Differential and Integral Calculus 1
MS-A0111
Hakula
Orlich/Ardiyansyah
Problem Sheet 4, 2020

NOTE[1]

The due date is published on the course pages. Homework can be submitted only digitally. Instructions on labeling the "papers" can be found on the course pages.

1 Introductory Problems (Home Exam)

INTRO 1 Evaluate the integral

$$\int x \cos x \, dx.$$

INTRO 2 Evaluate the integral

$$\int (x+3)e^{2x} \, dx.$$

INTRO 3 Obtain a reduction formula for $I_n = \int (\ln x)^n \, dx$, and use it to evaluate I_4.

INTRO 4 Evaluate the integral

$$\int \frac{1}{x^2 - 9} \, dx.$$

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2 Introductory Problems

INTRO 5 Find the area below $y = e^{-x}$ and above $y = 0$ from $x = 0$ to $x = \pi$.

INTRO 6 If f is differentiable on $[a, b]$ and $f(a) = f(b) = 0$, show that

$$\int_a^b (x - a)(b - x)f''(x)\,dx = -2\int_a^b f(x)\,dx.$$

INTRO 7 Solve

$$\frac{dy}{dx} = \frac{y}{2x}.$$

INTRO 8 Solve

$$\frac{dy}{dx} - \frac{2y}{x} = x^2.$$

3 Homework Problems

EXERCISE 1 Find the area of finite plane region bounded by the curve $y = \ln x$, the line $y = 1$, and the tangent line to $y = \ln x$ at x = 1.

EXERCISE 2 If f and g are two functions having continuous second derivatives on the interval $[a, b]$, and if $f(a) = g(a) = f(b) = g(b) = 0$, show that

$$\int_a^b f(x)g''(x)\,dx = \int_a^b f''(x)g(x)\,dx.$$

What other assumptions about the values of f and g at a and b would give the same result?

EXERCISE 3 Solve

$$\frac{dy}{dx} = \frac{3y - 1}{x}.$$

EXERCISE 4 Solve

$$\frac{dy}{dx} - y = e^x.$$