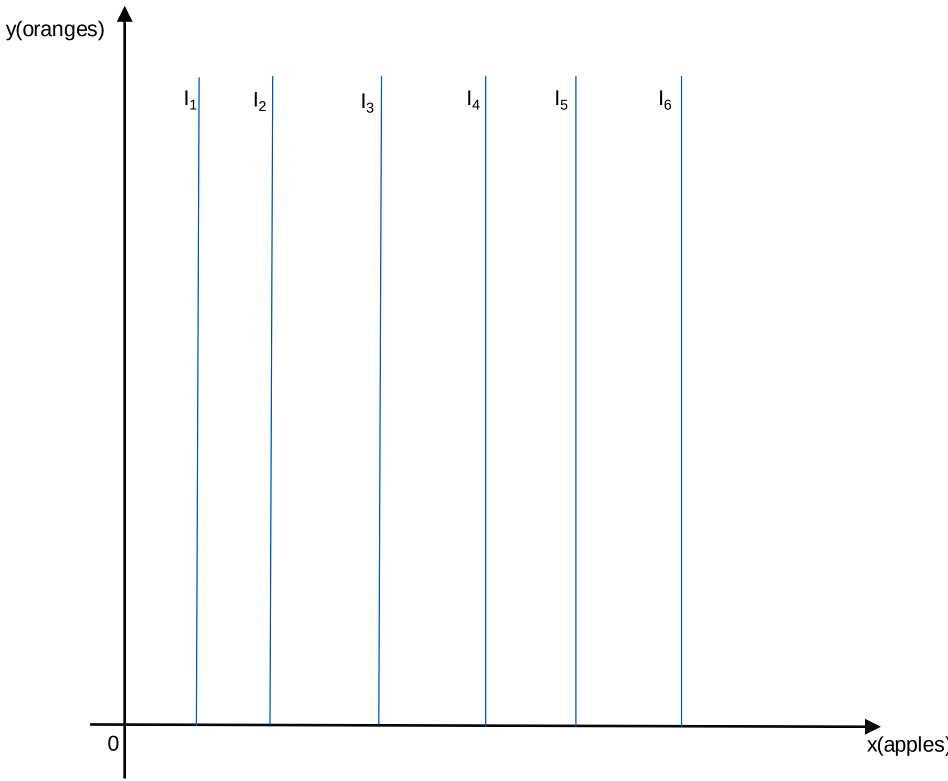
Exercise 1 c

Draw indifference curves for a consumer that hates oranges but likes apples.

Exercise 1 c

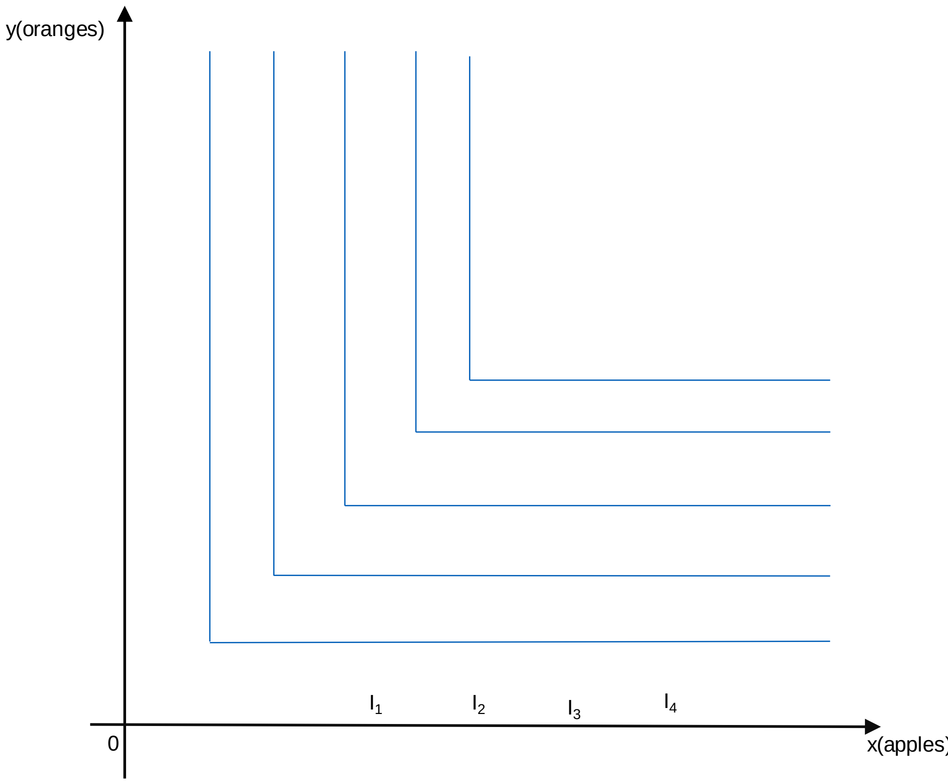


Draw indifference curves for a consumer that hates oranges but likes apples.

Exercise 1 d

Draw indifference curves for a consumer that likes fruit but only in a fruit salad where she puts exactly the same amount of apples and oranges.

Exercise 1 d



Draw indifference curves for a consumer that likes fruit but only in a fruit salad where she puts exactly the same amount of apples and oranges.

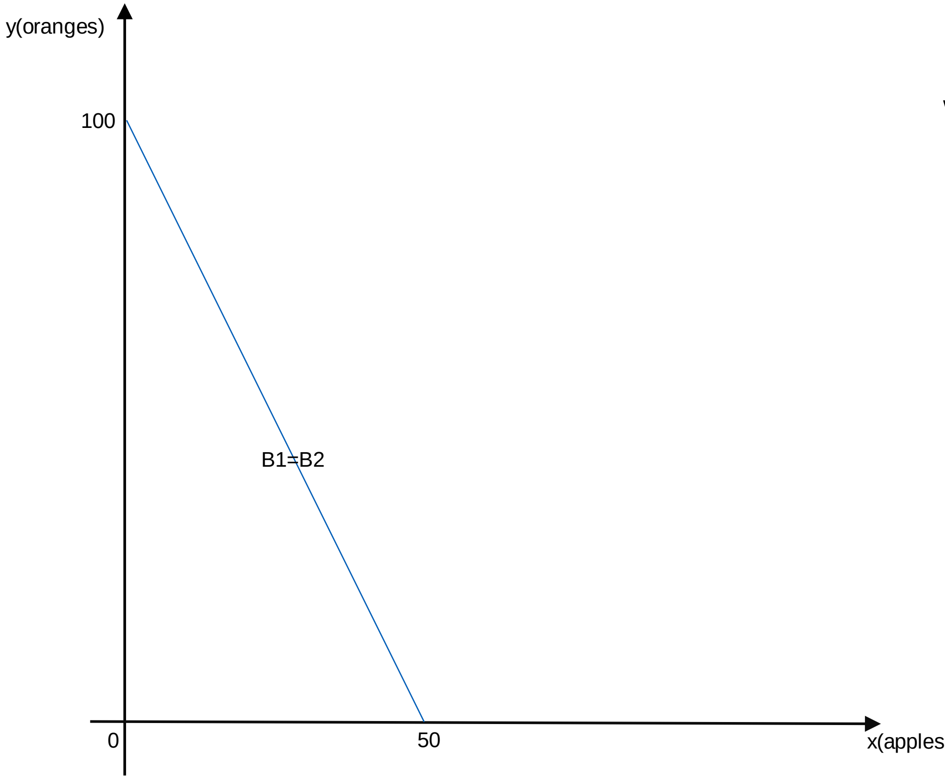
(perfect complements)

Exercise 1 e

Draw the budget set for a consumer that spends her budget *I* on apples and oranges. Put apples *x* on the x-axis and oranges *y* on the y-axis in a plane. Let p_x denote the price of apples and p_y the price of oranges so that the cost of consuming *x* and *y* is $p_x x + p_y y$.

Draw the budget set for $I = 200, p_x = 4, p_y = 2$. Draw also the budget set for $I = 100, p_x = 2, p_y = 1$.
What do you observe and how do you explain this?

Exercise 1 a



Draw the budget set B2: $I = 200, p_x = 4, p_y = 2$
Draw the budget set B1: $I = 100, p_x = 2, p_y = 1.$
What do you observe and how do you explain this?

Intercepts with the axes are:

$x_{B2} = \frac{200}{4} = 50$	$x_{B1} = \frac{100}{2} = 50$
$y_{B2} = \frac{200}{2} = 100$	$y_{B1} = \frac{100}{1} = 100$

B1 and B2 are the same line because budget is twice as much, but prices are also doubled.

Exercise 1 f

Consider the consumer whose *MRS* between apples and oranges depends on the ratio of her consumption so that $MRS = \frac{y}{x}$.

This means simply that for example at $x = 30, y = 60$ she considers each apple to be equally desirable as two oranges.

From the equations $MRS = MRT$ and the budget constraint $p_x x + p_y y = I$, solve the optimal consumption x^* and y^*.

Exercise 1 f

$$p_x x + p_y y = I$$
$$MRS = \frac{y}{x}$$

Let's derive *MRT* from the budget constraint: $y = \frac{I - p_x x}{p_y} = \frac{I}{p_y} - \frac{\boldsymbol{p_x}}{\boldsymbol{p_y}} x \Longrightarrow MRT = \frac{\boldsymbol{p_x}}{\boldsymbol{p_y}}$

Optimal consumption requires: $MRT = MRS.$ Therefore, at the optimal consumption: $\frac{\boldsymbol{p_x}}{\boldsymbol{p_y}} = \frac{y^*}{x^*} \rightarrow y^* = \frac{\boldsymbol{p_x}}{\boldsymbol{p_y}} \cdot x^*$

Let's substitute y^* with the expression from the budget constraint $y = \frac{I}{p_y} - \frac{\boldsymbol{p_x}}{\boldsymbol{p_y}} x$:

$$\frac{\boldsymbol{p_x}}{\boldsymbol{p_y}} \cdot x^* = \frac{I}{p_y} - \frac{\boldsymbol{p_x}}{\boldsymbol{p_y}} \cdot x^* \Leftrightarrow \frac{\boldsymbol{p_x}}{\boldsymbol{p_y}} \cdot x^* + \frac{\boldsymbol{p_x}}{\boldsymbol{p_y}} \cdot x^* = \frac{I}{p_y} \Leftrightarrow x^* = \frac{\frac{I}{p_y}}{2 \cdot \frac{\boldsymbol{p_x}}{\boldsymbol{p_y}}} = \frac{\boldsymbol{I}}{\boldsymbol{2p_x}}$$

Now, we can derive y^*

$$y^* = \frac{\boldsymbol{p_x}}{\boldsymbol{p_y}} \cdot x^* = \frac{\boldsymbol{p_x}}{\boldsymbol{p_y}} \cdot \frac{\boldsymbol{I}}{\boldsymbol{2p_x}} = \frac{\boldsymbol{I}}{\boldsymbol{2p_y}}$$

$$x^* = \frac{\boldsymbol{I}}{\boldsymbol{2p_x}}$$
$$y^* = \frac{\boldsymbol{I}}{\boldsymbol{2p_y}}$$

Exercise 2

Wages and choice to work

Exercise 2 a

Ann gets paid 10€/hour and works 8 hours/day.
She then gets a pay raise to 15€/hour.

Can you say with certainty what will happen to Ann's working hours as a result of this raise?

Exercise 2 a

Ann gets paid 10€/hour and works 8 hours/day.
She then gets a pay raise to 15€/hour.
Can you say with certainty what will happen to Ann's working hours as a result of this raise?

Before saying anything about how Ann will change her working hours after the change in hourly wage, <u>we need to know something about Ann's preferences</u>. Ann might want to **increase her free time** because now she can earn the same amount of money by working less **(income effect)**; on the other hand, as the opportunity cost of her free time has gone up, she might want to **work more** as a result of the wage increase **(substitution effect)**.

The net effect depends on the relative size of the two effects

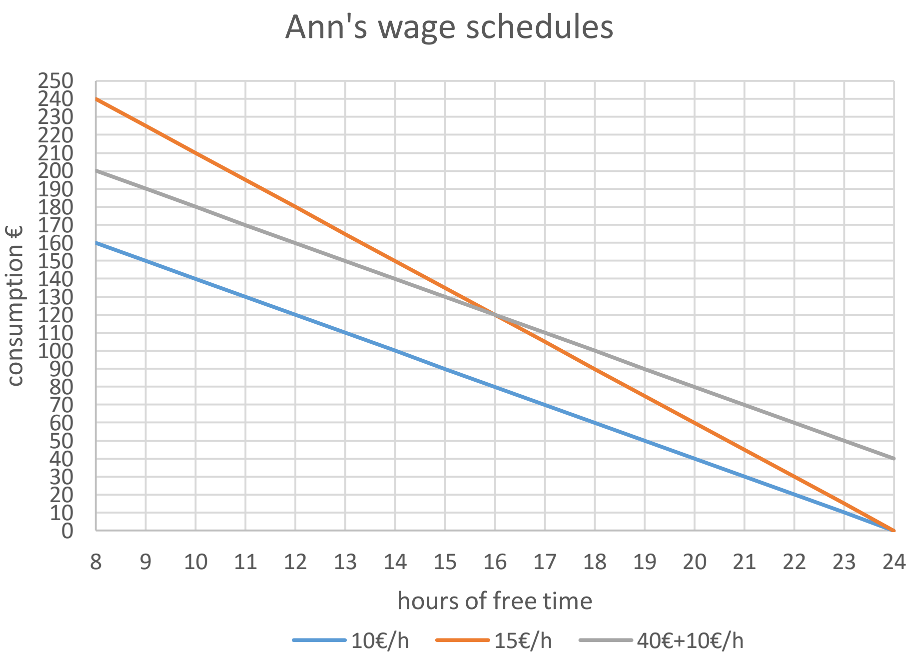
Exercise 2 b

Ann gets paid 10€/hour and works 8 hours/day.
She then gets a pay raise to 15€/hour.

Ann computes that at her old working hours, the boss ends up paying 40€ more per day.
She is tempted to go to the boss and ask for a different wage contract: a flat payment of 40€/day and the old wage of 10€/hour on top.

Draw the budget constraints under the different wage contracts.

Exercise 2 b



OLD PAY: 10€/hour x 8 hours/day = 80€/day
WITH RAISE: 15€/hour x 8 hours/day = 120€/day
NEW PROPOSAL: 40€/day + 10€/hour x 8 hours/day = 120€/day

Draw Ann’s budget constraints under the different wage contracts.

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Exercise 2 c

Should Ann ask for this alternative contract?

Exercise 2c

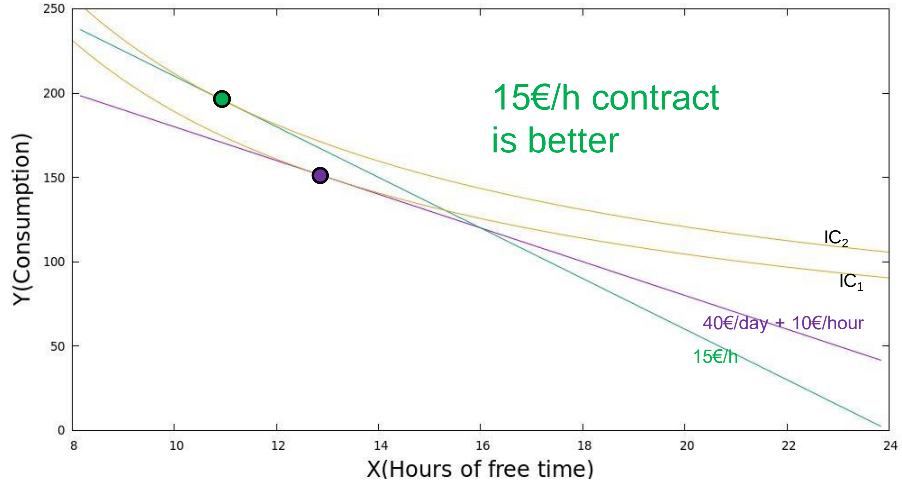
Should Ann ask for this alternative contract?

It really depends on her preferences.

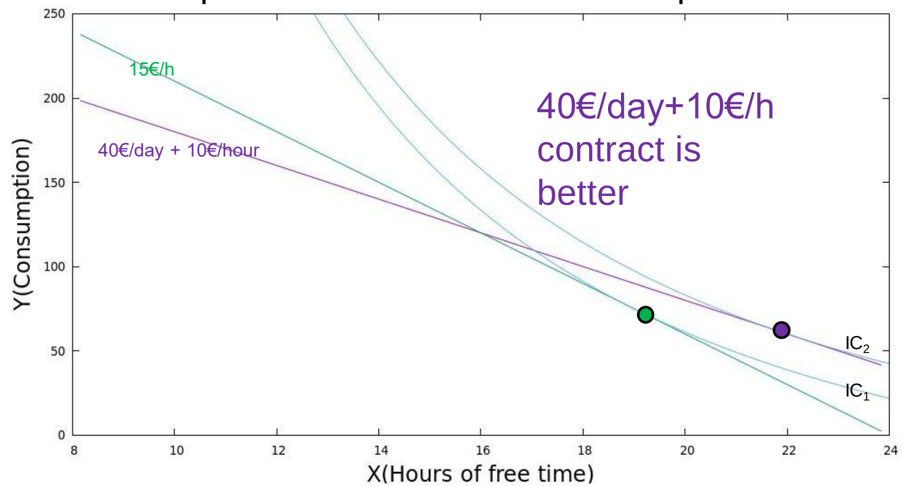
If she values consumption over free time (flatter curve), the contract with 40€/day and 10€/h is not good (she reaches a lower IC than under the 15€/h contract): she has essentially capped her maximum working hours and daily wage to the previous level (120€). With the 15€/h wage, she can work more than 8 hours and earn more.

If, on the other hand, she values free time more than consumption (steeper curve), the contract with 40€/day and 10€/h is a better deal than the 15€/h contract. In fact, she reaches a higher IC.

If Ann prefers consumption over free time



If Ann prefers free time over consumption



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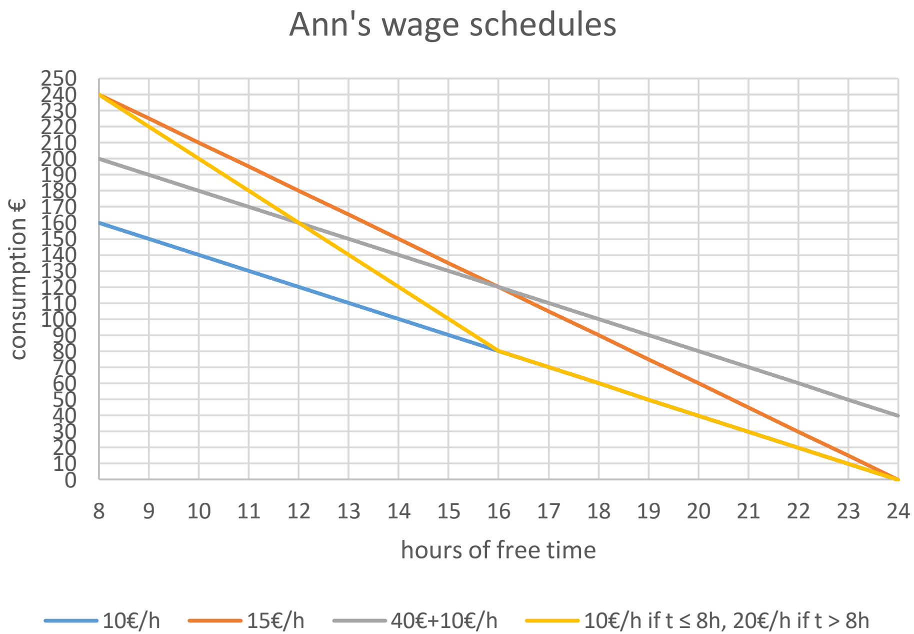
Exercise 2 d

Suppose that the boss wants to induce Ann to work more. Rather than raising the wage, the boss gives a bonus of 10€/hour for each hour of overtime work.

Draw Ann’s budget in this case.

Exercise 2 d



OLD PAY: 10€/hour
WITH RAISE: 15€/hour
NEW PROPOSAL: 40€/day + 10€/hour
OVERTIME BONUS: 10€/hour up to 8 hours + 20€/hour for overtime

Draw Ann’s budget constraint.

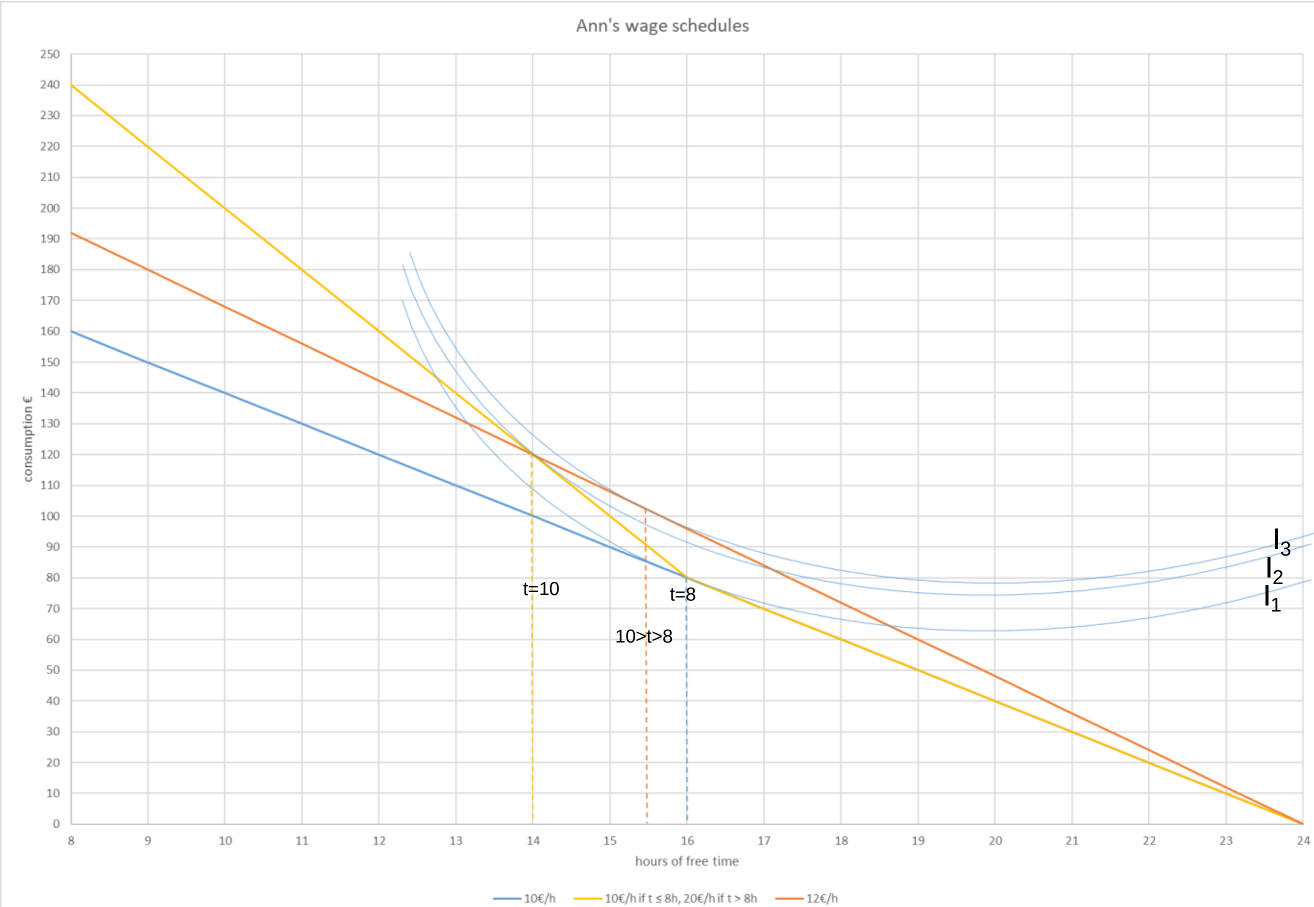
Exercise 2 e

Draw Ann’s indifference curve in such a way that is consistent with the choice of t=8 in the original budget set (10€/hour) and t=10 in the new budget set (10€/hour up to t=8 and 20€/hour for overtime).

What is Ann’s average hourly pay in the overtime bonus scheme if t=10?

How would Ann choose her working hours if she got paid this average wage for all of the hours that she works and no overtime bonus?

Exercise 2 e



Average pay when overtime is paid 20€/hour and Ann works 10 hours.

$$\frac{10 \cdot 8 + 20 \cdot 2}{10} = \frac{12€}{hour}$$

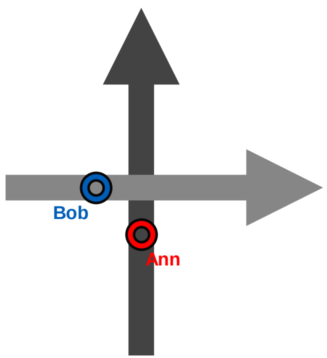
Choice h=8 (free time=16) if wage is 10€/h. Reaches I_1

Choice h=10 (free time=14) if overtime wage is 20€. Reaches I_2

Choice 8<h<10 if wage is 12€/h. Reaches I_3

Exercise 3

Games: crossroads



Two drivers come to the crossroads simultaneously from different directions. Each has to decide at the same time whether to continue driving or wait.

Exercise 3 a

- Who are the players?
- What are the strategies?
- What are the outcomes?
- What are reasonable payoffs?
- Draw a game matrix representing this situation.

Exercise 3 a

		Bob	
		Drive	Stop
Ann	Drive	-1, -1	2, 1
	Stop	1, 2	0, 0

- Who are the players? **Ann, Bob**
- What are the strategies? These are the available actions: **Drive, Stop**
- What are the outcomes? These are all the possible combinations of the actions taken by the players: **Drive, Drive; Drive, Stop; Stop, Drive; Stop, Stop**
- What are reasonable payoffs?

Choose the payoffs to reflect that:

- if there is a crash because nobody stops, both will be really unhappy (we can choose an even lower number);
- if both stop, nothing bad happens, but the two drivers are both wasting time;
- when one goes and the other stops, the one going is getting faster to their destination, and the one waiting will take longer to travel, but still less time than if they were both stopping.

Exercise 3 b

- Does either of the drivers have a dominant strategy?
- Are there Nash equilibria?

Exercise 3 b

		Bob	
		Drive	Stop
Ann	Drive	-1, -1	2, 1
	Stop	1, 2	0, 0

- Does either of the drivers have a dominant strategy? Is any player going to choose the same action regardless of what the other player is doing? **No, each of them will want to choose a different action depending on the action of the other.**

  If Bob Drives, Ann will get -1 if she Drives and 1 if she Stops.
  Since -1<1, Ann will want to Stop.
  If Bob Stops, Ann will get 2 if she Drives and 0 if she Stops. Since 2>0, Ann will want to Drive.
  Ann does not have a dominant strategy. She will want to stop if Bob drives, and will want to drive if Bob stops.

  The same reasoning applies to Bob, as their payoffs are symmetrical.

- Are there Nash Equilibria? When a player's action is the best response to the other player's action.
  If A: Drive → B: Stop; if B: Stop → A: Drive
  If A: Stop → B: Drive; if B: Drive → A: Stop
  So, [D,S] and [S,D] are Nash Equilibria

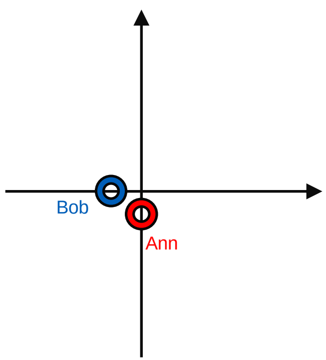
Exercise 3 c

If there are many equilibria, how should the players know which one to play?

Consider various traffic rules and arrangements to help in choosing a NE.
Why do we see different practical solutions to the problem?

Exercise 3 c

		Bob	
		Drive	Stop
Ann	Drive	-1, -1	2, 1
	Stop	1, 2	0, 0



- In such small crossroads, it is probably enough to apply the rule that "the car coming from the right side has the right of way" (or the other way around if we are in UK, India, etc…). In which case Ann would have the right of way, and she would take the action "Drive" knowing that Bob will "Stop" (because he knows that Ann will "Drive").
- Or similarly to above, there could be a road sign before the crossroads that tells Ann that she has the right of way and one that tells Bob that he has to give the right way (maybe the road on which Ann is driving in the main road and the other one is a side road).
- Alternatively, in larger and more trafficked roads, we could have a traffic light which alternates between red and green (of course when it shows red on one road, it shoes green on the other one). It would have the same effect.

- The flow of traffic at the crossroads should determine the coordination device used. When few cars are on the road, the first solution is enough (effective and cost-efficient). With heavy traffic, you avoid big jams by allowing traffic first in one direction and then in another at time intervals.

Exercise 4

Games: tax collection and tax fraud

- Tax officer decides whether to monitor or not monitor
- Tax payer decides whether to file honestly or fraudulently
- If the tax payer files fraudulently and gets caught, he pays a large fine

Exercise 4 a

Draw a game matrix for this situation.

Assume first that the payoffs are simply the monetary payoffs to the players.

Exercise 4

		Tax Payer	
		Honest	Cheat
Tax Officer	Monitor	-2, -21	40,-63
	Not Monitor	0, -21	0, -7

Taxes when honest	21
Taxes when dishonest	7
Fine if caught	42
Cost of monitoring	2

Exercise 4 a

Assume first that the payoffs are simply the monetary payoffs to the players.

Does this game have dominant strategies?
What about Nash Equilibria?

Exercise 4 b

		Tax Payer	
		Honest	Cheat
Tax Officer	Monitor	-2, -21	40,-63
	Not Monitor	0, -21	0, -7

Taxes when honest	21
Taxes when dishonest	7
Fine if caught	42
Cost of monitoring	2

There are no dominant strategies.
For tax officer, -2<0 but 40>0, so he will want to monitor if the tax payer cheats, and will not want to monitor if the tax payer is honest.
For the tax payer, -21>-63, but -21<-7, so he will want to be honest if the tax officer monitors, and will want to cheat if the tax officer does not monitor.

Nash Equilibrium: when a player's action is the best response to the other player's action.
If O: M → P: H; if P: H → O: NM; if O: NM → P: C; if P: C → O: M; and so on…
If P: H → O: NM; if O: NM → P: C; if P: C → O: M; if O: M → P: H; and so on…

So, there are no Nash Equilibria.

Exercise 4 c

How does the game change if you allow for the possibility that the tax payer might feel guilty about committing tax fraud?

Exercise 4c

		Tax Payer	
		Honest	Cheat
Tax Officer	Monitor	-2, -21	40,**-69**
	Not Monitor	0, -21	0, **-13**

Taxes when honest	21
Taxes when dishonest	7
Fine if caught	42
Cost of monitoring	2
Guilt when being dishonest	**-6**

<u>With this choice of payoffs, not much has changed.</u>

There are no dominant strategies.
For tax officer, -2<0 but 40>0, so he will want to monitor if the tax payer cheats, and will not want to monitor if the tax payer is honest.
For the tax payer, -21>-69, but -21<-13, so he will want to be honest if the tax officer monitors, and will want to cheat if the tax officer does not monitor.

Nash Equilibrium: when a player's action is the best response to the other player's action.
If O: M → P: H; if P: H → O: NM; if O: NM → P: C; if P: C → O: M; and so on…
If P: H → O: NM; if O: NM → P: C; if P: C → O: M; if O: M → P: H; and so on…

So, there are no Nash Equilibria.

Exercise 4c

		Tax Payer	
		Honest	Cheat
Tax Officer	Monitor	-2, -21	40,-79
	Not Monitor	0, -21	0, -23

I increase the guilt effect	
Taxes when honest	21
Taxes when dishonest	7
Fine if caught	42
Cost of monitoring	2
Guilt when being dishonest	-16

Now we have dominant strategies.
For the tax payer, -21>-79, and -21>-23, so he will want to be honest in every circumstance, whether the tax officer monitors or not.
The tax officer does not have a dominant strategy, because -2<0 but 40>0. He will want to monitor if the tax payer cheats, and will not want to monitor if the tax payer is honest.

Nash Equilibrium: when a player's action is the best response to the other player's action.
Since P has a dominant strategy, P:H always.
If O: M → P: H; if P: H → O: NM; if O: NM → P: H; if P: H → O: M.
If P: H → O: NM; if O: NM → P: H.
[NM,H] is a Nash Equilibria.

Exercise 5

Games: dinner with a friend

Five menus at a given price are valued by the diners as follows:

	Price	Valuation
Menu 1	30	33
Menu 2	40	44
Menu 3	50	52
Menu 4	60	59

Exercise 5 a

Given prices and valuations of the menus, which menu would the diner choose if he was paying his own bill?

Exercise 5a

When each person pays for themselves, each friend chooses the menu that gives the maximum payoff.

Payoff(M) = Price of menu – Valuation of menu

Payoff(M1) = 33 – 30 = 3

Payoff(M2) = 44 – 40 = 4

Payoff(M3) = 52 – 50 = 2

Payoff(M4) = 59 – 60 = -1

M2 is the best choice, as it gives the larges payoff.

Exercise 5 b

Draw the game matrix in the case in which the two friends split the bill equally, regardless of which menus they pick.
They choose simultaneously the menu without agreeing in advance.

Exercise 5b

		B			
		M1	M2	M3	M4
A	M 1	33-30=3, 33-30=3	33-35=-2 44-35=9	33-40=-7 52-40=12	33-45=-12 59-45=14
	M 2	44-35=9 33-35=-2	44-40=4, 44-40=4	44-45=-1 52-45=7	44-50=-6 59-50=9
	M 3	52-40=12 33-40=-7	52-45=7 44-45=-1	52-50=2 52-50=2	52-55=-3 59-55=4
	M 4	59-45=14 33-45=-12	59-50=9 44-50=-6	59-55=4 52-55=-3	59-60=-1, 59-60=-1

Total bill ; **bill per person**					
		B			
		M1	M2	M3	M4
A	M1	60; **30**	70; **35**	80; **40**	90; **45**
	M2	70; **35**	80; **40**	90; **45**	100; **50**
	M3	80; **40**	90; **45**	100; **50**	110; **55**
	M4	90; **45**	100; **50**	110; **55**	120; **60**

Calculate payoffs from each combination of menu choices by subtracting the bill that each person has to pay (given also the menu chosen by the other person) from the valuation for the menu chosen by each player.

E.g.: if A chooses M2 and B chooses M4, A gets 44-((40+60)/2)=-6; B gets 59-((60+40)/2)=9

Exercise 5 c

Do the players have dominant strategies?
If yes, is the dominant strategy equilibrium socially desirable?
If no, what kind of Nash equilibria does the game have?

Exercise 5 c

		B			
		M1	M2	M3	M4
A	M1	3, 3	-2, 9	-7, 12	-12, 14
	M2	9, -2	4, 4	-1, 7	-6, 9
	M3	12, -7	7, -1	2, 2	-3, 4
	M4	14, -12	9, -6	4, -3	-1, -1

M4 is a dominant strategy for both players. Whatever the other player chooses, the payoff from M4 is always higher than the payoff from any other menu.

For player A, payoff from M4:
If B picks M1: 14 > 3, 9, 12
If B picks M2: 9 > -2, 4, 7, 9
If B picks M3: 4 > -7, -1, 2
If B picks M4: -1 > -12, -6, -3
The same applies to player B, for whatever choice of player A.

Given this, the Nash equilibrium is (M4, M4) which gives payoff (-1, -1).

This is a lot worse than the socially desirable payoffs if each friend was paying for their own meal and chose M2 (4, 4).

Exercise 5 d

Do you think that in this type of situation, there should be other concerns to include in the subjective payoffs?

Exercise 5 d

Should we include other aspects in the subjective payoffs?

- Maybe splitting the bill in half adds a positive feeling which is reflected in higher payoff….
- Maybe one feels a bit guilty taking the most expensive option, so we could reduce the payoff from picking M4.