

## Pedagogical Material in MyCourses

0. INTRODUCTION
1. ELASTICITY
2. VISCOELASTICITY (+ basics of creep)
3. PLASTICITY

### Reading – Textbooks:

- Lemaitre and Chaboche – *Mechanics of Solid Materials*
- Ottosen & Ristinmaa – *The Mechanics of Constitutive Modeling*
- W.F. Chen, D.J. Han – *Plasticity for Structural Engineers* (only chapters 1-5)



# Course of materials modelling in other universities



TECHNISCHE  
UNIVERSITÄT  
DARMSTADT

Technische Universität Darmstadt

9.10.2016

Aalto-university,  
summer 2016

## 13-02-0003-v| Werkstoffmechanik

### Veranstaltungsdetails

**Lehrende:** Prof. Dr.-Ing. Michael Vormwald; Dipl.-Ing. Melanie Fiedler

**Veranstaltungsart:** Vorlesung

**Orga-Einheit:** FB13 Bau- und Umweltingenieurwissenschaften

**Anzeige im Stundenplan:** Werkstm. (V)

**Fach:**

**Anrechenbar für:**

**Semesterwochenstunden:** 3

**Unterrichtssprache:** Deutsch

**Min. | Max. Teilnehmerzahl:** - | -

### Lehrinhalte:

- Klassifizierung der Phänomene des Deformations- und Festigkeitsverhaltens
- Lineare Elastizität
- Isotropie, Anisotropie (Orthotropie, transversale Isotropie)
- Elastoplastizität
- Idealplastizität, Isotrope und kinematische Verfestigung
- Viskoelastizität, Viskoplastizität
- Werkstoffgesetze für Stahl, Beton, Glas, Holz, Kunststoffe und Geomaterialien
- Numerische Umsetzung



### Content:

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- **Elasticity – kimmoisuus tai elastisuus**  
(linear, hyper-elasticity, isotropy, anisotropy, orthotropy)
- **Viscoelasticity - viskoelastisuus**
- **Viscoplasticity or creep – viskoplastisuus ... viruminen**
- **Failure hypotheses - lujuushypoteesit**
- **Plasticity - plastisuus**  
associative, non-associative
- **Damage - vauriotuminen**  
damage-plasticity ex. Concrete Damage Plasticity, Model in Abaqus

... for comparison of the courses contents at two universities to show the relevance of such course content for CIV-engineers

Today subject

Ref: thanks go to an exchange student for providing the course content-list above

# CONTENT

- 1. INTRODUCTION
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- 3. VISCOELASTICITY
- 4. PLASTICITY

## Elasticity in Solids

### Definitions

### Thermodynamical framework

### Elastic Solids

Isothermal Cauchy-elastic material  
Green-Elastic or Hyper-elastic Materials  
*Examples of Non-Linear Elasticity*  
*Hysteresis during loading and unloading*

### Equations of Elasticity

### Material Symmetries

Degree of symmetry

### Linear Elasticity – Matrix Formulation

#### Anisotropy

#### Isotropy

*Limits on Elastic Parameters Values*

#### Orthotropy

#### Transversal isotropy

*Limits on Elastic Parameters Values*

### Nonlinear isotropic Hooke formulation

*Generalized Hooke's Law – Examples of problems*

Orthotropic case – A worked example

### Good to know: layered composite (transverse orthotropy)

*Transformation of Stress and Strain Components*

*Example exercises for training*

...



Minimum Master level

Minimum L-level

### Nonlinear isotropic Hooke formulation

#### Some general aspects

Why splitting volumetric and deviatoric (shearing)?

### Thermo-elasticity

### Hyperelasticity

Rubber or rubber-like Elasticity  
Terminology and some definitions  
Thermodynamics of rubber – enthalpic and entropic forces

### Some classical models

- Neo-Hookean model
- Mooney-Rivlin model
- Yeoh model
- Ogden model

Example of Rubber Elasticity In Abaqus

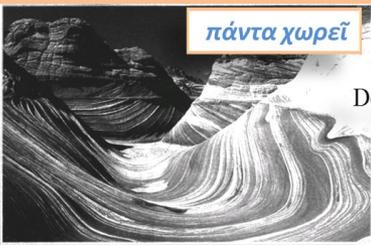
### W. Gilbert's experiment

### On thermodynamics of elastomers

### Homework

**Appendix 1**  
Stress invariants (Recall)

**Appendix 2**  
On Thermodynamics of Rubber  
Enthalpic and Entropic forces



πάντα χωρεῖ

$$De = \frac{\tau_c}{\tau_p}$$

small: fluid  
large: solid

"The mountains flowed before the Lord"  
(The Song of Deborah, Bible)  
דבורה

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Otaniemi, February 2017

# Viscoelasticity

$$\mathcal{F}(\sigma, \dot{\sigma}, \varepsilon, \dot{\varepsilon}, T, \dot{T}) = 0$$

## Content

- Experimental observations: evidence of viscoelastic behavior
- Stress relaxation at constant strain
- Creep at constant stress
- Strain-rate dependence
- Constitutive models in the rate form:
  - Maxwell model
  - Kelvin-Voight model
  - Standard linear solid model
  - Burgers model
  - Generalized Maxwell model
  - Kelvin chain model



## Reading: Textbooks

- Lemaitre and Chaboche – *Mechanics of Solid Materials*. **Chapter 4.3**
- Ottosen & Ristinmaa – *Introduction to time-dependent material behaviour*. **Chapter 14**

Lemaitre & Chaboche textbook as an e-book:  
<http://proquestcombo.safaribooksonline.com.libproxy.aalto.fi/book/physics/9781107384712>

### Primary creep:

$$\varepsilon = A(\sigma) \cdot t^{1/\beta}$$

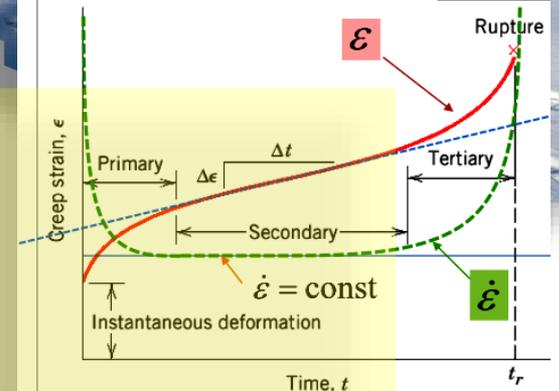
$$2 < \beta < 4$$

$$\varepsilon = A(\sigma) \cdot [t/t_{REF}]^{1/\beta}$$

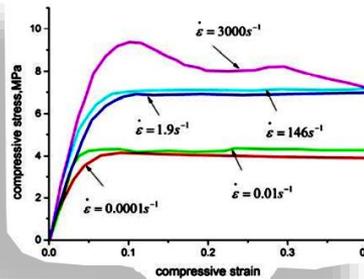
### Secondary creep:

$$\dot{\varepsilon} = K_2 \left[ \frac{\sigma}{\sigma_{Ref}} \right]^n \exp\left(-\frac{Q_c}{RT}\right)$$

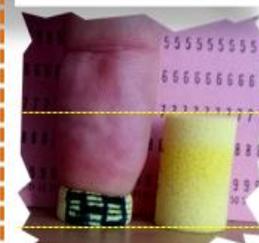
### Constant stress and temperature



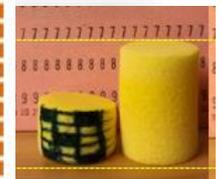
**Experimental:** Compressive responses of balsa wood at static, intermediate, and high strain rate



### Initial loading



Partial recovery



# Content

## Motivation

- Course of materials modelling in other universities
- Literature & textbooks
- Some historical notes on engineering plasticity
- Motivation: How engineering Plasticity is seen in Abaqus, Ansys, Lusas?
- Stress invariants
- Examples of Failure of Structures
- What is failure? Types of failures, failure envelopes and failure criteria

## Plasticity

### Failure hypothesis or Yield criteria

#### Plasticity Isotropic & Isothermal Rate-Independent

- Examples
- Some basic physics for Engineering Plasticity
- Plastic basic behavior in simple tension & compression

#### Modelling of uniaxial behavior in plasticity – simplified models

- Elastic-Perfectly Plastic Model
- Elastic-Linear Work-hardening model
- Elastic-Exponential Hardening model
- Ramberg-Osgood model
- **Tangent- and plastic modulus**
- **Hardening rules**
- **Elastic-plastic behaviour – cyclic loading**
  - **Worked uniaxial example – analytical & Abaqus**
- Loading history dependency and **strain hardening effects**
- Homework: Uniaxial Elastic-plastic behaviour : ex #1 ex #2
- Some examples of solved problems in Plasticity
  - ✓ **Plastic limit load and displacement-force relation in bending**

# Engineering Plasticity

Continued...

## Engineering Plasticity

### Classical theory – fundamentals

#### The three ingredient of the classical plasticity theory

- **Yield criteria**
- **Flow rule**
- **Hardening rule**

#### Yield Criteria

##### Pressure independent Yield criteria

- Tresca yield Criterion
- Von Mises yield Criterion

##### Pressure dependent Yield criteria

- Mohr-Coulomb Criterion
- Drucker-Prager Criterion
- Ottosen (1977) developed a 4-parameters failure criterion for concrete
- Hoek-Brown failure criterion
- Mohr-Coulomb yield criterion
- The Cam-Clay model

(good to know) Example of material Behavior of Clay and Silt in Otaniemi

#### Other types of failure criteria

- Maximum Principle Stress Criteria (Rankine)
- Maximum Principal strain (St. Venant)

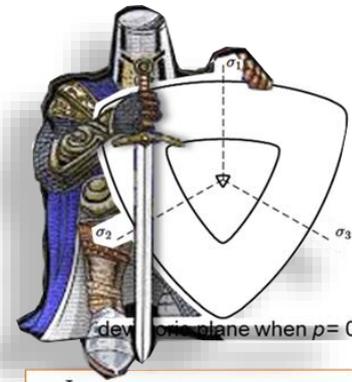
#### Anisotropic yield criteria

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## Appendices

Appendix 1: Stress invariants

Appendix 2: Recommended compulsory reading



$$A \frac{J_2}{\sigma_c} + \Lambda \sqrt{J_2} + B I_1 - \sigma_c = 0,$$

Continued

## Hardening – notions

- Hardening Rules
- Examples of simple rheological models for Rate-independent plasticity
- Examples of hardening rules in Abaqus – how they looks like?

## Flow rules

### Flow rule & Consistency condition

Plastic strain increment  
 Principle of maximum plastic work  
 Normality rule  
 Consistency Condition

### Associative and Non-associate Plasticity

**Convexity of the criterion**  
**Normality of the plastic flow**  
 Some application examples of associated and non-associated plasticity

### Incremental Stress-Strain Relationships

Example of a flow rule for isotropic hardening  
 Examples of hardening rules in Abaqus – how it looks like?

EOL

