Aalto University
School of Business

Intermediate Microeconomics

Consumer Theory, Producer Theory

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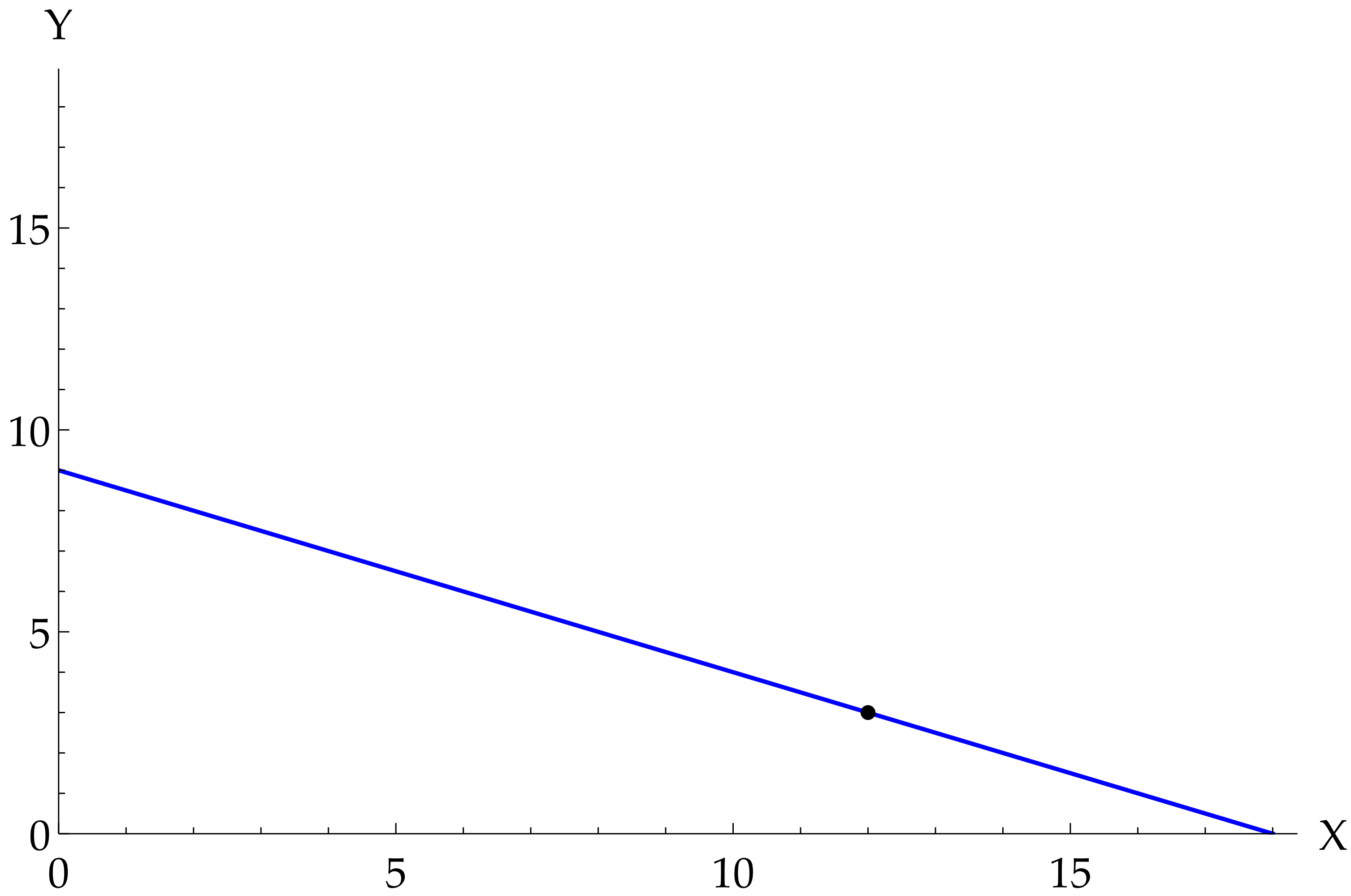
ECON-2100

Consumers

- Indifference curves, budget sets
  (*samahyötykäyrät, budjettijoukot*)
- Price level measurement
- Utility function
  (*hyötyfunktio*)
- Welfare measurement: Compensating variation
  (*kompensoiva variaatio*)
- Income effects
  (*tulovaikutukset*)

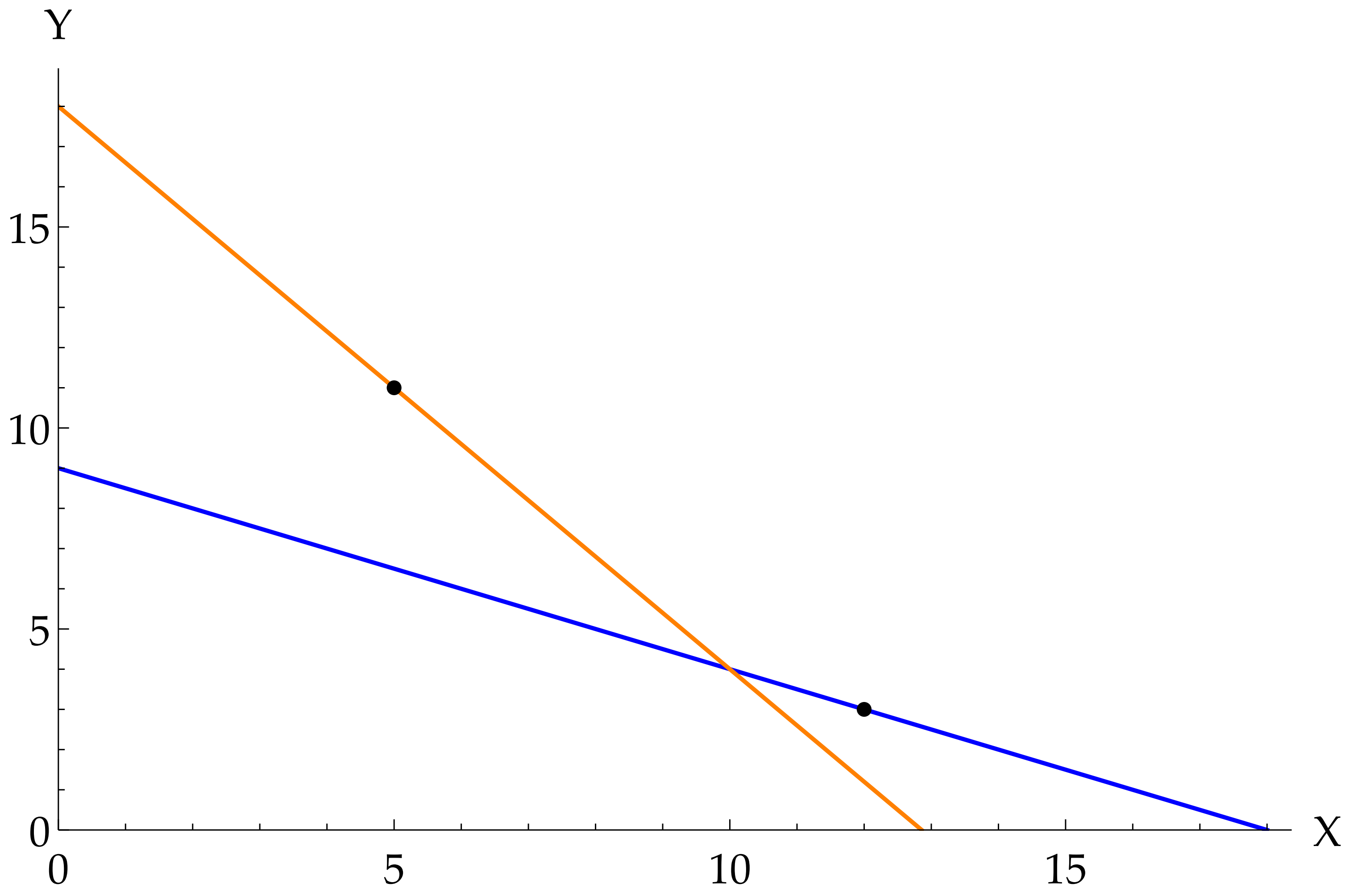
Budget sets



Budget curve shows bundles $\{x, y\}$ afforded by given "budget" M: $B_y(x) = \frac{M}{p_y} - \frac{p_x}{p_y} x$

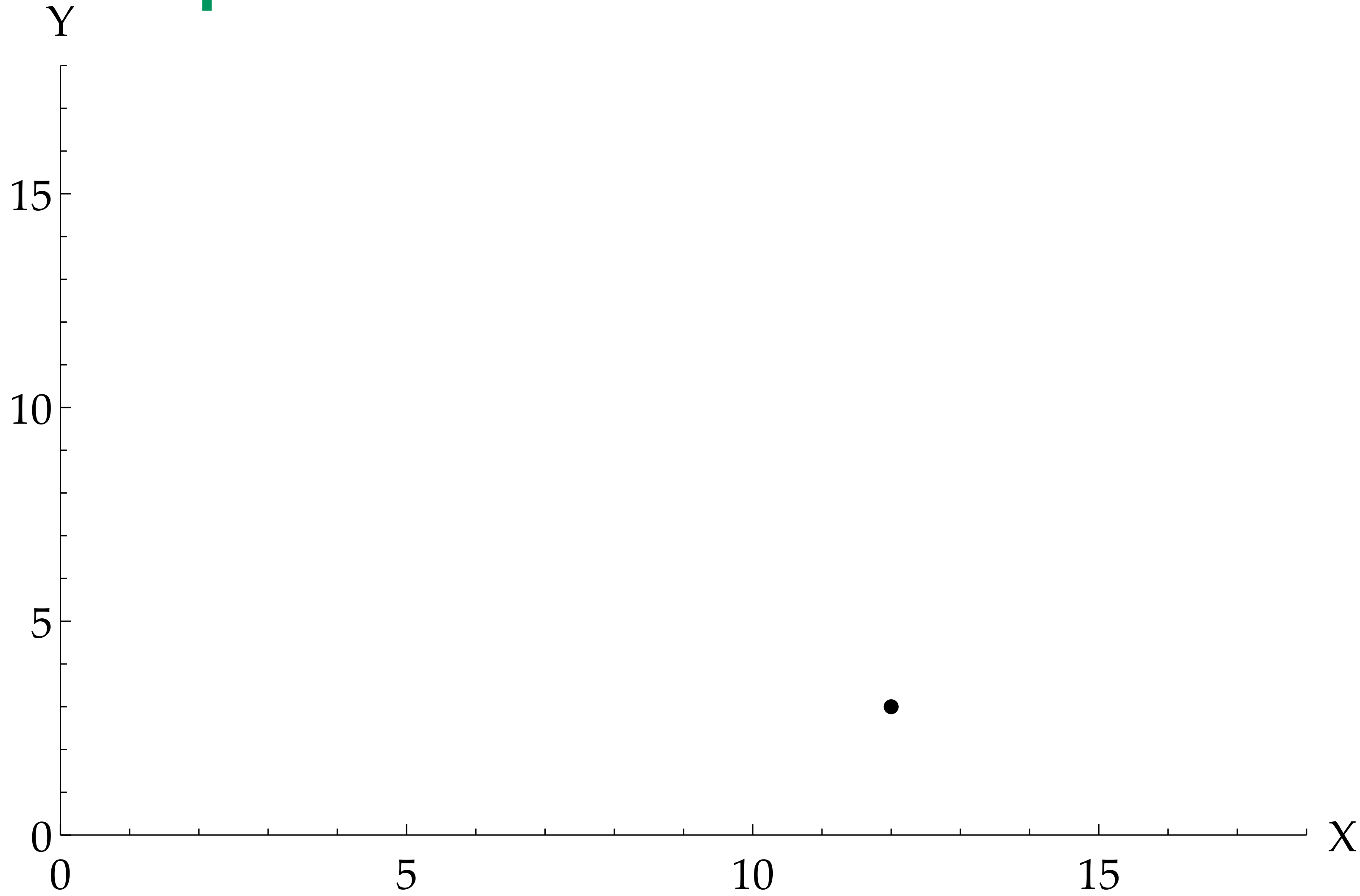
Example: $p_x = 5$, $p_y = 10$, $M = 90 \Longrightarrow B_y(x) = \frac{90}{10} - \frac{5}{10} x = 9 - 0.5x$

Budget sets



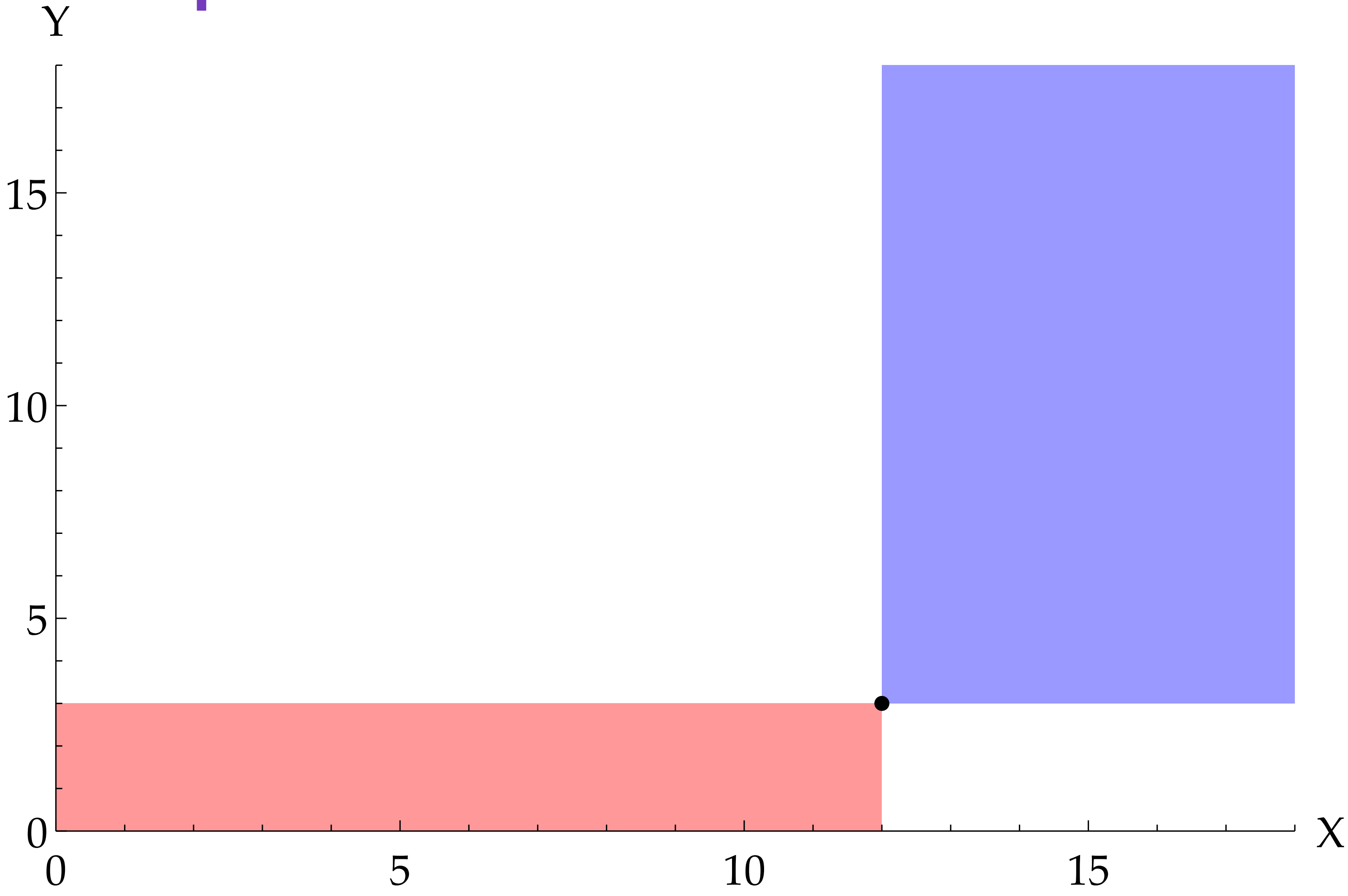
Example 2: $p_x = 7$, $p_y = 5$, $M = 90 \Longrightarrow B_y(x) = 18 - 1.4x$
For example, bundle $\{x, y\} = \{5, 11\}$ costs exactly $M = 90$ at these prices.

Consumption bundle



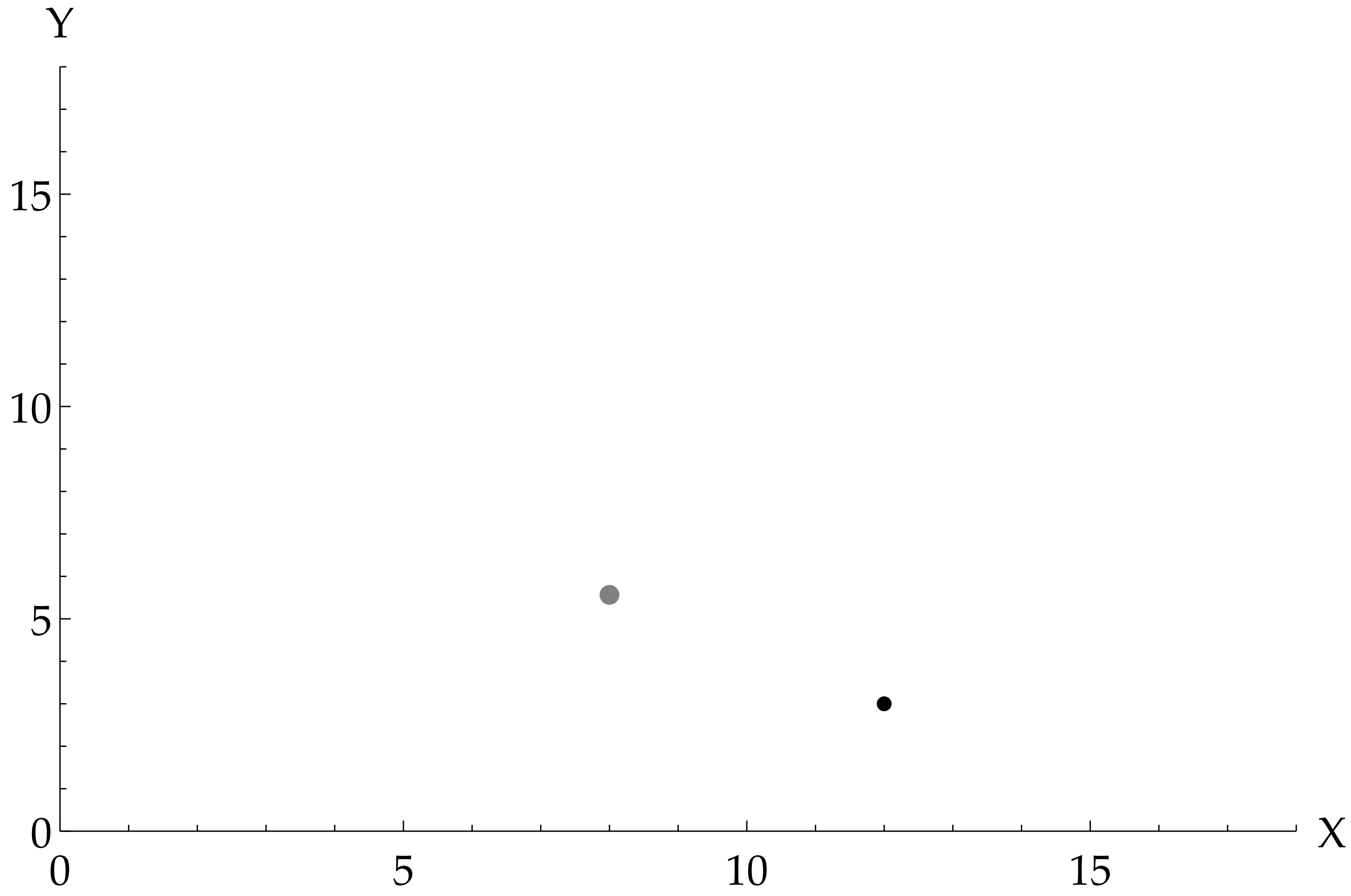
Consumption bundle $\{x, y\} = \{12, 3\}$

Consumption bundle



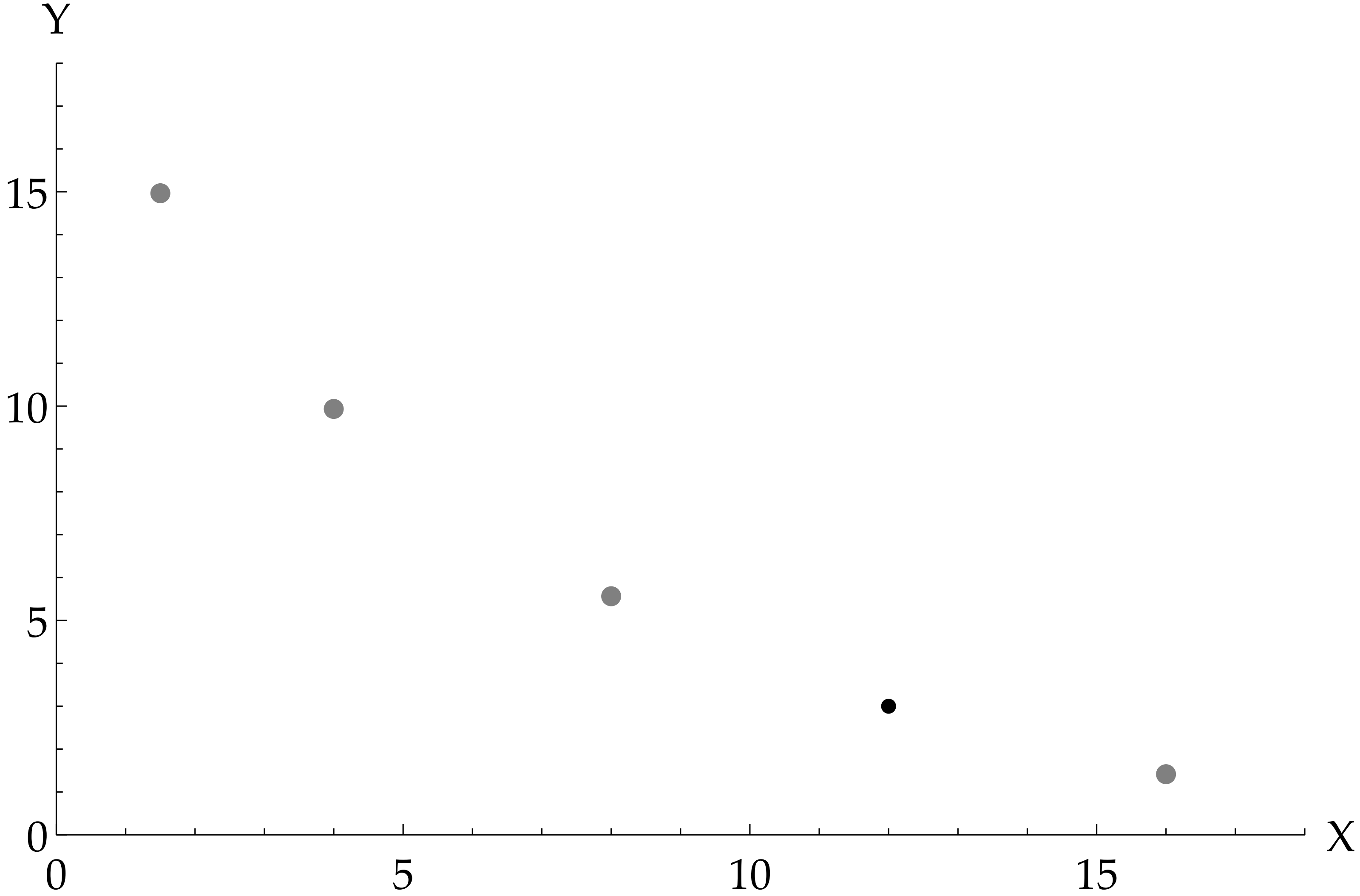
Consumption bundle $\{x, y\} = \{12, 3\}$
Some consumption bundles are surely better or worse than $\{x, y\}$.

Indifference curves



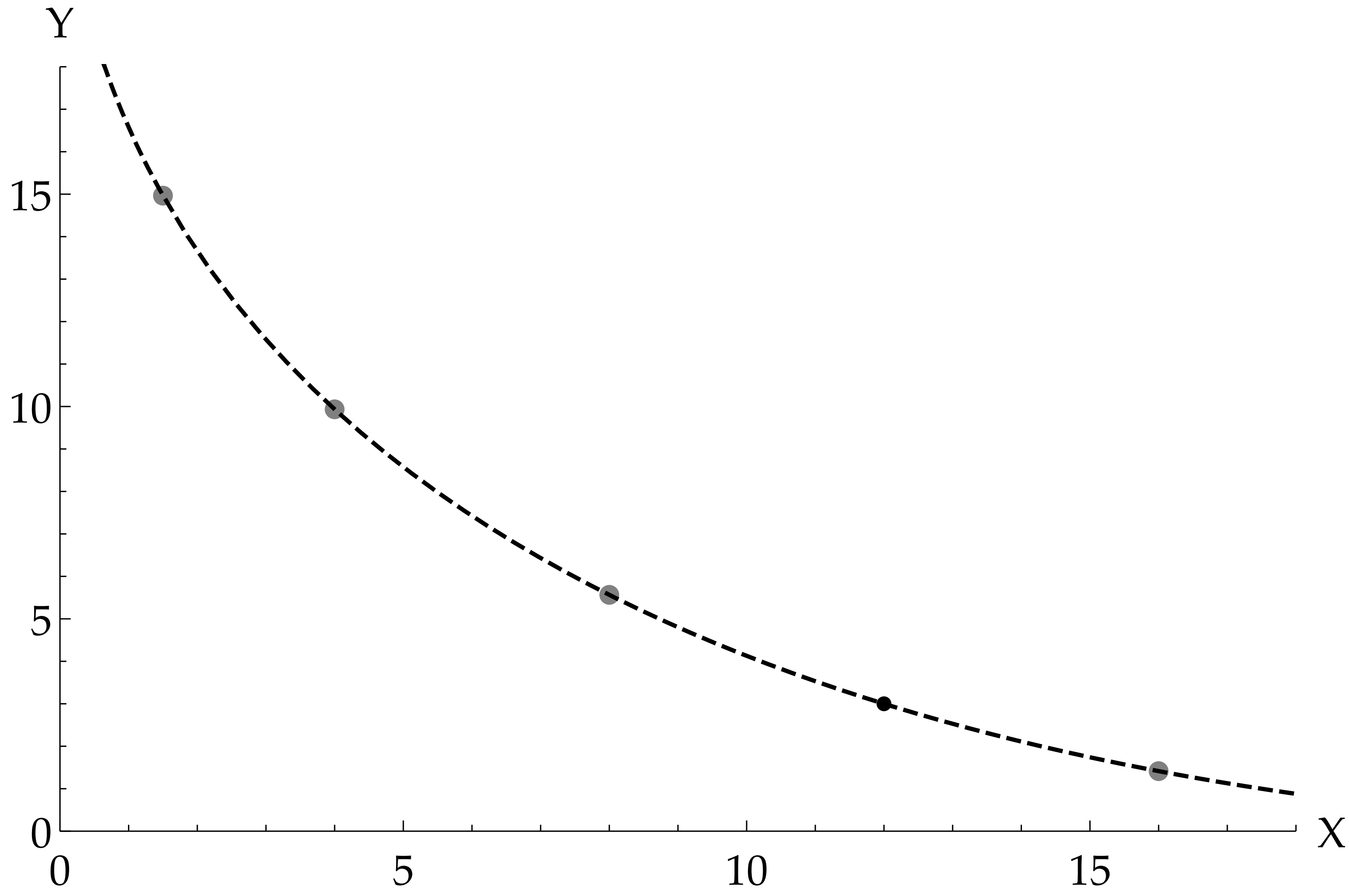
Some bundles are equally preferable. Consider some $x < 12$.
At high enough $y \geq 3$ there is a point of indifference with $\{12, 3\}$.

Indifference curves



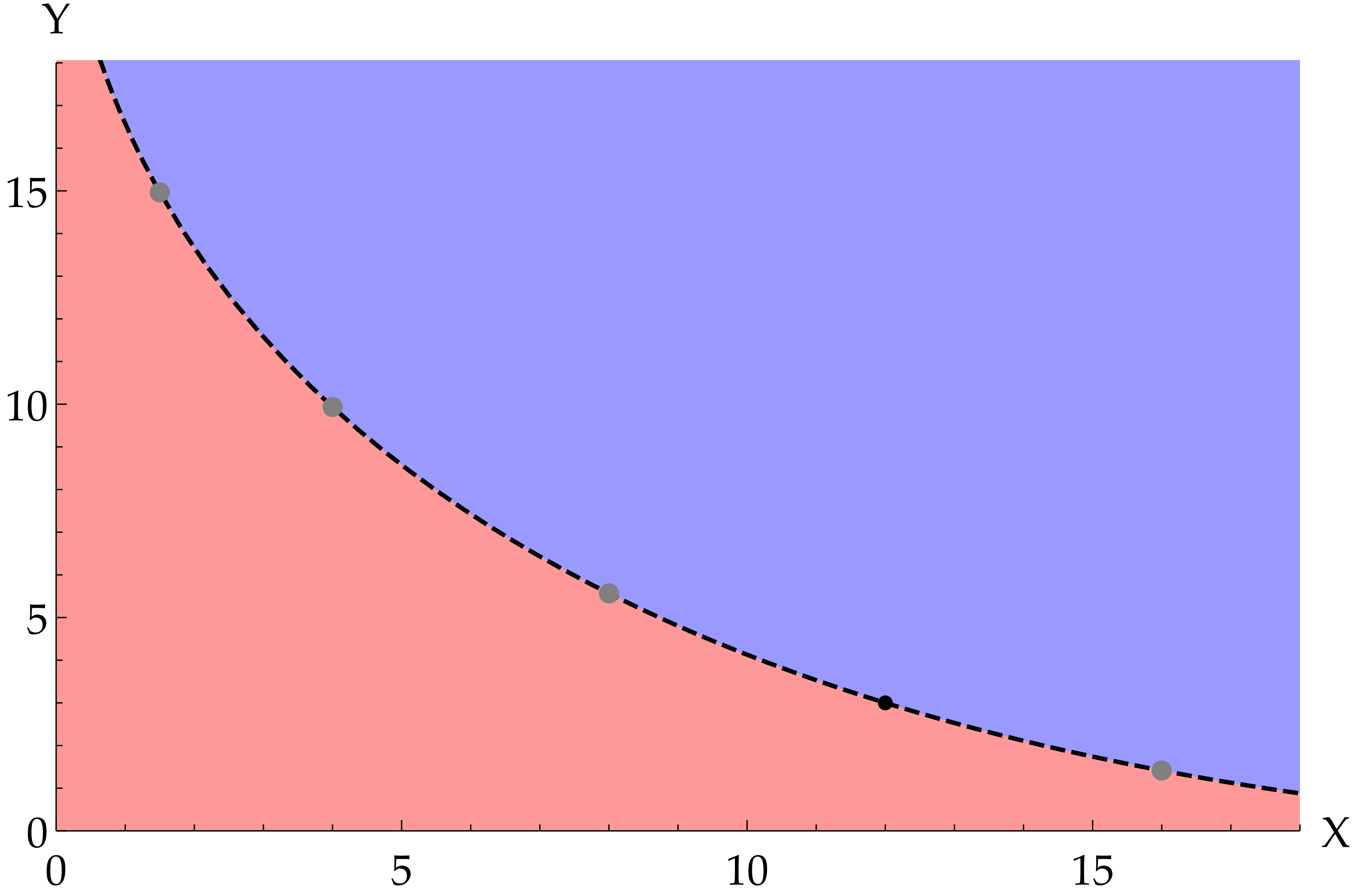
Some bundles are equally preferable. Consider any $x \geq 0$.
At some y there is a point of indifference with $\{12, 3\}$.

Indifference curves



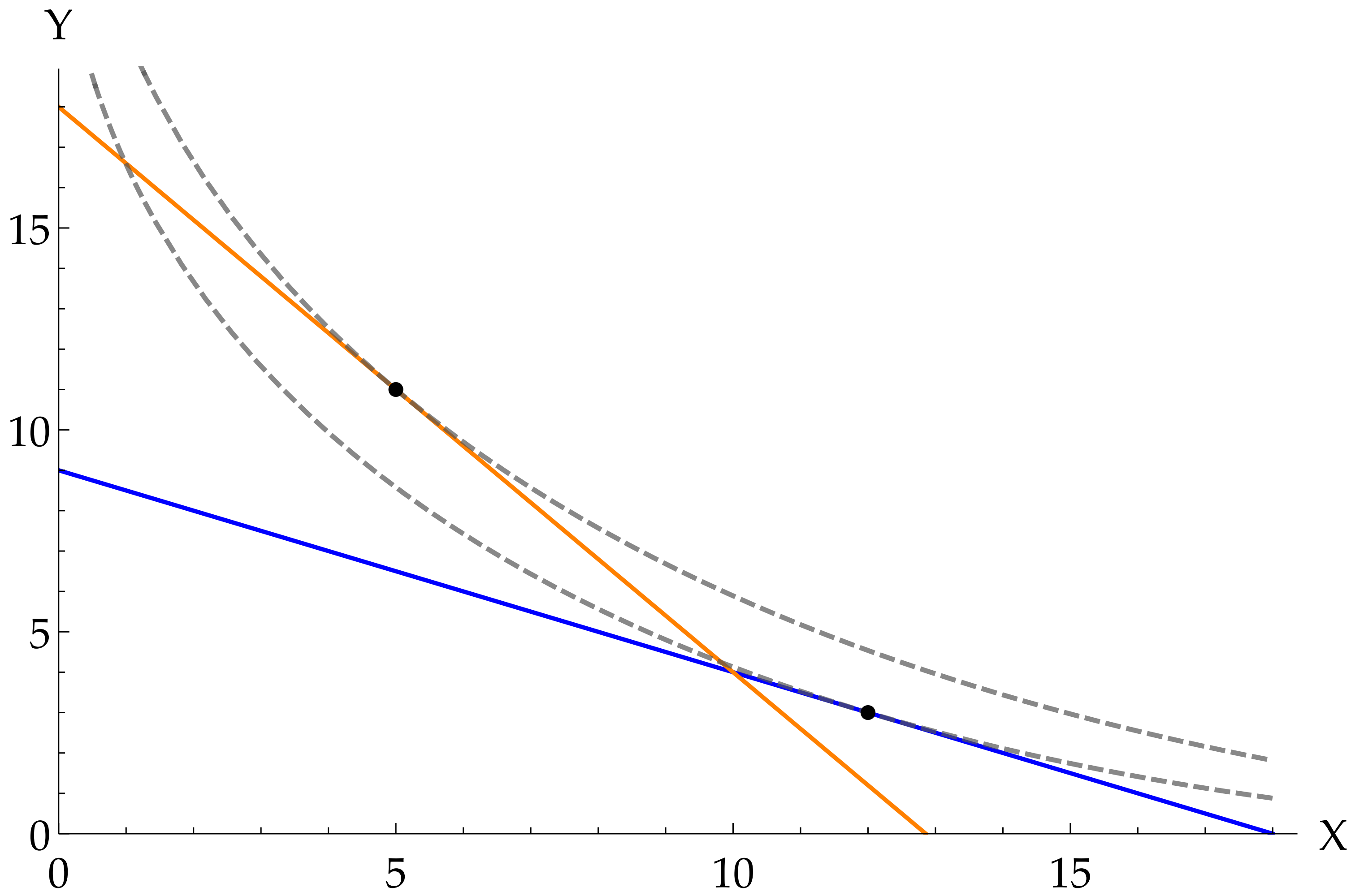
Some bundles are equally preferable. Consider any $x \geq 0$.
At some y there is a point of indifference with $\{12, 3\}$.

Indifference curves



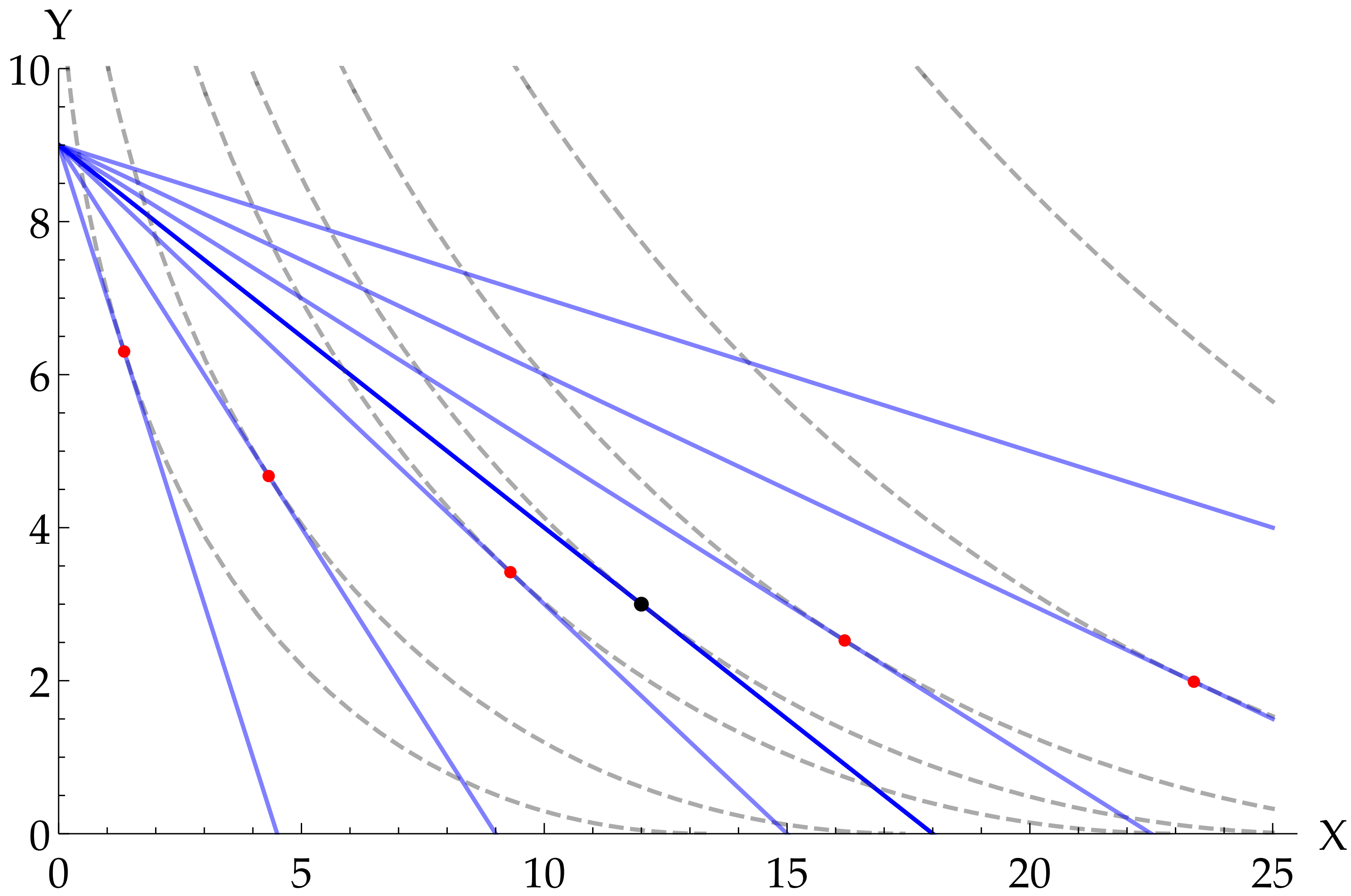
Bundles on an indifference curve share better-than and no-better-than sets.

Consumer choice



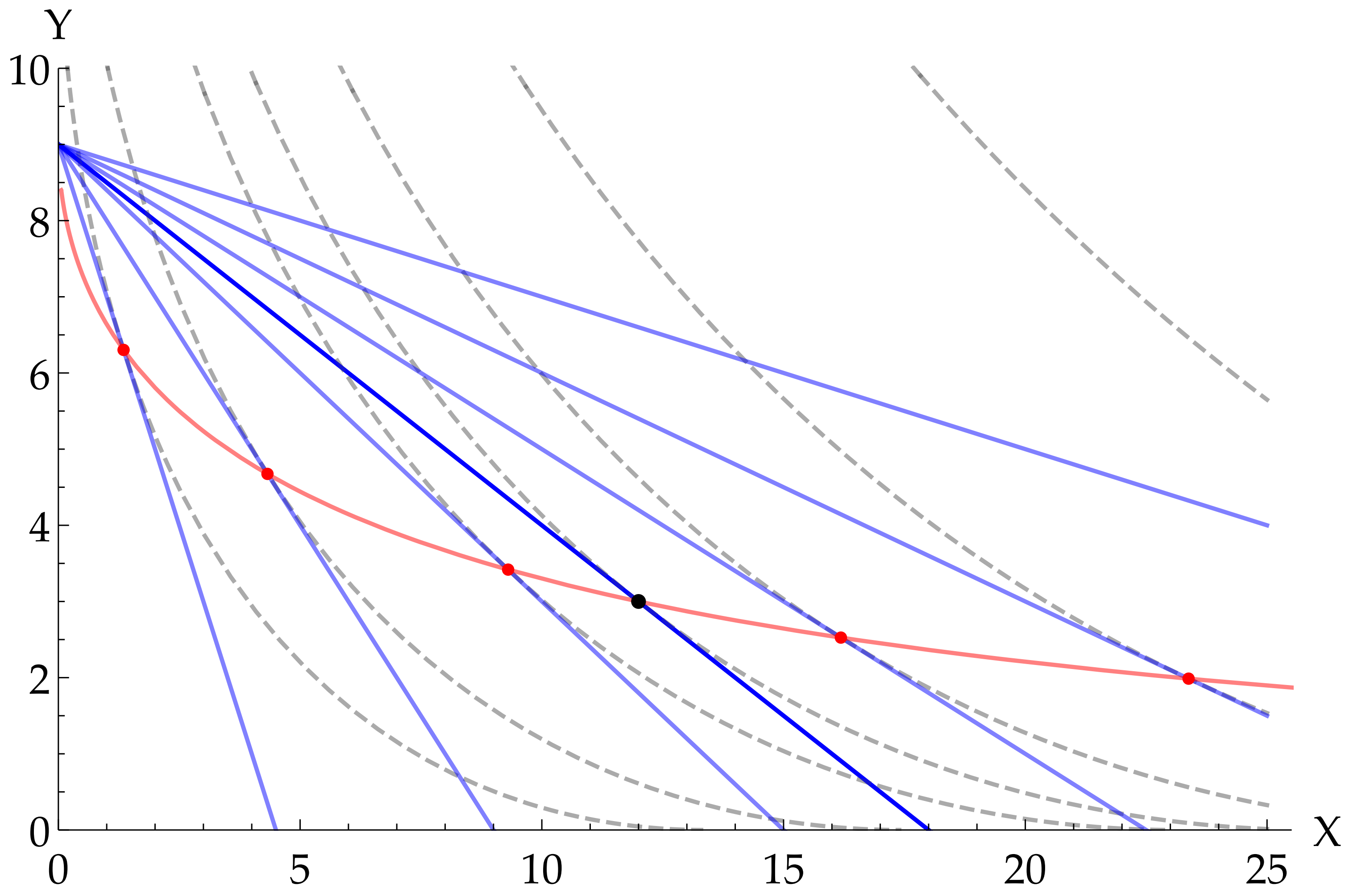
Budget sets and preferences with optimal bundles, where slope of indifference curve and budget line equal: $-\frac{\partial u}{\partial x} / \frac{\partial u}{\partial y} = -p_x/p_y$

Consumer demand



Optimal $\{x, y\}$ bundle at all prices p_x and given $\{p_y, M\}$ shows demand for x.
Slope of budget curve $-p_x/p_y$ varies.

Consumer demand



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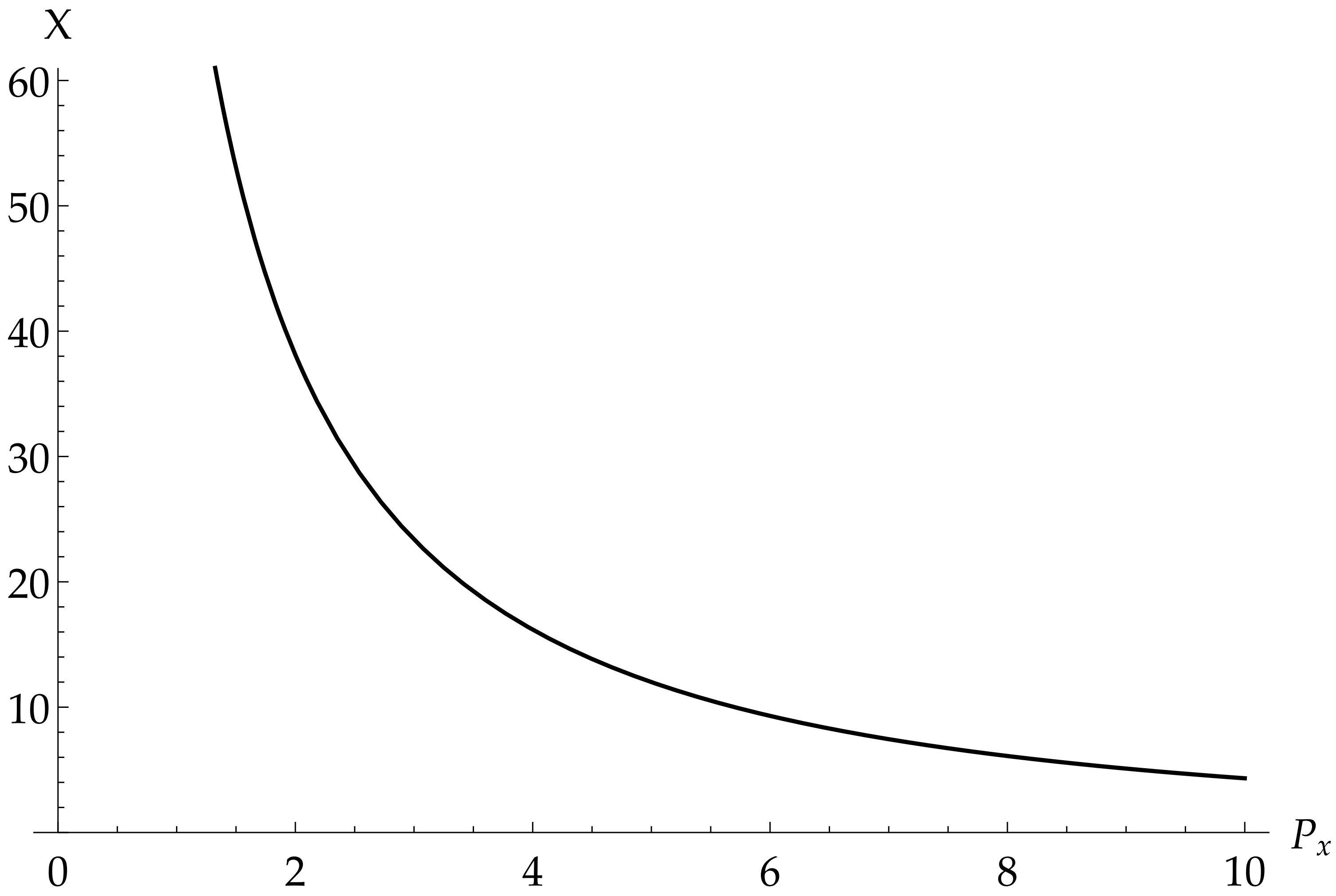
Consumer demand



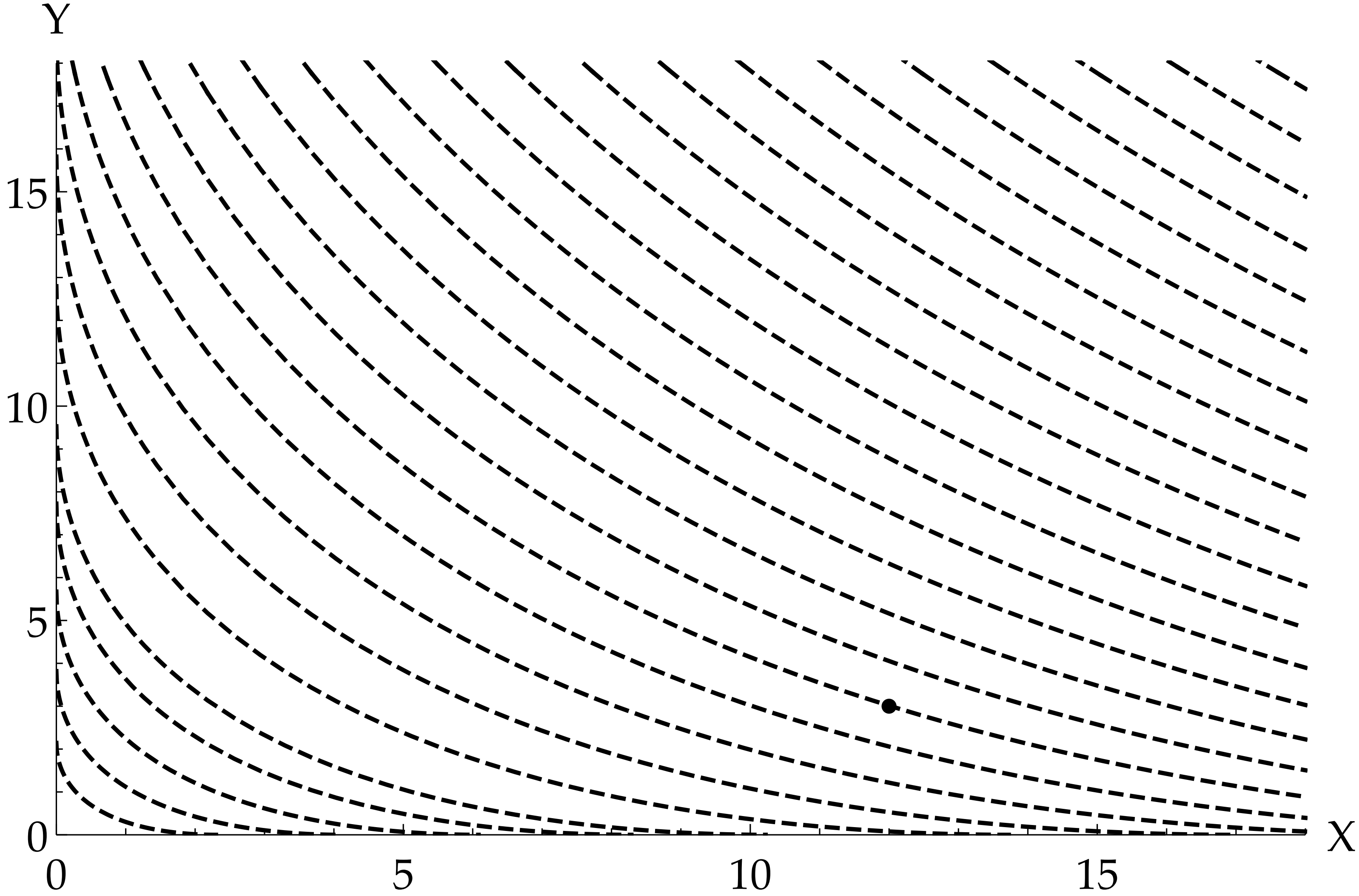
Demand for x, holding constant p_y and M

Utility functions

- Preferences over n goods can be described by a utility function
  ...if preferences are "sensible": Complete, Reflexive, Transitive
- Non-satiation often assumed (probably always in this course)
  Bundle that has more of everything is strictly preferred
- Utility function $u(\mathbf{x})$ orders all possible bundles $\mathbf{x} \in \mathbb{R}^n$.
  If $u(\mathbf{x}) > u(\tilde{\mathbf{x}})$ then $\mathbf{x}$ strictly preferred to $\tilde{\mathbf{x}}$
- Units of u are meaningless: $v(u(\mathbf{x}))$ for any monotonic increasing v describes the same preferences as u
- Indifference curves are contours (=level curves) of the utility map

What would it mean for two indifference curves to cross?

Indifference curves



Example: CES-utility $u(x, y) = (\alpha x^\rho + (1-\alpha) y^\rho)^{\frac{1}{\rho}}$, drawn for $\alpha = 0.49$, $\rho = 0.53$
Each indifference curve $y = \bar{y}(x, \bar{u})$ is solved from $u(x, y) = \bar{u}$ for some level $\bar{u}$

Demand and Utility

- Utility function implies demand curves for all goods
- For example , with two goods x, y, prices p_x, p_y, budget m

$$x^d(p_x, p_y, m) = \underset{x \in [0, m/p_x]}{\arg\max}\; u\left(x, \frac{m - p_x x}{p_y}\right)$$

  Sometimes known as “Marshallian” or “uncompensated” demand

- “Hicksian demand” minimizes expenditure for a given utility level

$$x^h(p_x, p_y, \bar{u}) = \underset{x \geq 0}{\arg\min} \left\{ p_x x + p_y \bar{y}(x, \bar{u}) \right\}$$

  where $\bar{y}$ gives the indifference curve at utility level $\bar{u}$

More careful treatment in *Mathematics for Economics* (31C01100)

On Price Level Measurement

- Stores/countries/years A and B. Which one is cheaper?
  Easy if one is cheaper for all goods.
- What was the inflation rate last year?
  Easy if all prices changed by same percentage rate
- Beyond easy cases, price indices are needed
- A price index aggregates all prices into one number
- If ideal price index has lower value then the consumer is better off
  Any reasonable price index depends consumer preferences
- Measurement of Real GDP (and thus of economic growth)
  depends crucially on the measurement of price level

Price Level Example: Store A vs B

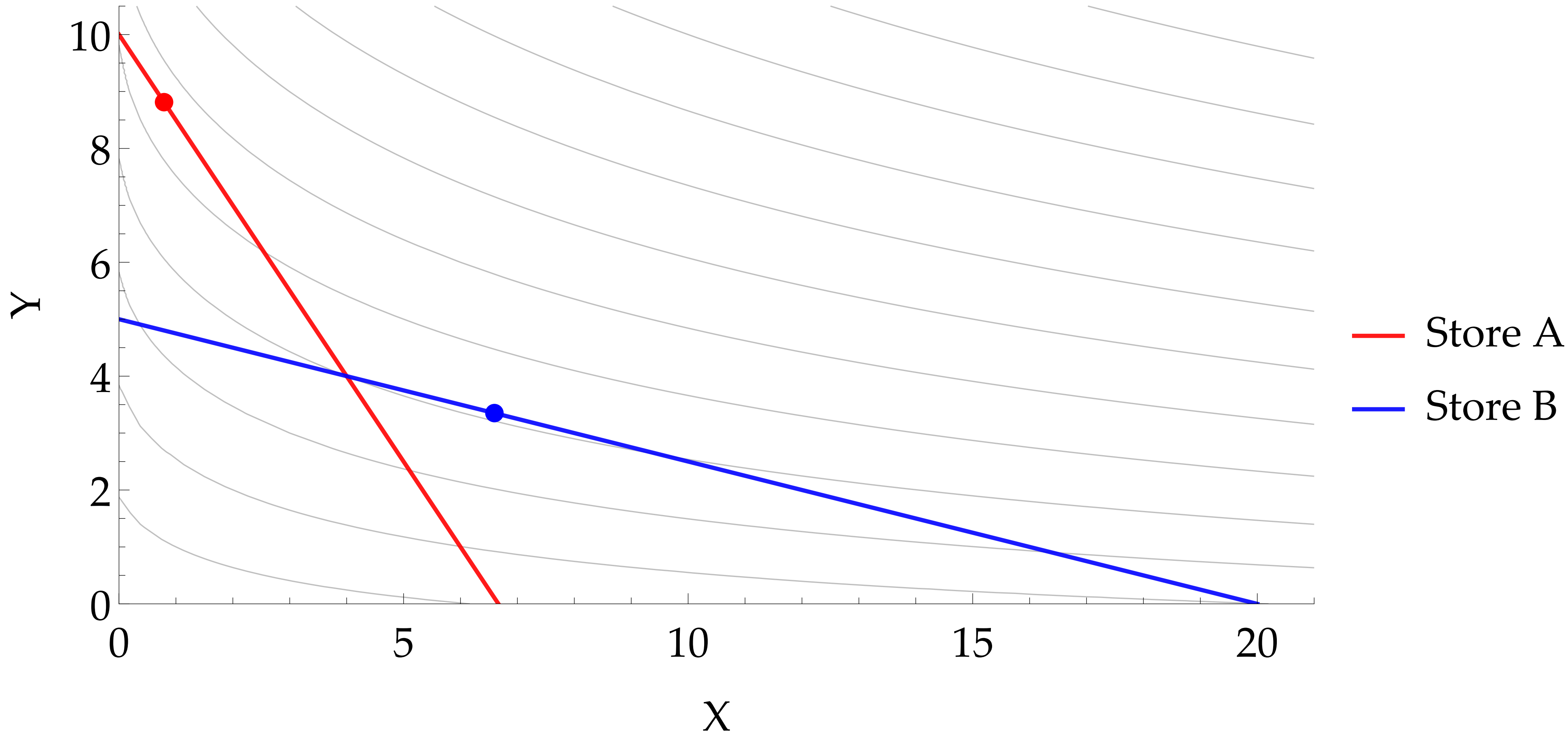
- Claim by Store A: Spend €100 at our store, then go see how much the basket you chose costs at Store B. We are cheaper!
- Claim by Store B: Spend €100 at our store, then go see how much the basket you chose costs at Store A. We are cheaper!

In economics “cheaper” enables higher utility at same expenditure

Ideal cost-of-living index. Choose base store, say A.
What is the cost of the cheapest basket at store B that gives the same utility as the preferred €100 basket at store A.

Laspeyres and Paasche indices need only price and quantity data, ideal index value is in between

Price Level Example: Store A vs B

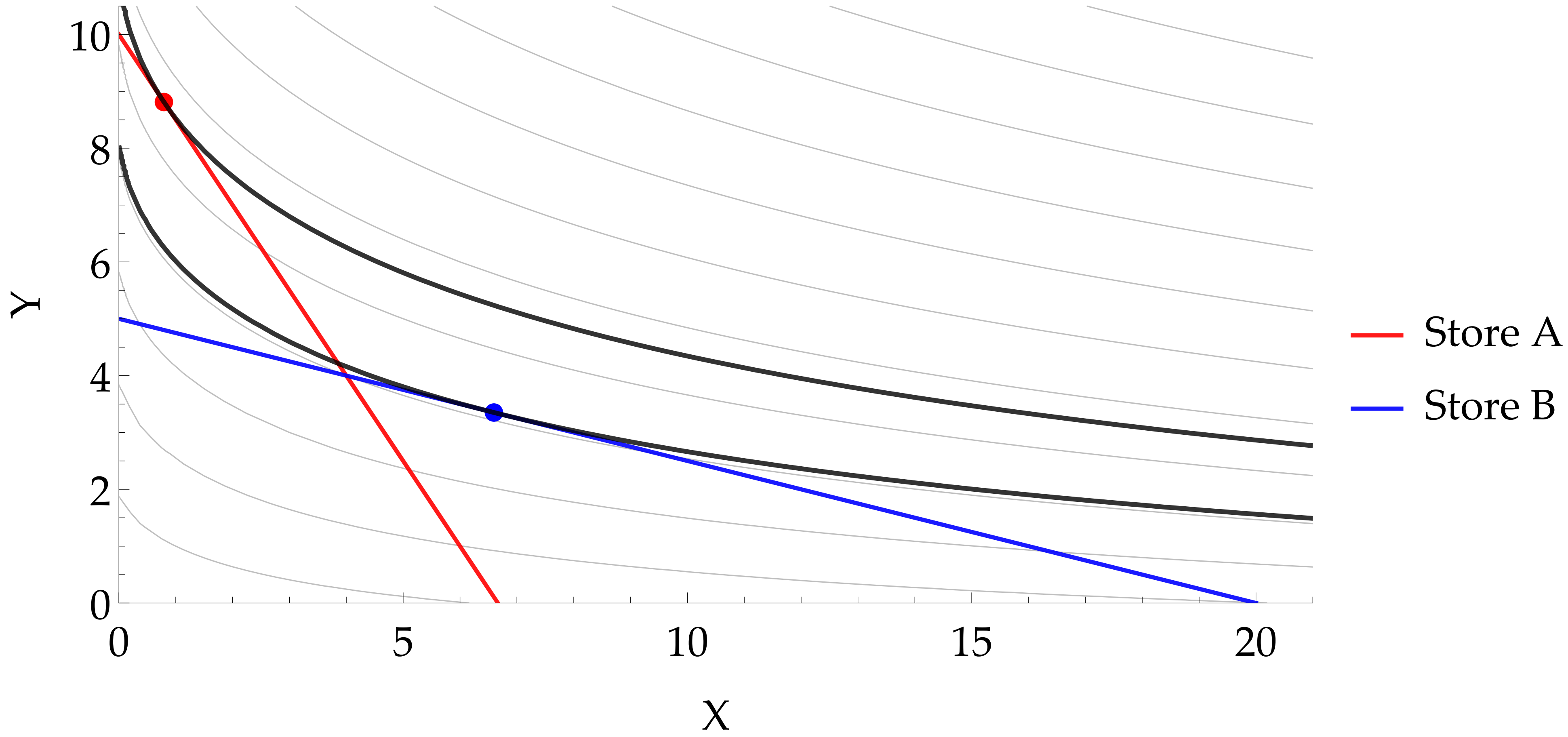


M=100

Prices at Store A: $P_x^A = 15$, $P_y^A = 10$.

Prices at Store B: $P_x^B = 5$, $P_y^B = 20$.

Price Level Example: Store A vs B

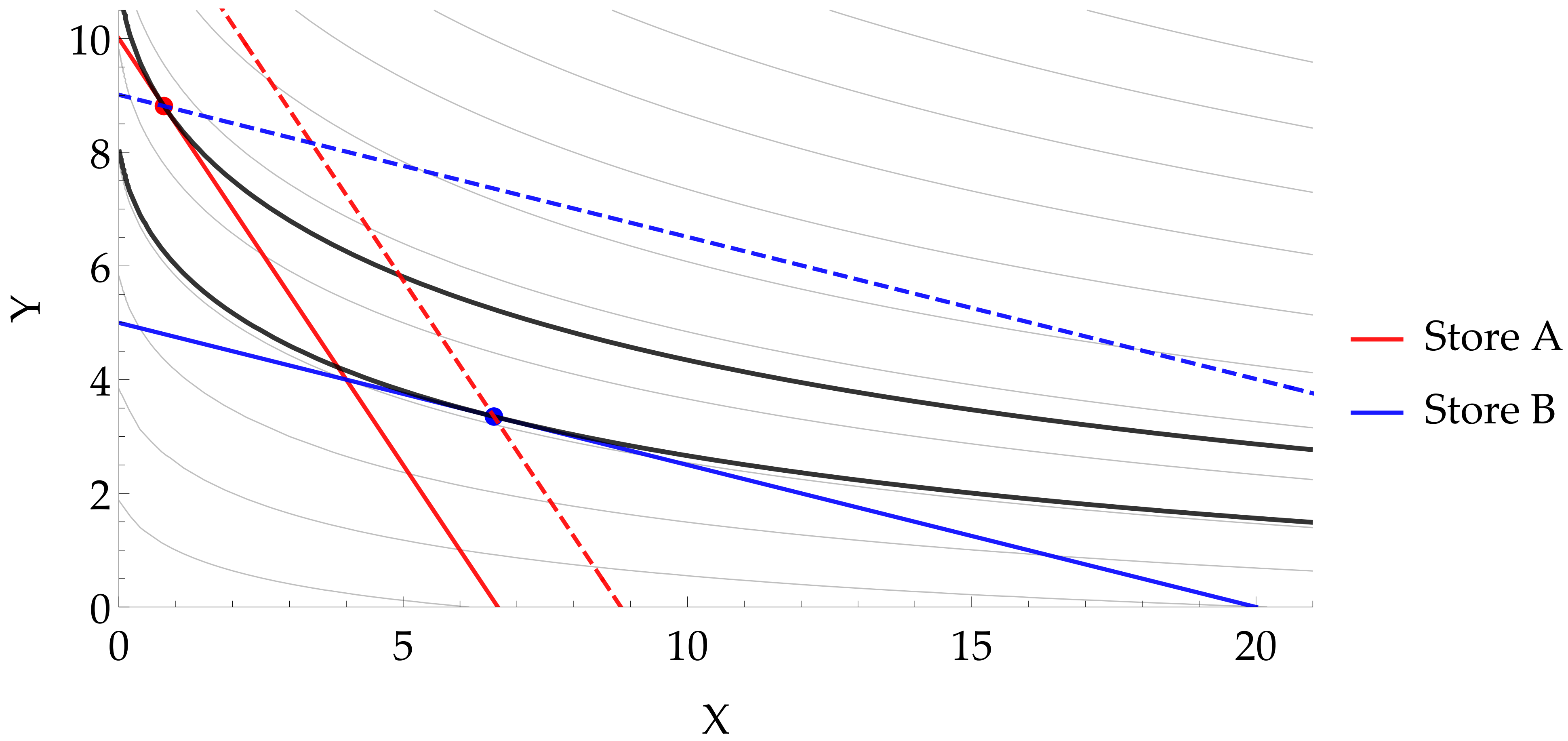


M=100

Prices at Store A: $P_x^A = 15$, $P_y^A = 10$. Basket $x^A = 0.8$, $y^A = 8.8$.

Prices at Store B: $P_x^B = 5$, $P_y^B = 20$. Basket $x^B = 6.6$, $y^B = 3.35$.

Price Level Example: Store A vs B



Cost of Basket A in Store A = Cost of Basket B in Store B = 100

Cost of Basket A in Store B $P_x^B x^A + P_y^B y^A = 180$

Cost of Basket B in Store A $P_x^A x^B + P_y^A y^B = 132.5$

Measuring Consumer Welfare

What is the impact of an external change (anything other than preferences or wealth) on a consumer's welfare?

- Compensating variation (CV)
  How much should consumer wealth change ex post for original utility level to be restored?
- Equivalent variation
  How much should consumer wealth have had to change ex ante, for it to result in same change in utility level as the change?

Compensating Variation Example: Store A vs B

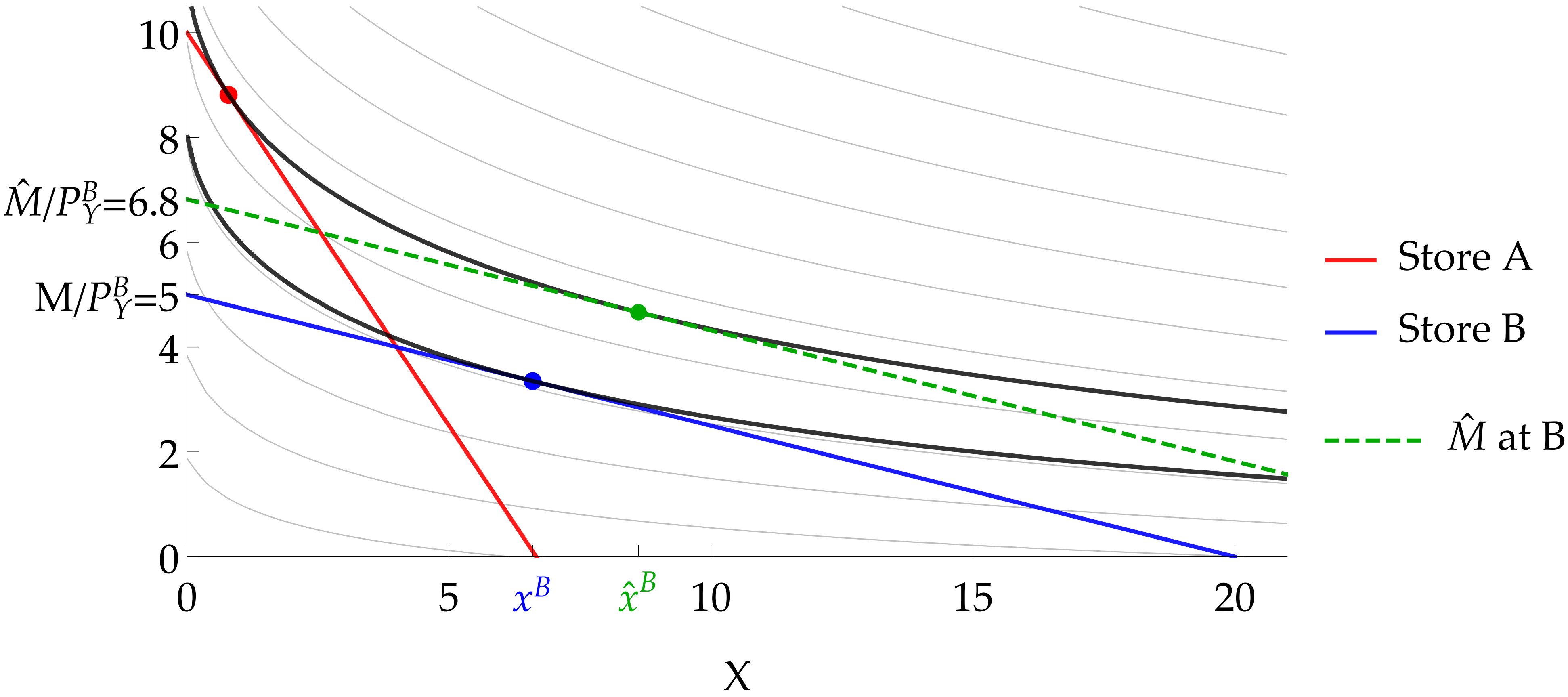
What is the CV of having to move from Store A to Store B?
(For this consumer, at initial wealth 100)

Cost of $\{x^A, y^A\}$ in Store B is 180, but that would give higher utility than 100 enables at store A.

For CV we need the cost of cheapest bundle at B that enables utility level $u(x^A, y^A)$. CV is the difference between this hypothetical bundle $\{\hat{x}^B, \hat{y}^B\}$ and initial wealth 100.

CV would be negative if the consumer were better off at Store B.

Compensating Variation Example: Store A vs B



Bundle $\{\hat{x}^B, \hat{y}^B\} = \{8.6, 4.7\}$ costs $\hat{M} = 136.4$ at Store B,
so CV of losing access to Store A is 36.4
(for a consumer with these preferences and initial expenditure 100)

Income effects

As income rises the demand for. . .

- inferior goods decreases (*inferiorinen hyödyke*)
- normal goods increases (*normaalihyödyke*), and. . .
  - less than proportionally for necessities (*välttämättömyyshyödyke*)
  - more than proportionally for luxury goods (*ylellisyyshyödyke*)

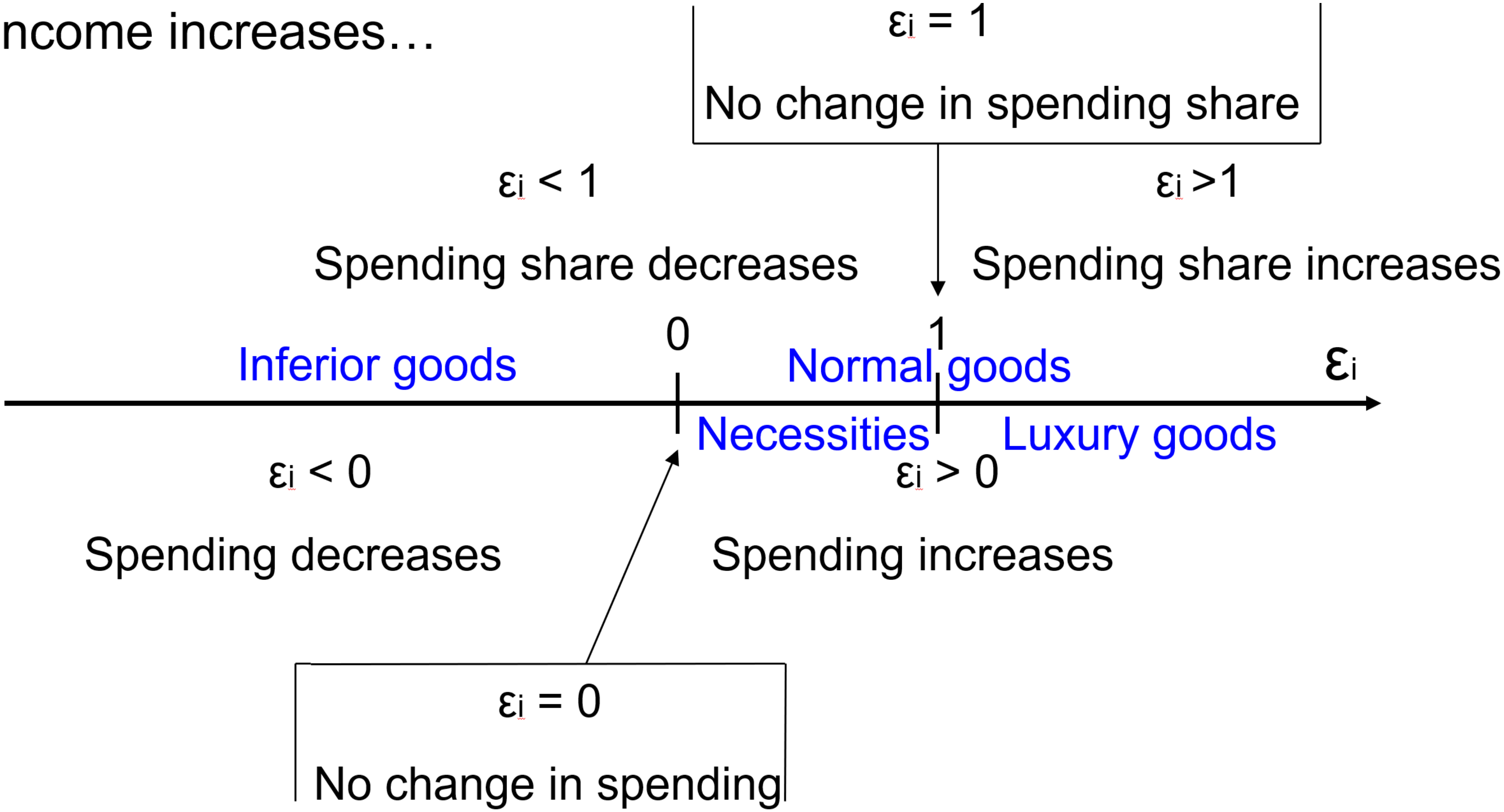
Income elasticity of demand:

$$\varepsilon^i = \frac{\text{\% change in quantity}}{\text{\% change in income}}$$

As prices are now constant, % change in quantity equals % change in expenditure

Income effects

As income increases…



Some estimated income elasticities

Estimates from U.S. data

Restaurant meals	1.40
Electricity	0.20
Ice cream	0.84
Macaroni	−0.41
Milk	0.15
Cars	2.56
Public transit	−0.36
Education	1.05

By group	Food	Beverages
Poorest 15%	0.73	0.97
Middle 70%	0.58	0.97
Richest 15%	0.27	0.97

Implications for cyclicality by industry?

Income and Substitution Effects

A price change Δp results in a change of quantity demanded Δq. This can be decomposed to income and substitution effects

- Income effect: price increase makes the consumer, in effect, poorer. How would quantity change, if only this income reduction and no change in price?
- Substitution effect: How would quantity change, if income also changed to exactly offset the Income effect

(Vice versa for a price decrease)

Income and Substitution Effects

A consumer faces a price decrease for p_x between years $1, 2$

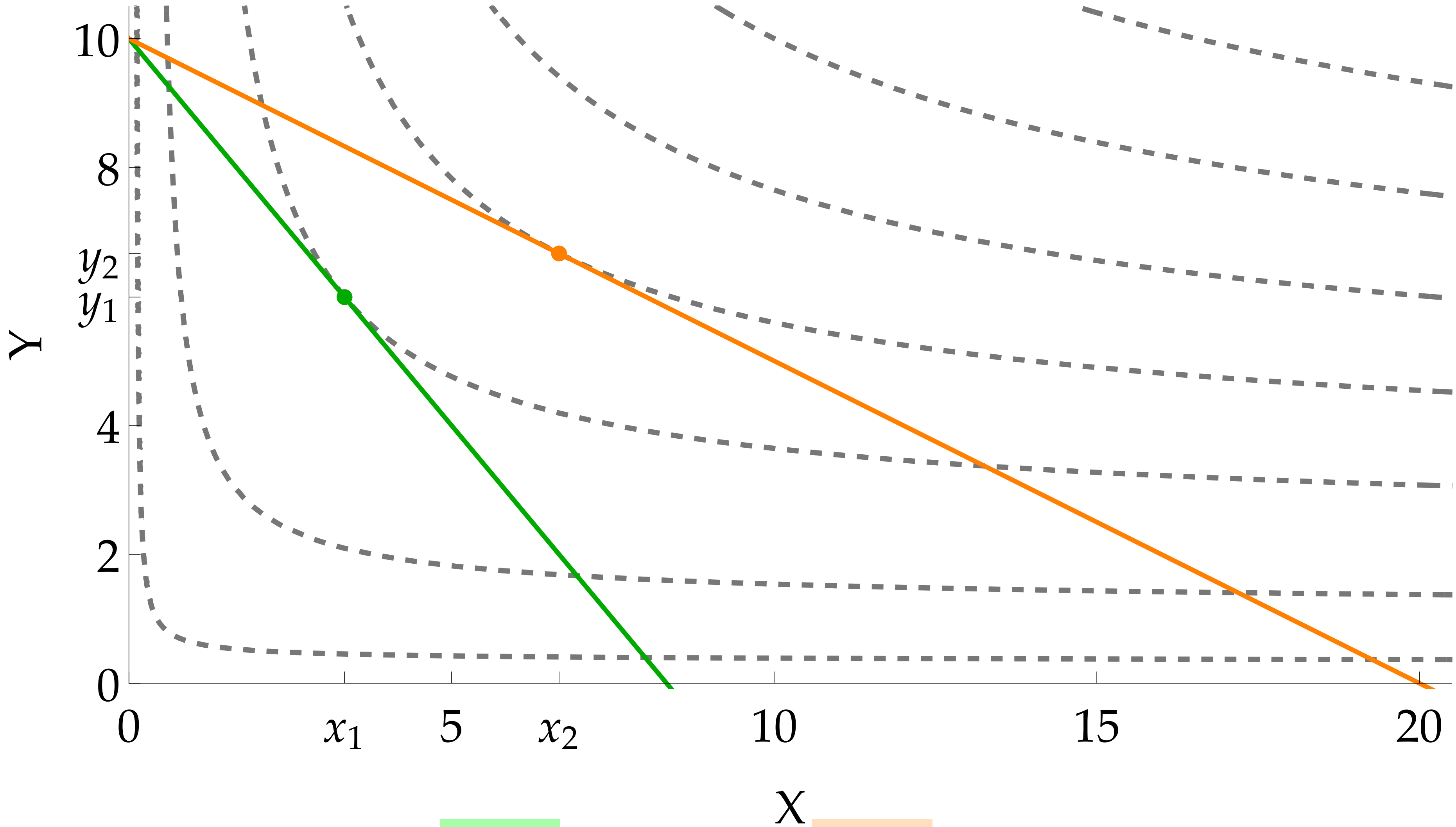
Split the change $\Delta x = x_2 - x_1$ to income and substitution effects

Since $\Delta p_x = p_{x,2} - p_{x,1} < 0$ the substitution effect is positive

Lower price level $\rightarrow$ consumer becomes effectively richer.
Sign of income effect depends on whether x is normal or inferior.

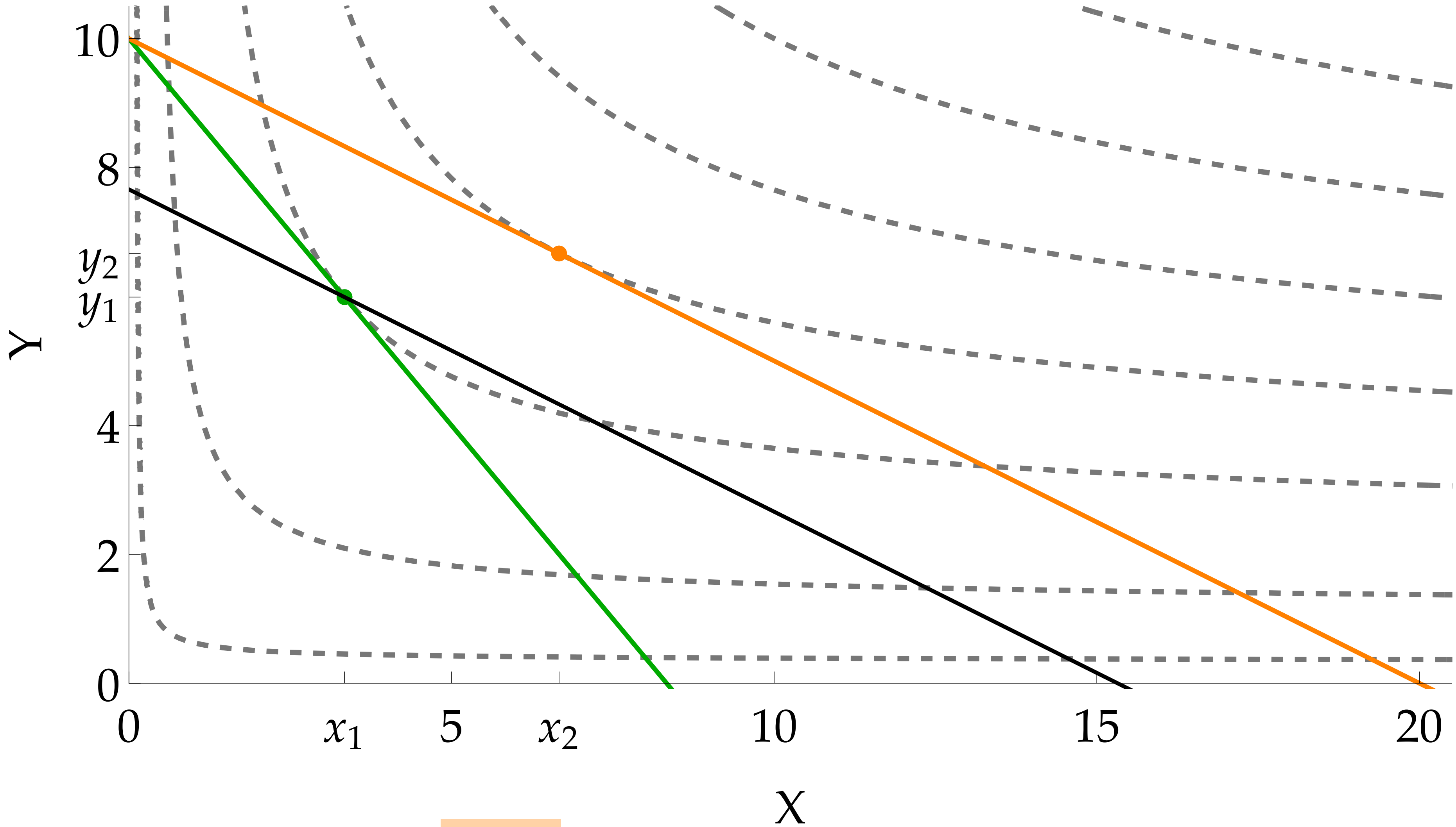
It is in principle possible for income effect to be so negative that consumer reduces consumption despite lower price (Giffen good)

Income and Substitution Effects: Example



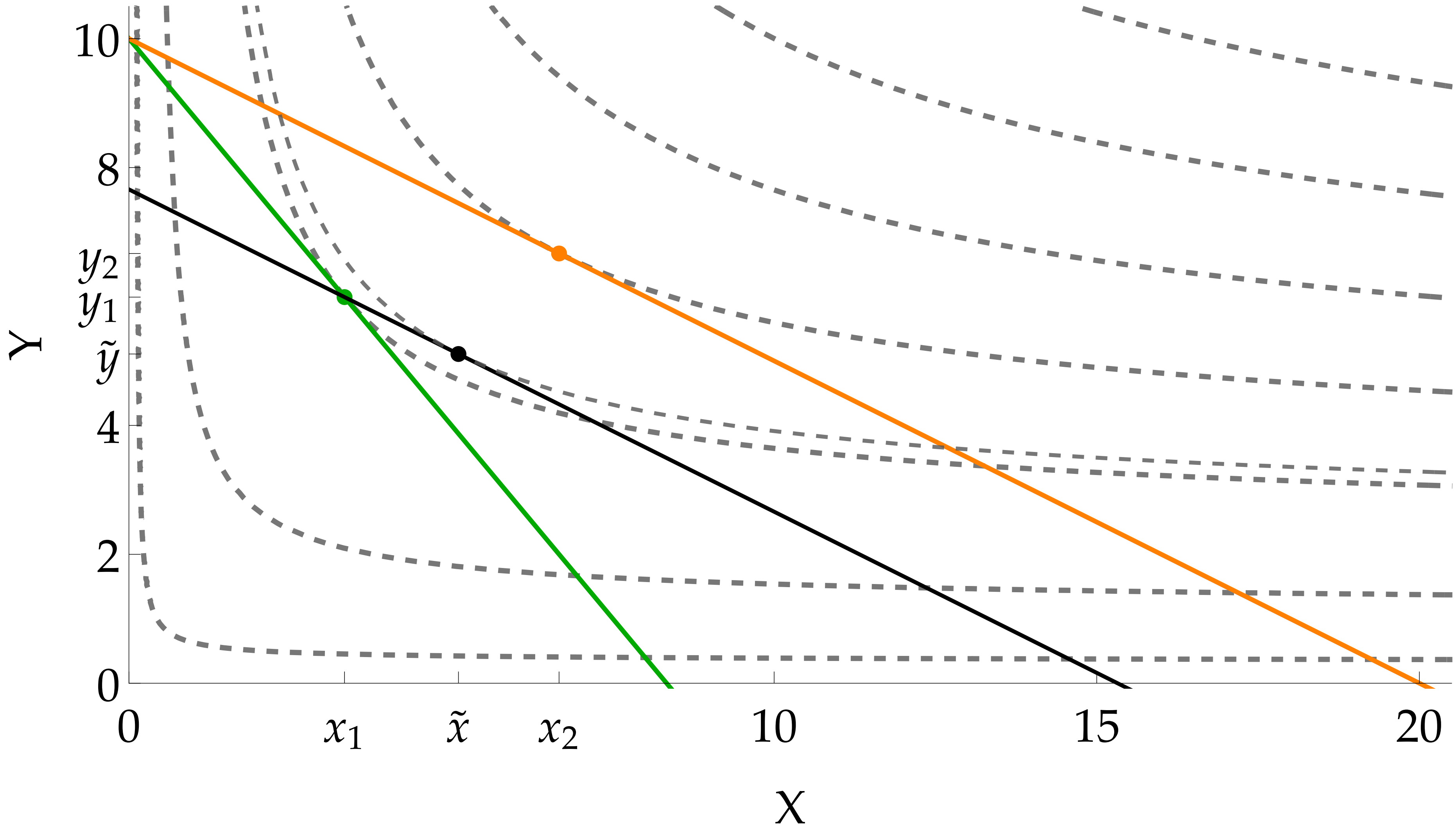
Budget lines with old slope $-\frac{p_{x,1}}{p_y}$ and new slope $-\frac{p_{x,2}}{p_y}$

Income and Substitution Effects: Example



Budget line with new slope $-\frac{p_{x,2}}{p_y}$ and hypothetical $\tilde{M}$ to afford old bundle $\{x_1, y_1\}$

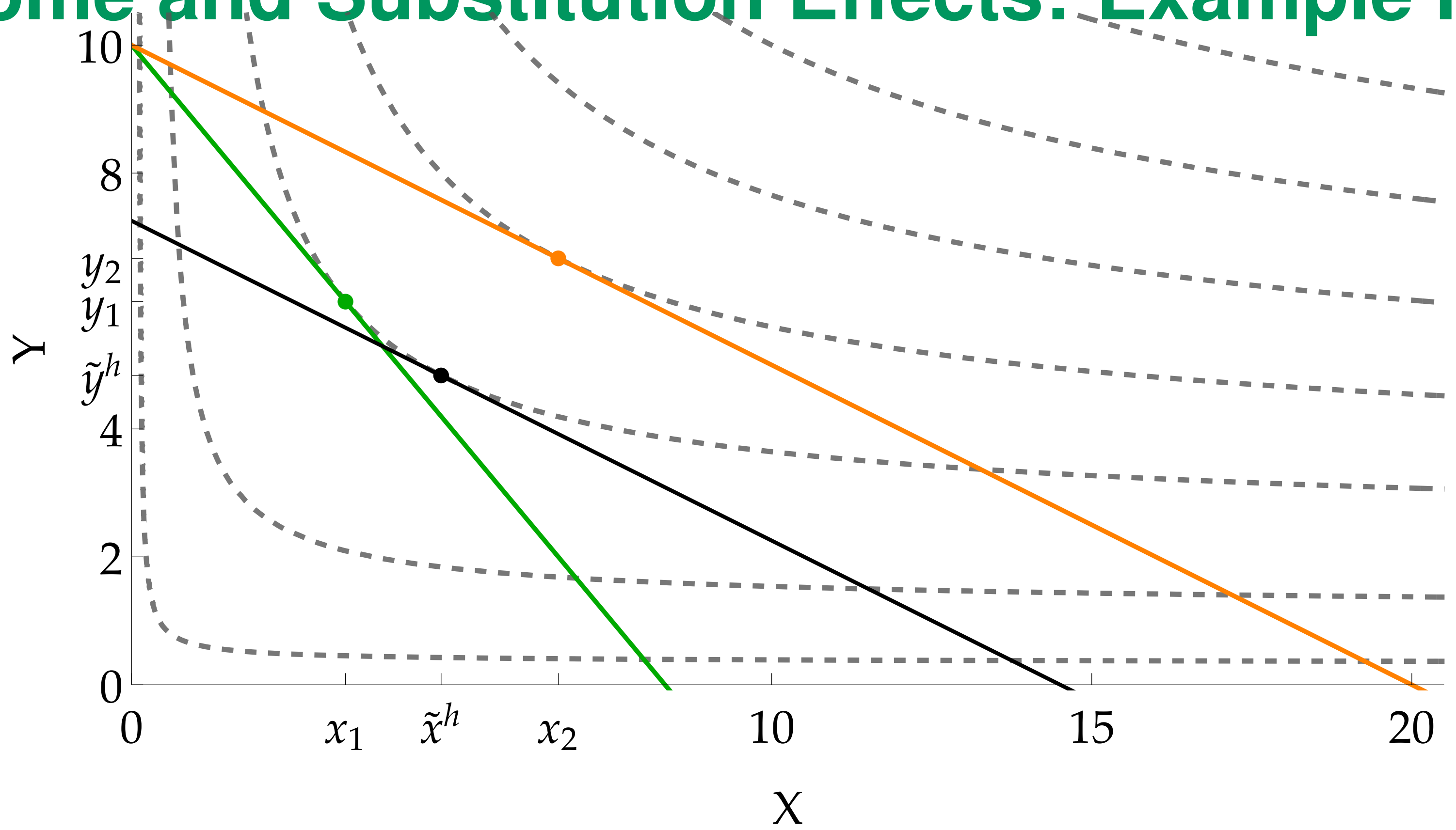
Income and Substitution Effects: Example



Substitution effect $\Delta x^s = \tilde{x} - x_1$

Income effect $\Delta x^n = x_2 - \tilde{x}$

Income and Substitution Effects: Example Hicks



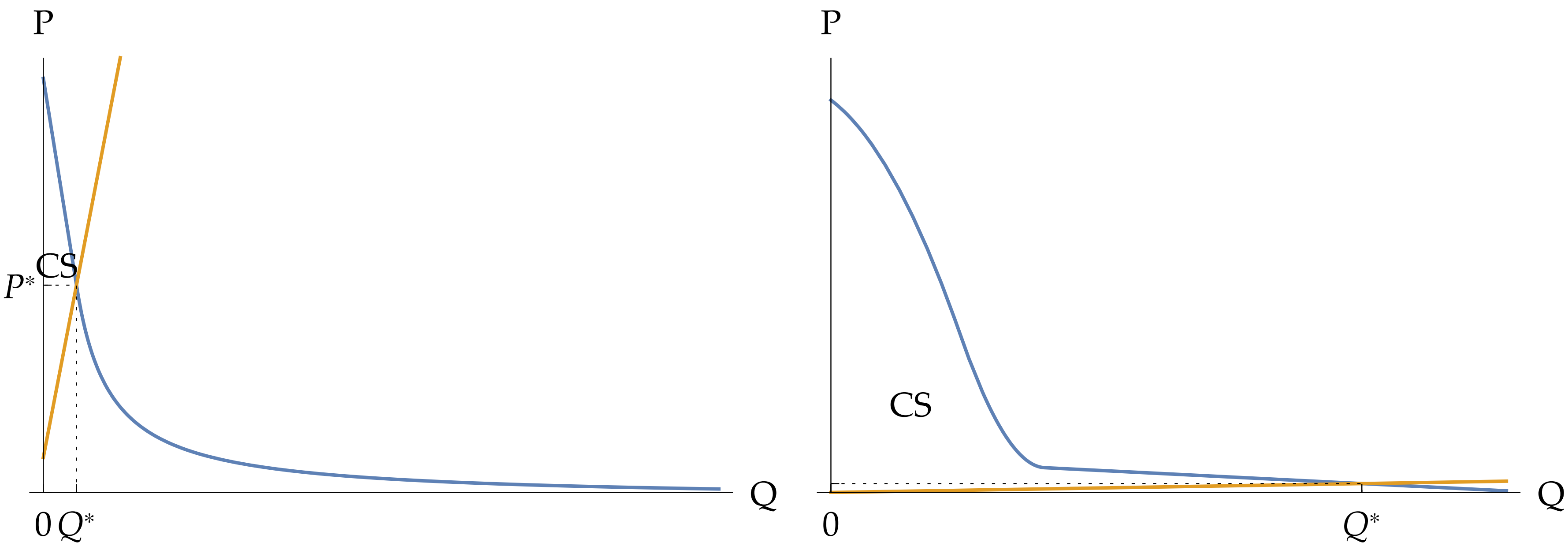
A slightly different version of the Income/Substitution effect decomposition starts by keeping utility at initial level for the hypothetical bundle. This is known as the Hicksian substitution effect. For small changes the results are the same.

This difference is similar to how the discrete approximation of elasticity is different depending on how we choose the divisor in percentage calculation (value before, after, or middle). In the limit of a very small change (point elasticity) it's all the same.

Diamond Water Paradox

Water is much more useful than diamonds.

Why are diamonds more expensive?



Demand and supply for diamonds (left) and water (right)

Welfare created by a good is described by surpluses, not price

Price is related to marginal value, not total value

Producers

- Isoquants, isocost curves
  (*samatuotoskäyrät, samakustannuskäyrät*)
- Production function, cost function
  (*tuotantofunktio, kustannusfunktio*)
- Input demand
  (*panoskysyntä*)
- Price-taker's supply decision

Production function

- Production function gives the maximal level of output q that can be produced with any combination of inputs $\mathbf{x} \in \mathbb{R}^n$

$$q = f(\mathbf{x}) = f(x_1, x_2, \ldots, x_n)$$

- With two inputs K and L, input prices r and w, profits are

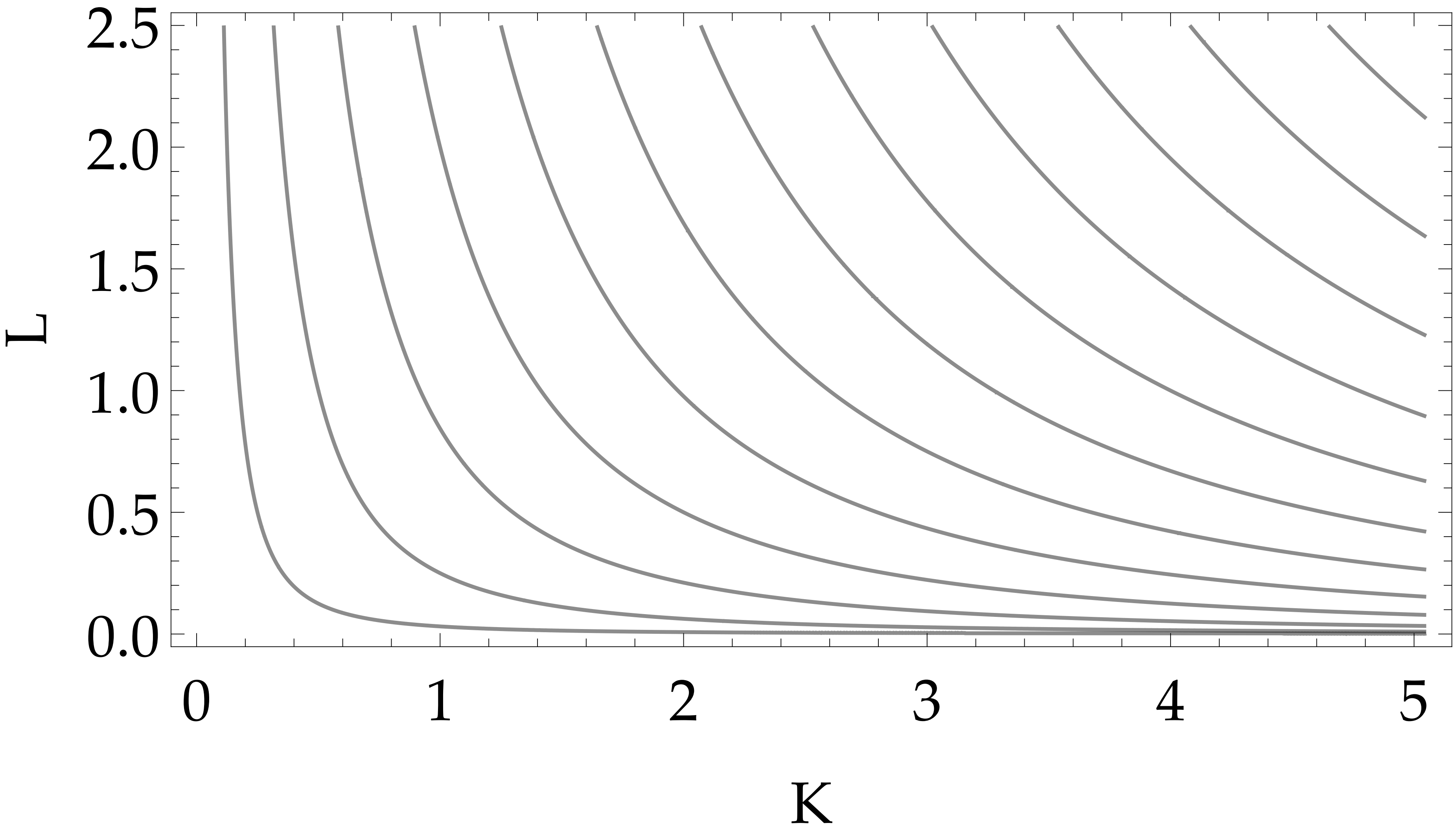
$$pf(K, L) - rK - wL$$

- Price-taking profit-maximizing firm has the profit function

$$\Pi(p, r, w) = \max_{K \geq 0, L \geq 0} \{pf(K, L) - rK - wL\}$$

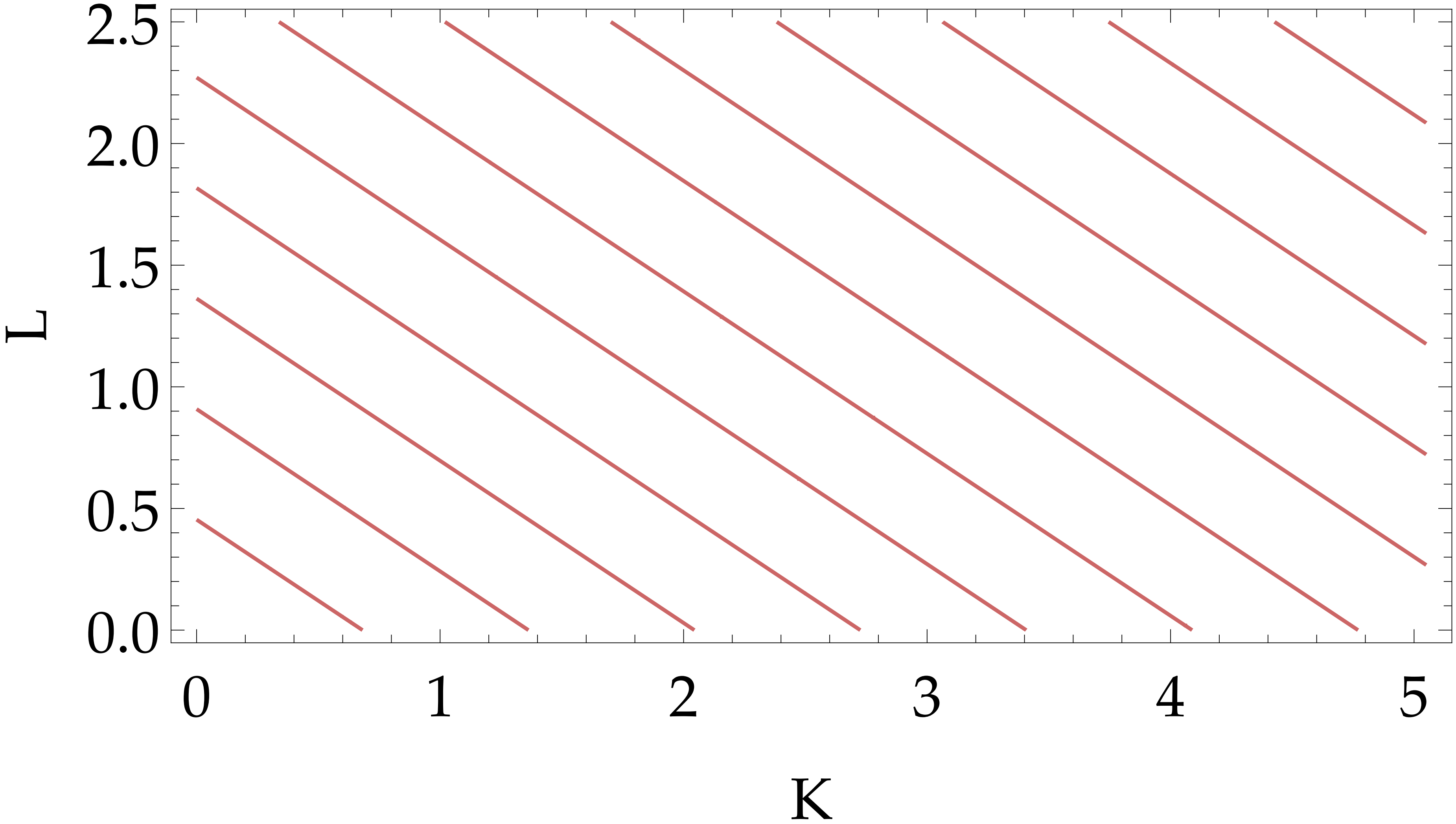
- Isoquants are the level curves of the production function

Production function example: Cobb-Douglas



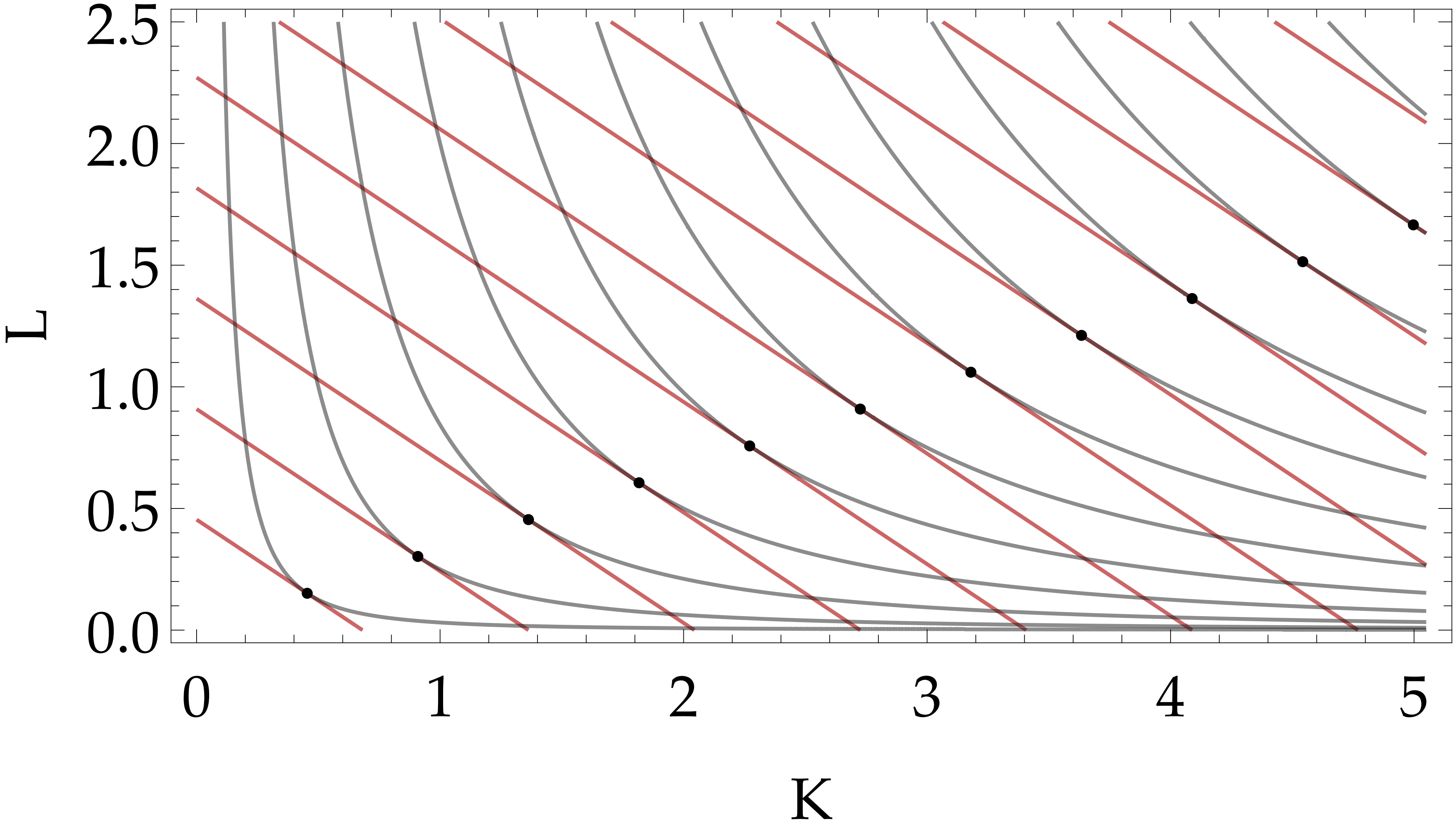
Cobb-Douglas production function $f(K, L) = K^{\alpha} L^{1-\alpha}$, here drawn for $\alpha = 2/3$
Each isoquant $L = \bar{L}(K, \bar{q})$ is solved from $f(K, L) = \bar{q}$ for some $\bar{q}$

Isocost curves: example



Total cost $rK + wL$ is the same for input combos along an isocost curve
Here drawn for $w = 3$, $r = 2$, so slope $-r/w = -0.67$.

Profit maximization



Profit maximization requires efficiency in production: costs minimized for any output

At optimal input combo slopes of isoquant and isocost equal: $-\frac{\partial f}{\partial K} / \frac{\partial f}{\partial L} = -r/w$

Cost function, Input demand

- Cost function gives the minimal cost at which output q can be produced (at given input prices)

For example, with two inputs K, L, prices r, w

- Cost function is

$$\mathrm{TC}(q,r,w) = \min_{K\geq 0}\{rK + w\bar{L}(K,q)\}$$

- Input demand is

$$K^d(q,r,w) = \arg\min_{K\geq 0}\{rK + w\bar{L}(K,q)\}$$

$$L^d(q,r,w) = \bar{L}(K^d(q,r,w),q)$$

More careful treatment in *Mathematics for Economics* (31C01100)

Price-taker's supply decision

Data: market price p, cost function $\mathrm{TC}(q)$

Profits: $\Pi(q) = pq - \mathrm{TC}(q)$

1. Find q such that $\mathrm{MC}(q) = p \Longrightarrow q^s$
2. If multiple solutions, choose the one with highest profits *
3. Are profits $\Pi(q^s)$ positive? If not produce nothing

This results in a supply function $q^s(p)$, which is increasing.
If $\mathrm{FC} > 0$ then choke price must be strictly positive.

* With lumpy costs, TC has jumps at several levels of q'
$\mathrm{MC}(q')$ not defined, so check them as candidate solutions