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CHEM-E2200: Polymer blends and composites

Long-fibre composites and laminates

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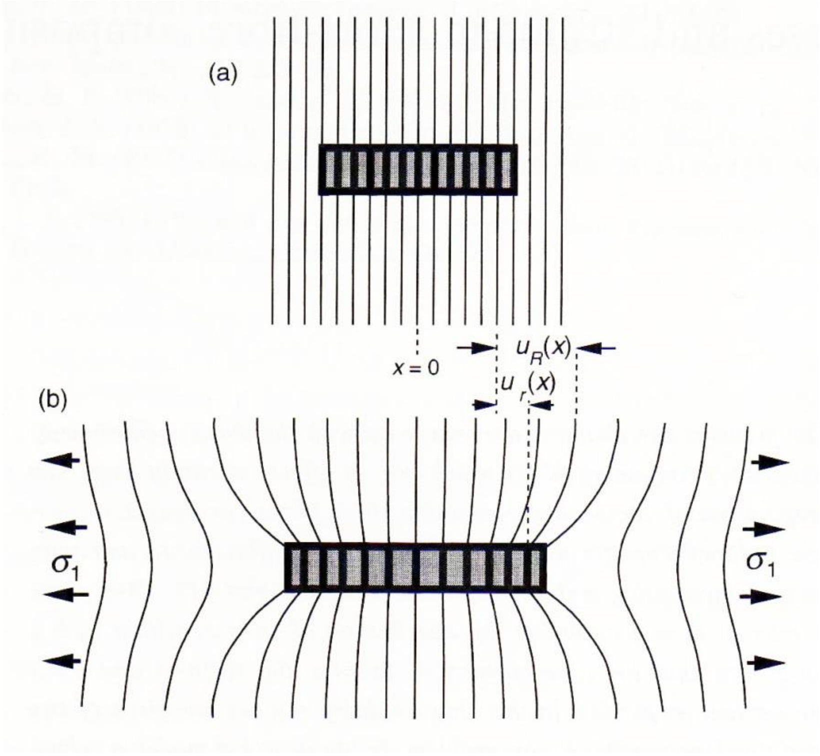
28th September 2020

Today

- Axial and transverse elastic deformation in long-fibre composites
- Off-axis deformation in long-fibre composites
- Multi-ply laminates
- Strength and failure in long-fibre composites
- Compressive failure in composites

Elastic deformation of long-fibre composites

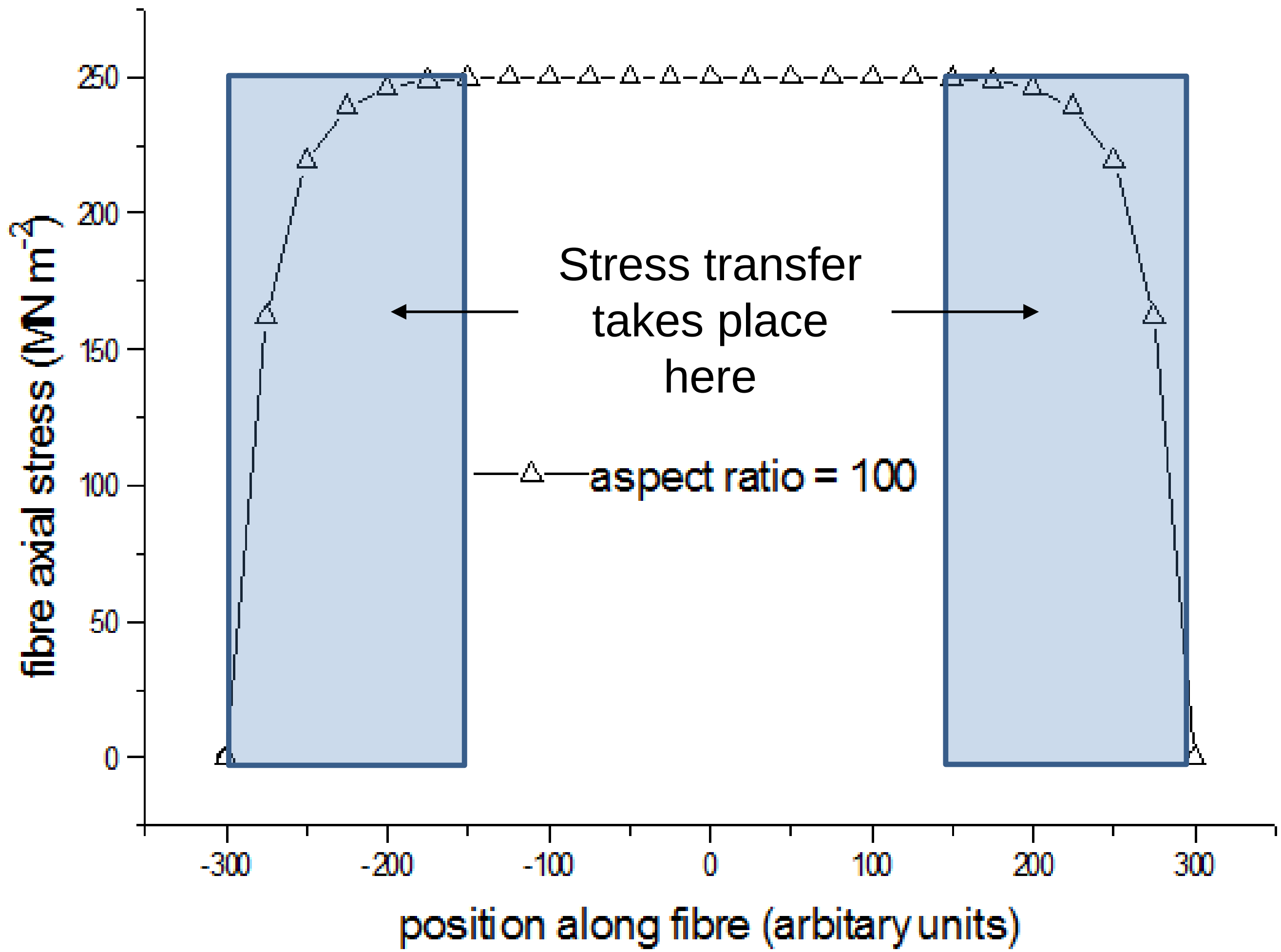
Shear-lag model



(Source: Hull & Clyne 1996)

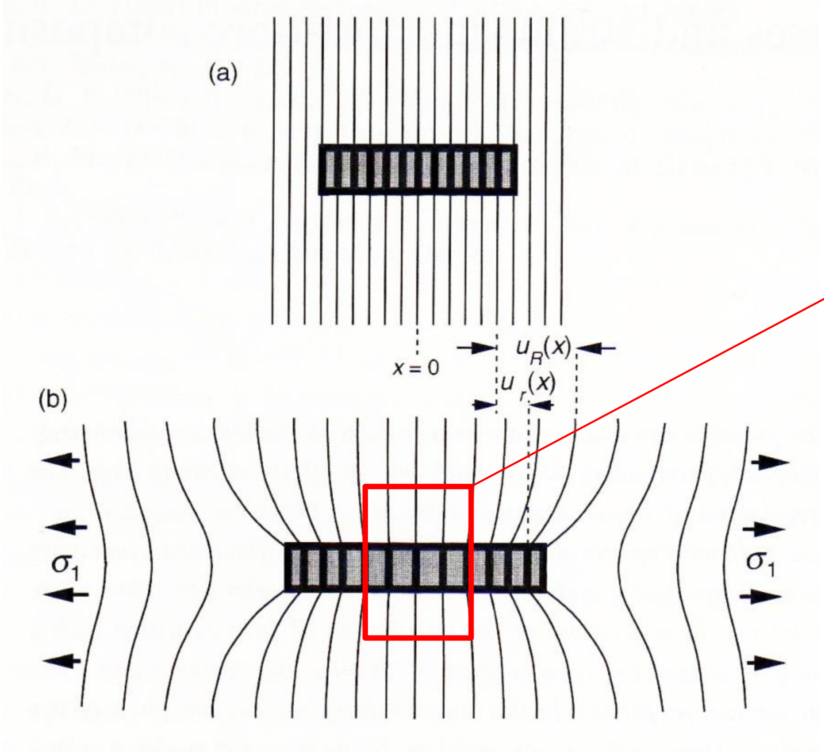
Distortion of a **compliant** matrix around **stiff** reinforcement

Aspect ratio > stress transfer aspect ratio



(Assumptions: fibre modulus: 50 GPa; matrix modulus: 3.5 GPa; axial strain: 0.5%)

Shear-lag model



Reinforcement and matrix undergo equal strain where no stress-transfer is taking place

(Source: Hull & Clyne 1996)

Distortion of **compliant** matrix around **stiff** reinforcement

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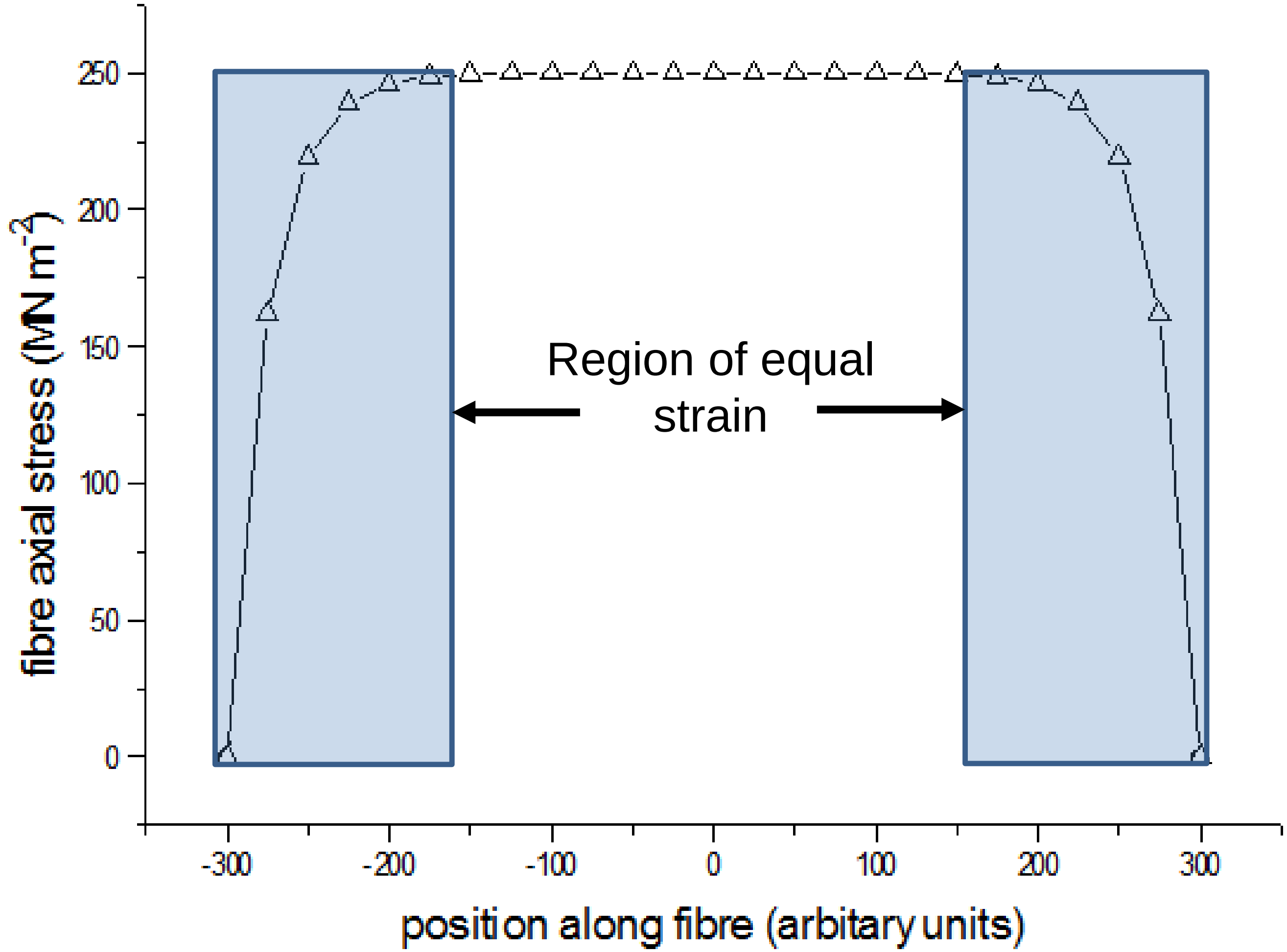
Aspect ratio > stress transfer aspect ratio



(Assumptions: fibre modulus: 50 GPa; matrix modulus: 3.5 GPa; axial strain: 0.5%)

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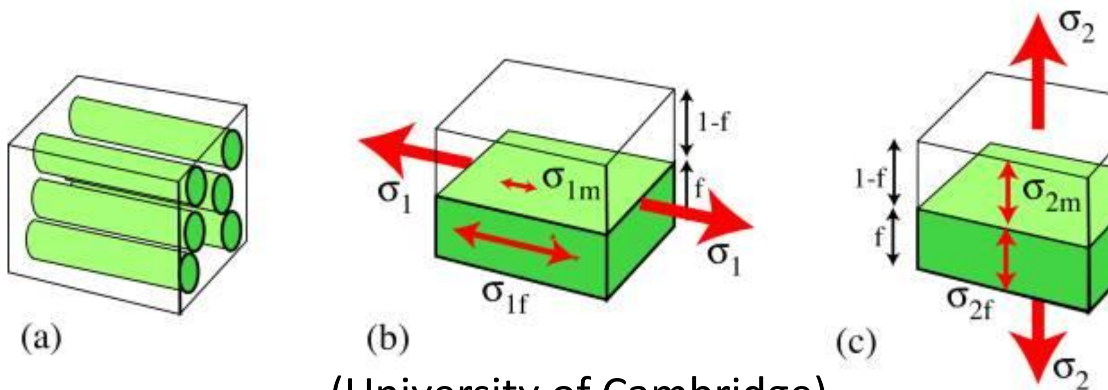
Elastic deformation of long fibre composites

- If the fibre reinforcement is of very high aspect ratio (for all intents and purposes it can be regarded as infinitely long), then the reinforcement and matrix undergo the same **strain** (when the stress is applied parallel to the fibre axis)
- The composite can be thought of as two ‘slabs’ of material (reinforcement and matrix) that both undergo **equal** strain
- Or that under go **equal stress**, if the composite is loaded perpendicular to the fibre axis



(University of Cambridge)

Axial stiffness of long fibre laminates: the Voigt or ‘equal strain’ model

- Axial composite stress can be given by:

$$\sigma_1 = V_f \sigma_f + (1 - V_f)\sigma_m$$

- Since the axial strain in the reinforcement, ε_f, matrix, ε_m, and thus composite, ε_1, are equal, we can substitute and replace for stress and derive an expression for the axial stiffness of a UD laminate as a function of the Young’s moduli of the phases and volume fraction. This is known as the ‘Rule of mixtures’ and can be used to predict the axial stiffness of a composite

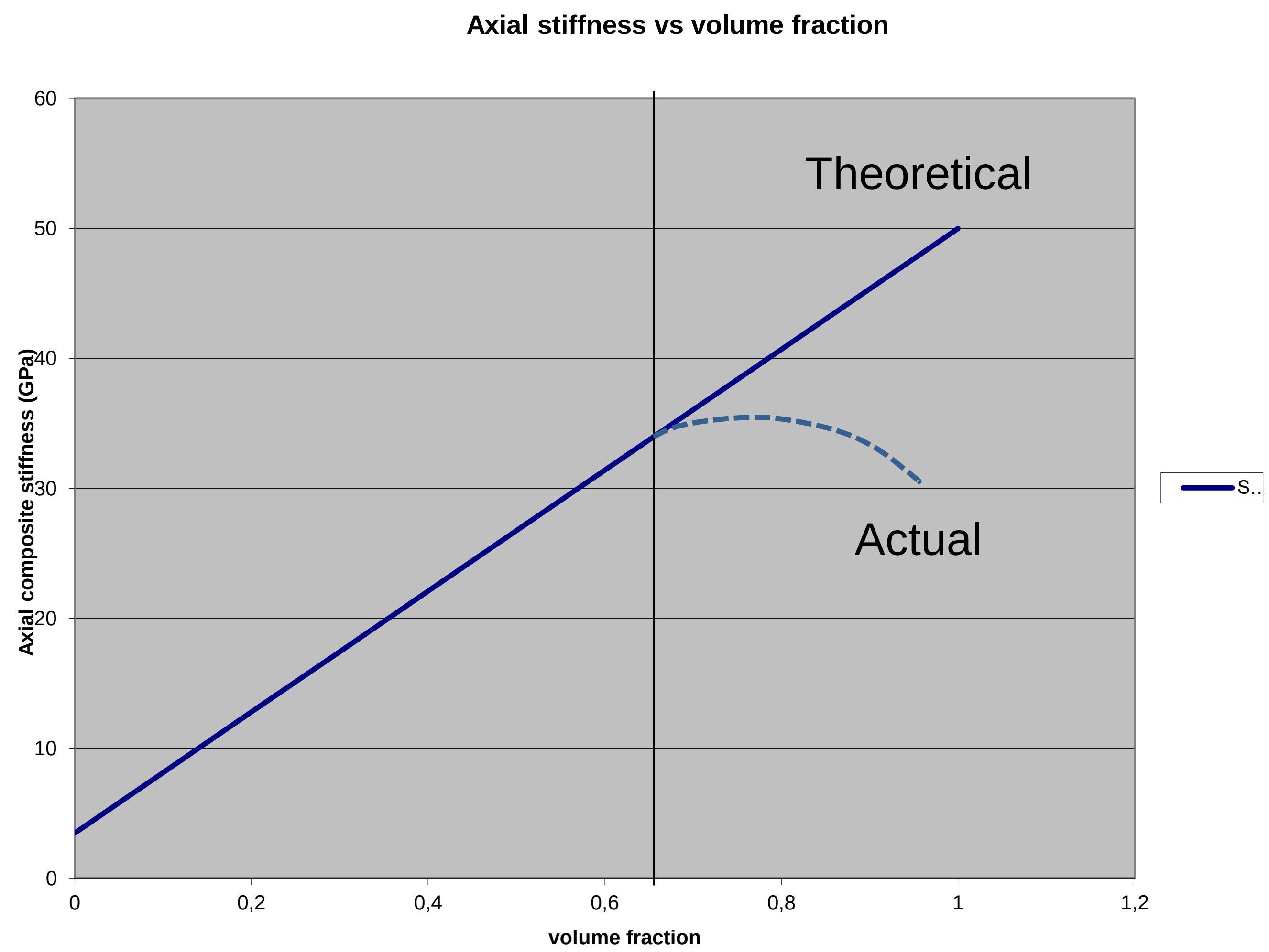
$$E_1 = V_f E_f + \left(1 - V_f\right) E_m$$

E_1 is the axial modulus of the laminate

E_f is the modulus of the fibre (assuming that the fibre possesses isotropic properties)

E_m is the modulus of the matrix

Axial stiffness



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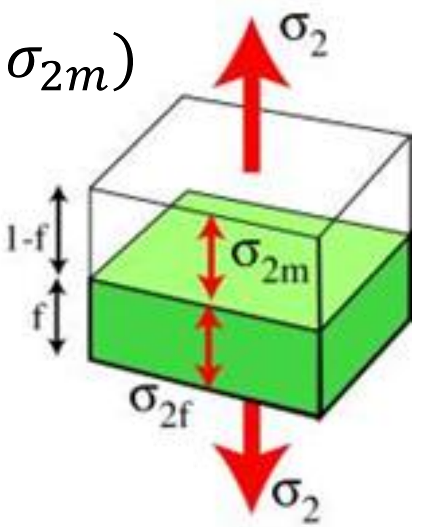
Transverse stiffness – Reuss model

- The equal stress (or **Reuss** model) - also known as the inverse rule of mixtures - gives a prediction for the transverse stiffness of a long fibre composite
- Transverse strain equal to the volume weighted sum of the strains of the 'slabs' of fibre and matrix:

$$\varepsilon_2 = V_f . \varepsilon_f + (1 - V_f) . \varepsilon_m \qquad (NB\colon\ \sigma_2 = \ \sigma_{2f} = \sigma_{2m})$$

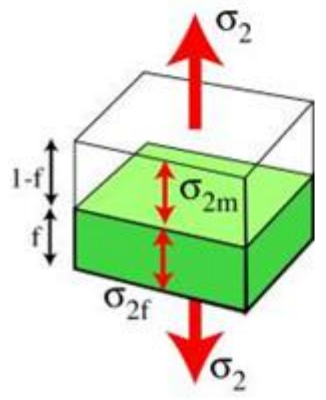
$$E_2 = \left(\frac{V_f}{E_{2f}} + \frac{(1 - V_f)}{E_{2m}} \right)^{-1}$$



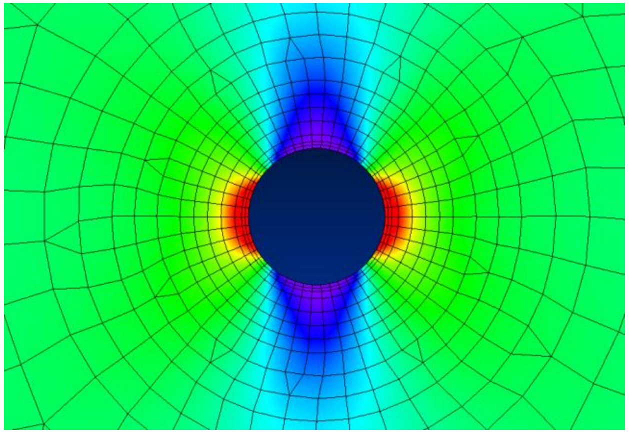
(Source: University of Cambridge)

Transverse stiffness – Reuss model

- It is a 'series' model and does not account for the regions acting in parallel
- No does account for the distortions around circular fibres (circular holes result is a x3 stress concentration in a uniaxially loaded plate)
- It is therefore an oversimplification, but does give a lower bound for transverse stiffness



(Source: University of Cambridge)



(Source: http://www.fracturemechanics.org/hole.html)

Transverse stiffness – Halpin-Tsai

- A semi-empirical prediction developed by Halpin and Tsai (1967), gives a better prediction of transverse laminate stiffness

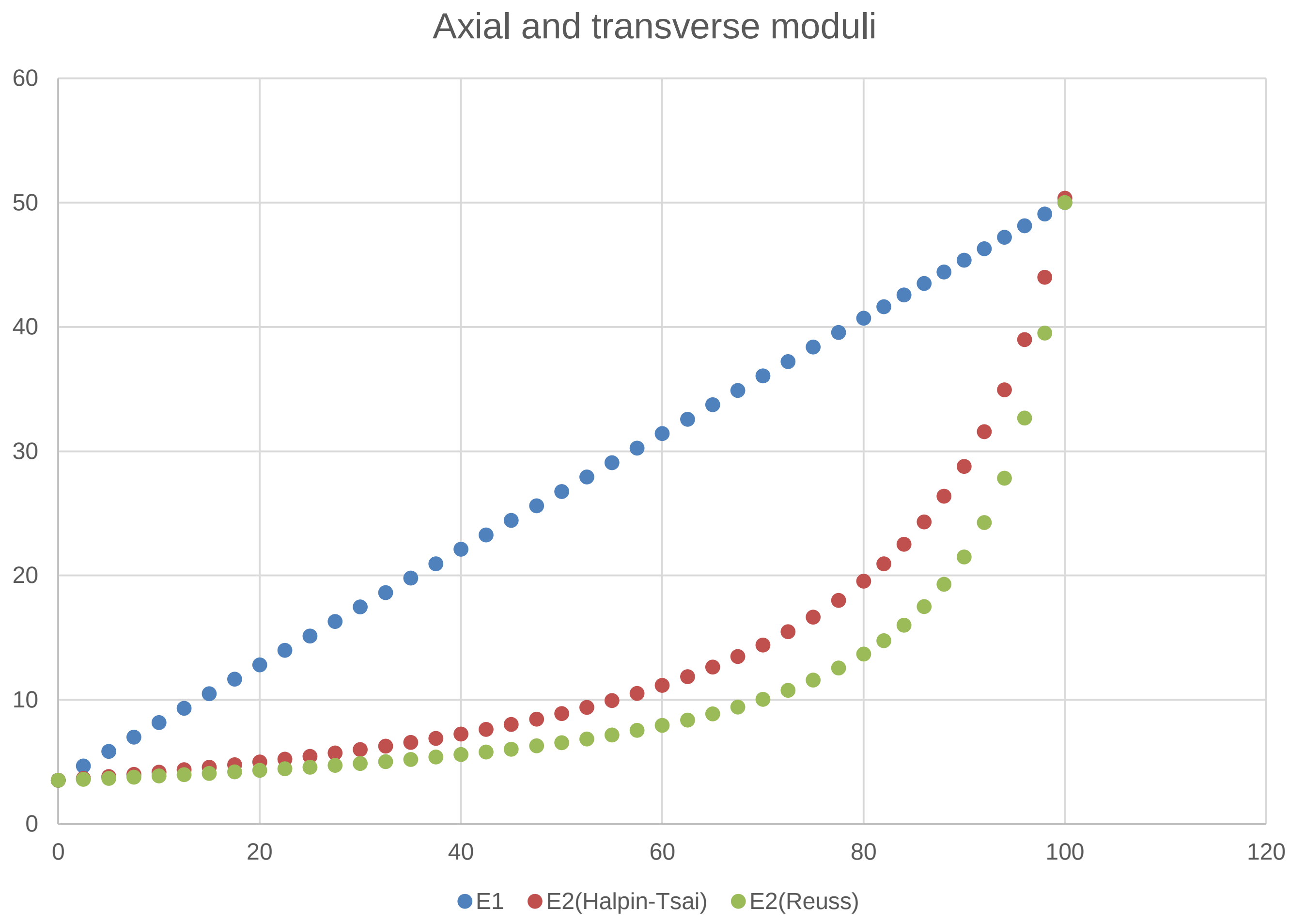
$$E_2 = \frac{E_m\left(1+\xi\eta V_f\right)}{\left(1-\eta V_f\right)}$$

Where: $$\eta = \frac{\left(\dfrac{E_f}{E_m}-1\right)}{\left(\dfrac{E_f}{E_m}+\xi\right)}$$

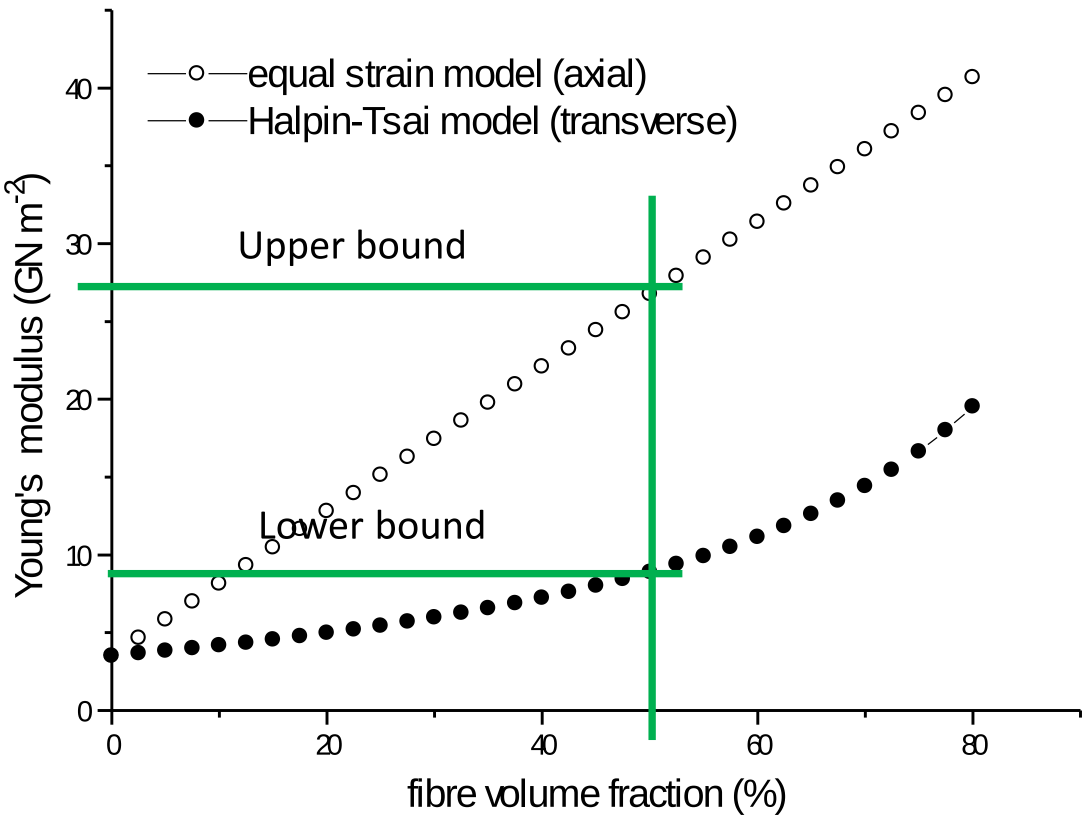
E_2 is the transverse laminate modulus

ξ is an adjustable parameter generally of the order of unity.

Axial / transverse stiffness

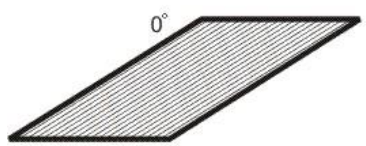


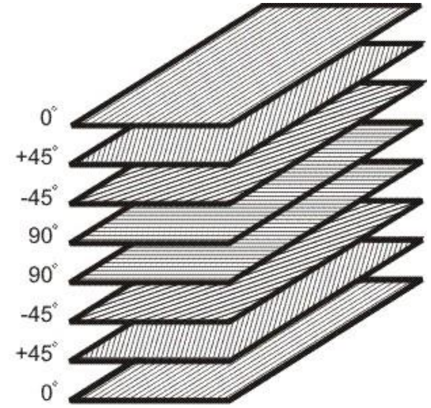
Axial / transverse stiffness

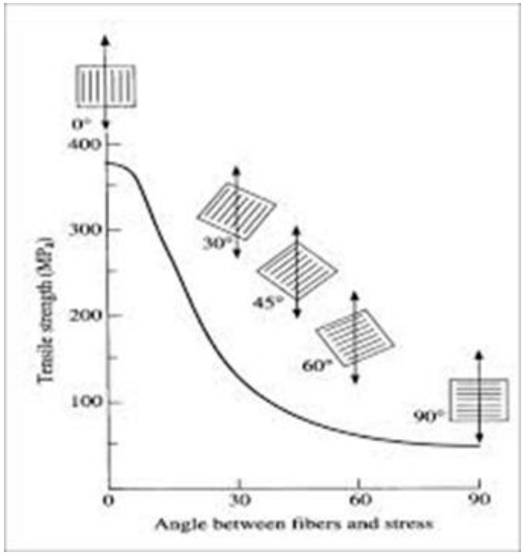


Effect of loading angle

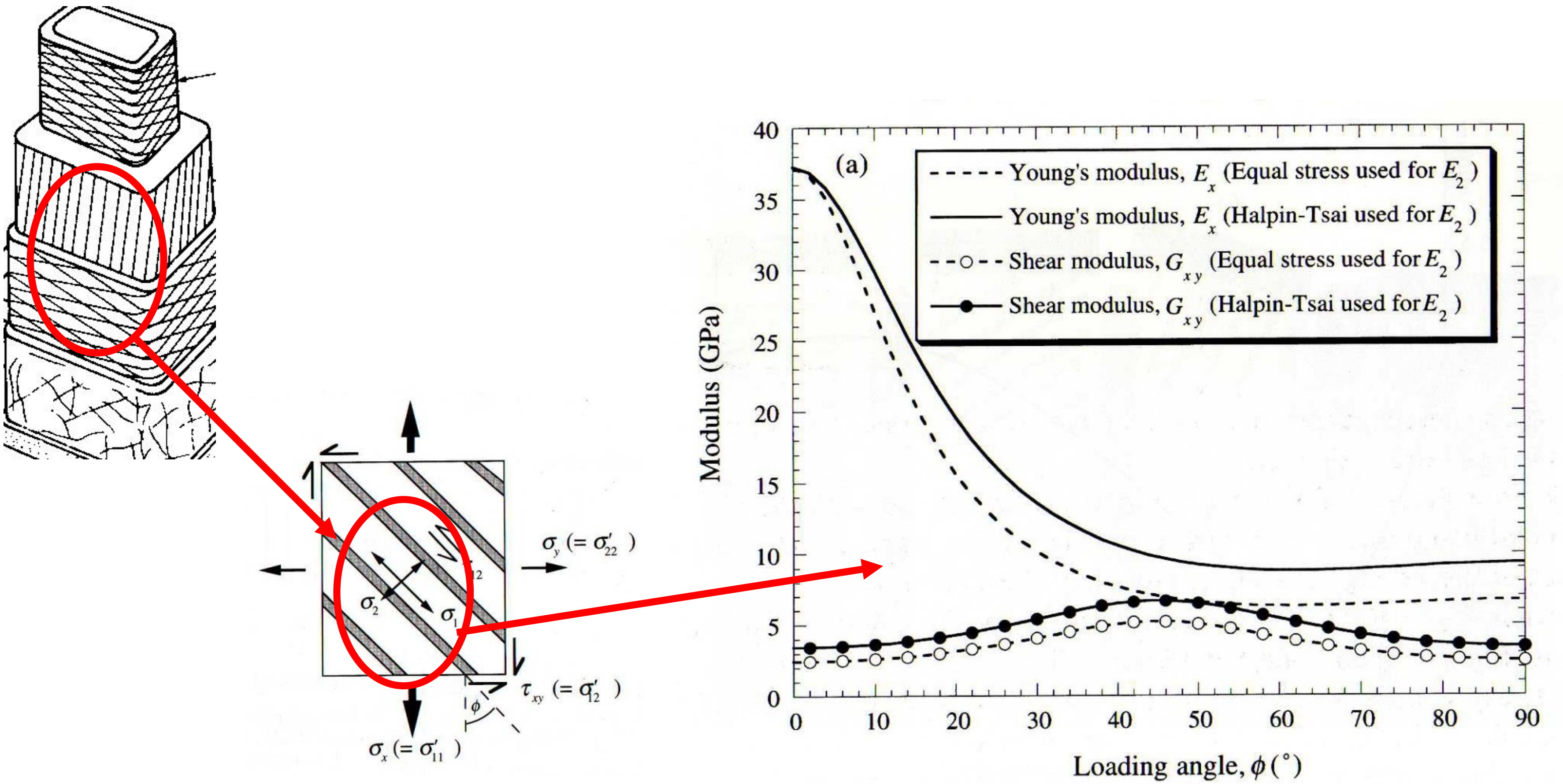
- The Voight and Reuss models provide upper (axial) and lower (transverse) bounds for the stiffness of a long-fibre composite laminate if the properties of the constituents (phases) and the fibre volume fraction are known
- But what happens at angles intermediate between the two bounds?







Predicting the stiffness of real laminates



(Source: Hull & Clyne, 1996)

Shear stiffness – Hapin-Tsai

- A semi-empirical prediction developed by Halpin and Tsai (1967), gives a better prediction of shear stiffness

$$G_{12} = \frac{G_m\left(1 + \xi\eta V_f\right)}{\left(1 - \eta V_f\right)}$$

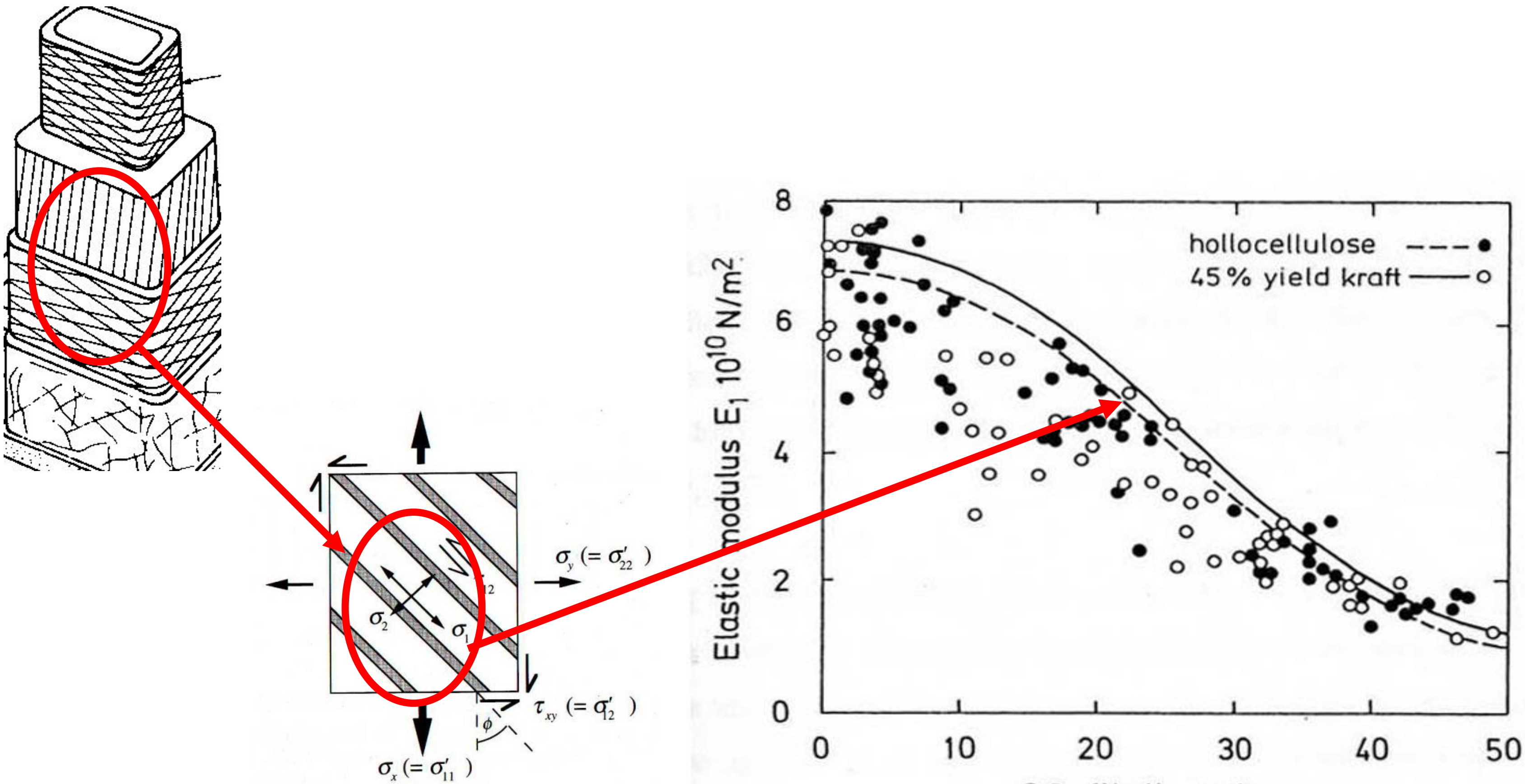
Where:

$$\eta = \frac{\left(\frac{G_f}{G_m} - 1\right)}{\left(\frac{G_f}{G_m} + \xi\right)}$$

G_{12} is the shear modulus

ξ is an adjustable parameter generally of the order of unity.

Predicting the stiffness of real laminates



(Source: Hull & Clyne, 1996)

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Randomly oriented fibre

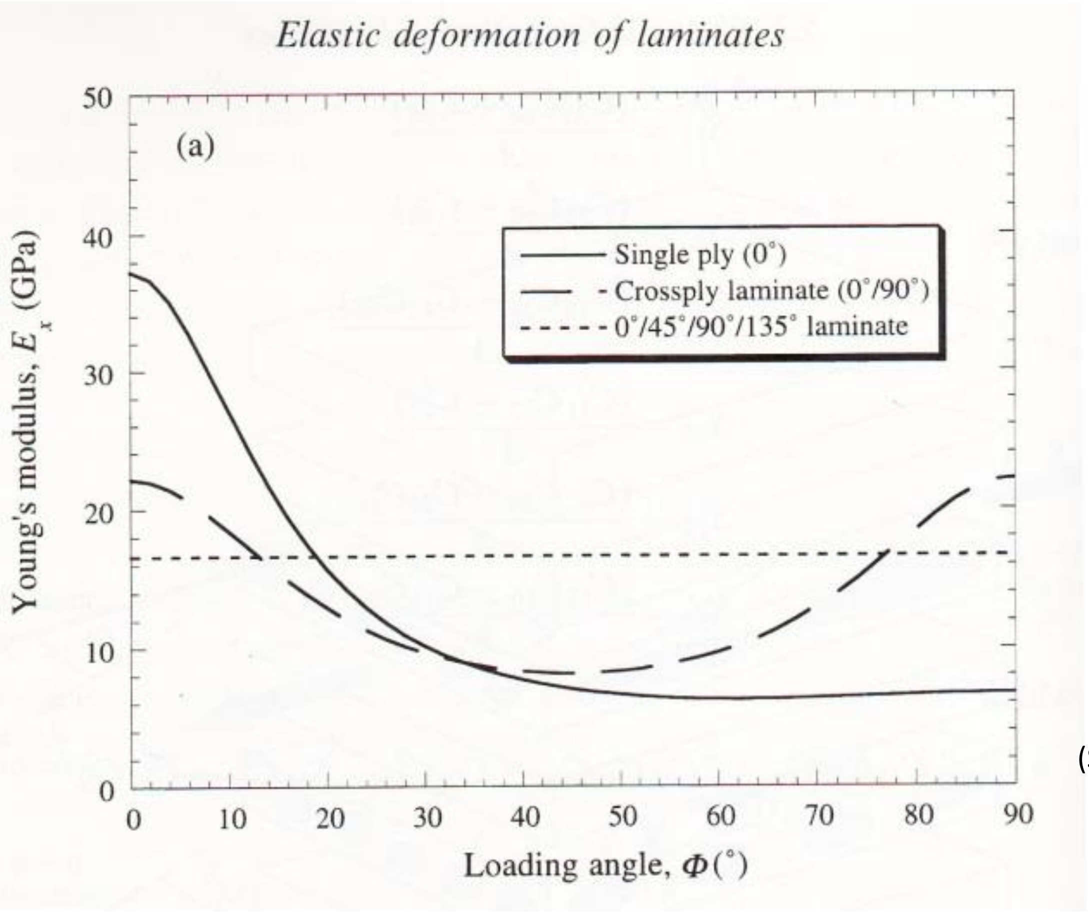
- When fibres are randomly oriented in one plane (e.g. chopped strand mat - C.S.M. reinforcement), then the composite stiffness is given by:

$$E_r \cong \frac{3}{8} E_1 + \frac{5}{8} E_2$$

Where: E_1 is the axial laminate stiffness
E_2 is the transverse laminate stiffness

Essentially the composite stiffness is a 'average' of the axial and transverse stiffness

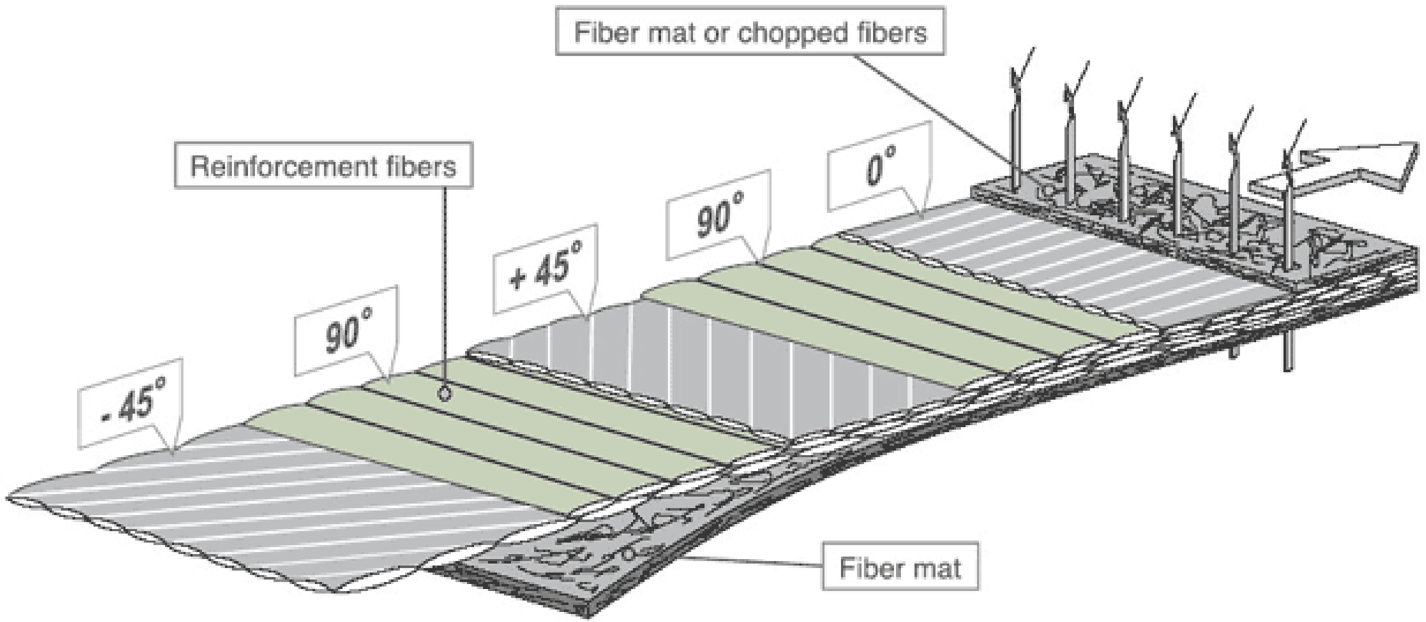
Stiffness and orientation



(Source: Hull & Clyne 1996)

As the loading angle changes from axial to transverse, the stiffness of the composite decreases

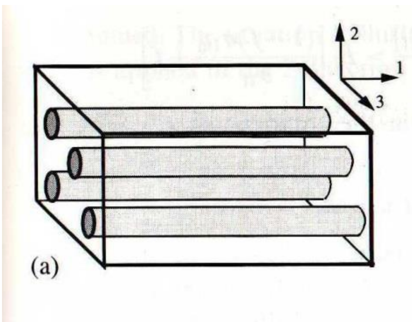
Cross-/multi-ply



(Source: http://www.lbie.com/vyarn.htm)

Strength and failure of laminates

Failure in laminates



(Source: Hull & Clyne 1996)

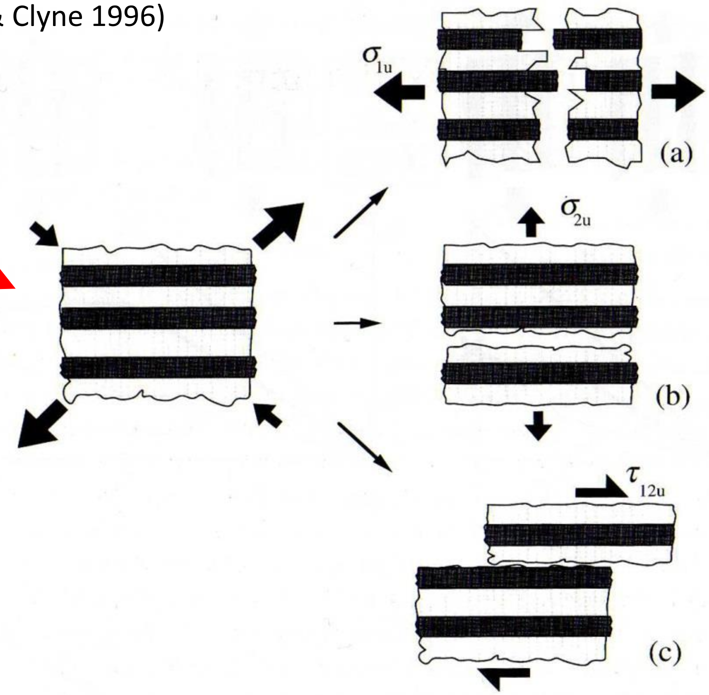


Fig. 8.1 Schematic illustration of how an arbitrary stress state in a lamina gives rise to failure as a result of exceeding critical values of (a) axial tensile stress σ_{1u}, (b) transverse tensile stress σ_{2u} and (c) shear stress τ_{12u}.

- Under an arbitrary state of stress, failure can occur in a number of ways

Failure under axial loading parallel to the fibre axis

- Under axial loading there are ‘competing’ effects from the matrix and reinforcement
- At low reinforcement volume fractions, the addition of reinforcement weakens the matrix
- At higher volume fractions the role of the reinforcement becomes dominant

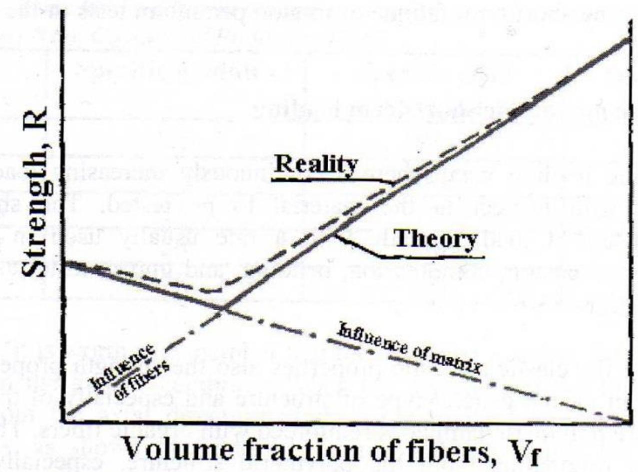
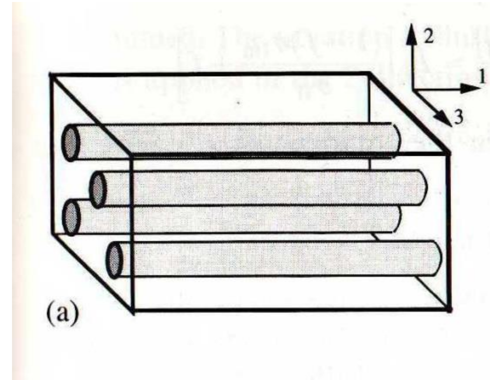


Fig. 185. *Axial strength as a function of the volume fraction of fibers in a composite Reinforced unidirectionally with continuous fibers (schematic presentation).*

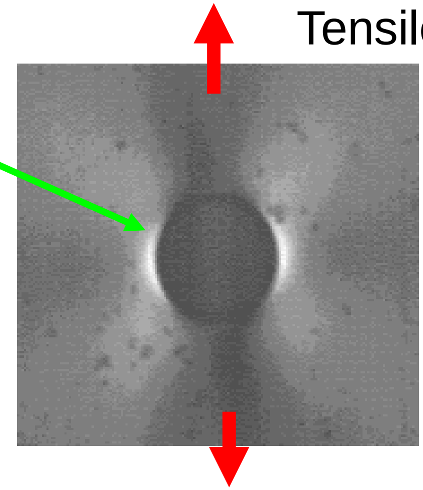
(Source: Kettunen, 2006)

Tensile failure perpendicular to the fibre axis

- Fibres act as a series of ‘holes’ in the matrix
- Holes act as ‘stress concentrators’
- This can result in a weakening of the matrix – unless the interface (bond) between the reinforcement and the matrix is sufficiently strong



Tensile stress



Photoelastic model (Hughes, 2001)

Axial tensile failure

- If it is assumed that both reinforcement and matrix behave in a brittle manner then, two distinct failure scenarios may be envisaged. The first occurs when the fibre strain to failure, ε_{fu} is less than that of the matrix failure strain, ε_{mu}. The second occurs when the failure strain of the matrix is less than that of the fibre. Simplified models for the prediction of axial laminate strength (σ_{1u}) have been developed utilising this approach (Hull & Clyne, 1996).

Axial tensile failure

- Where fibre failure precedes matrix fracture (i.e. $\varepsilon_{fu} < \varepsilon_{mu}$) it can be shown that the failure stress of the lamina can be expressed as follows (Hull & Clyne, 1996; Piggott, 1980):

$$\sigma_{1u} = V_f \sigma_{fu} + (1 - V_f) \sigma_{mfu}$$

Where: σ_{mfu} is the matrix stress at the onset of fibre fracture

Predicting the strength of laminates

- When the fibre failure strain exceeds that of the matrix (e.g. a glass fibre reinforced unsaturated polyester composite), cracking of the latter occurs prior to axial composite failure. As the composite is loaded, matrix cracking initiates once the failure strain ε_{mu} is reached. As cracking continues, load is progressively transferred to the fibres. If the load is carried entirely by the fibres, then the failure stress σ_{1u} is given by:

$$\sigma_{1c} = V_f . \sigma_{fu}$$

Failure under shear

- Dependent upon the orientation of the reinforcement (i.e. unlikely to occur ‘across the fibre’ but likely to occur parallel to the fibre)
- Likely to be close to the shear yield strength of the matrix material

Failure under off-axis loading

- It is clear that the strength of a unidirectional composites is highest when it is loaded parallel to the fibre direction (axial tensile strength) and lowest when it is loaded perpendicular to this direction (transverse tensile strength)
- At intermediate angles, failure may occur by a combination of axial tensile, transverse tensile or shear loads
- The ‘Tsai-Hill’ criterion can be used to predict the failure of a unidirectional laminate under off-axis loading and furthermore can provide, by inspection of the relative magnitudes of the terms, an indication of the likely failure mode (Hull & Clyne, 1996)

Tsai-Hill criterion

$$\sigma_{\phi} = \left[\frac{\cos^2\phi\left(\cos^2\phi - \sin^2\phi\right)}{\sigma^2{}_{1u}} + \frac{\sin^4\phi}{\sigma^2{}_{2u}} + \frac{\cos^2\phi\sin^2\phi}{\tau^2{}_{12u}} \right]^{-1/2}$$

Where:

σ_{ϕ} is the off-axis tensile failure stress of the lamina

ϕ is the loading angle

σ_{1u} is the axial tensile strength of the lamina

σ_{2u} is the transverse tensile strength of the lamina

τ_{12u} is the shear strength of the lamina

Relationship between the strength of a laminate and loading angle

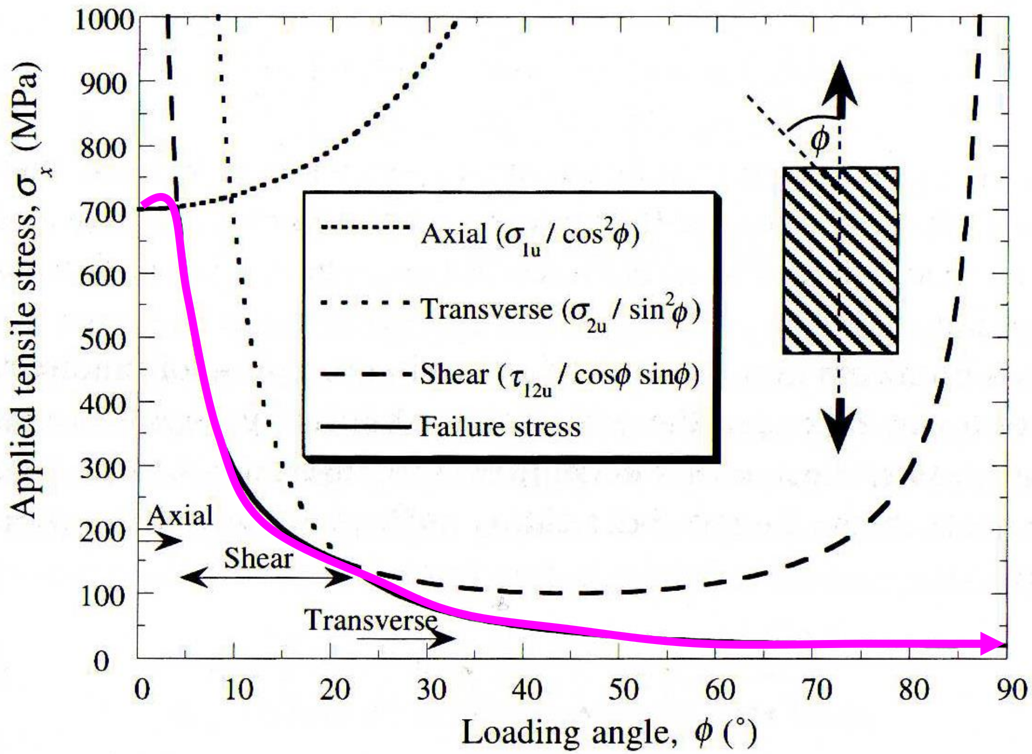
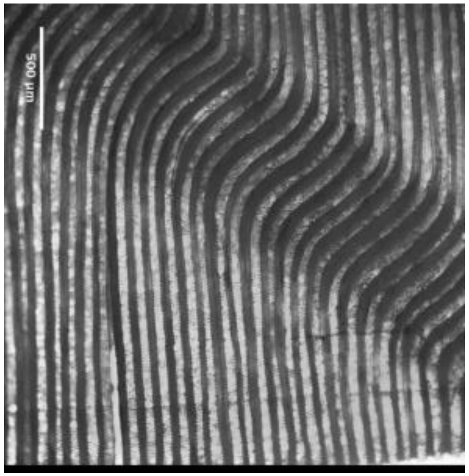


Fig. 8.22 Predicted dependence on loading angle ϕ of the applied stress for the onset of different failure modes for a polyester/50% glass lamina, according to the maximum stress criterion.

(Source: Hull & Clyne 1996)

Failure of unidirectional polymer matrix composites in compression



Kinking in Dyneema
(Source: http://faculty.engineering.asu.edu/ppe ralta/?page_id=95)

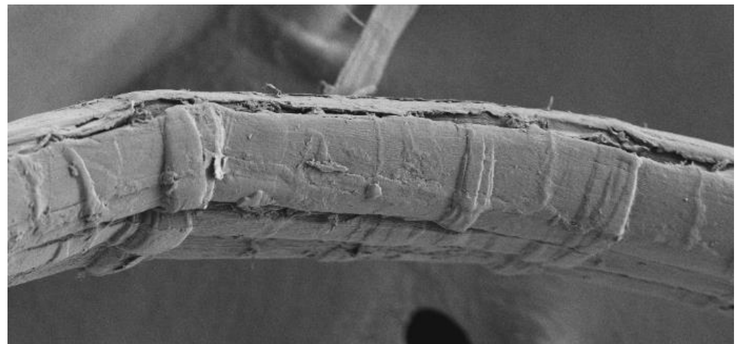
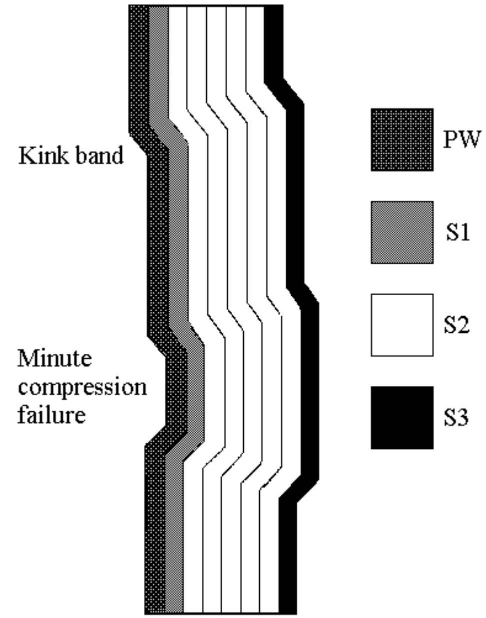


Fig. 1: Kink bands in hemp (*Cannabis sativa* L.) fibre



Cell wall failure
(Source: Wardop & Dadswell, 1947)

Compression failure (and why trees are good at standing up!)

Polymer matrix composites often fail through kinking, when overloaded in axial compression parallel to the axis of the fibres (Hull & Clyne, 1996). Argon (1972), proposed a model for the initiation of kinks in such materials. At the onset of instability, it was predicted that the compressive stress, σ_{comp} , would be given simply by:

$$\sigma_{comp} = {}^{\tau_s}\!/\!{}_{\Delta\Phi}$$

Where:

τ_s is the plastic shear strength of the matrix

$\Delta\Phi$ is the average misorientation angle of the reinforcing elements (in radians)

Literature

- Argon, A.S. (1972). Fracture of Composites. Academic: New York
- Hull, D. and Clyne, T.W. (1996). *An Introduction to Composite Materials*. Cambridge University Press, Cambridge, UK
- Piggott, M.R. (1980). Load-Bearing Fibre Composites. Pergamon, Oxford.
- Wardop, A.B. and Dadswell, H.E. (1947). Contributions to the Study of the Cell Wall. 5. The Occurrence, structure and Properties of Certain Cell Wall Deformations. Commonwealth of Australia C.S.I.R. Bulletin No. 221.