Aalto University School of Science
Department of Mathematics and Systems Analysis Korte

MS-C1350 Partial differential equations, fall 2020

Pre-lecture assignment for Tue 27 Oct 2020

Please answer YES or NO, unless otherwise stated.

1. Assume that Ω is a bounded open set in $\mathbb{R}^n$ with a smooth boundary. Consider the Dirichlet problem

$$\begin{cases} -\Delta u = f & \text{in} \quad \Omega, \\ u = g & \text{on} \quad \partial\Omega. \end{cases}$$

   (a) $\int_\Omega |\nabla u(x)|^2 \, dx = \int_\Omega u(x) f(x) \, dx + \int_{\partial\Omega} \frac{\partial u}{\partial \nu}(x) g(x) \, dS(x)$.

   (b) $\int_\Omega f(x) \, dx = -\int_{\partial\Omega} \frac{\partial u}{\partial \nu}(x) \, dS(x)$.

   (c) If $g(x) = 0$ for every $x \in \partial\Omega$, then $u(x) = 0$ for every $x \in \Omega$.

   (d) The problem has a unique solution.

2. Assume that Ω is a bounded open set in $\mathbb{R}^n$ with a smooth boundary. Consider the Neumann problem

$$\begin{cases} \Delta u = 0 & \text{in} \quad \Omega, \\ \dfrac{\partial u}{\partial \nu} = h & \text{on} \quad \partial\Omega. \end{cases}$$

   (a) The zero function is a solution to the problem with the zero boundary values.

   (b) The zero function is the only solution to the problem with the zero boundary values.

   (c) The solution to the problem with the zero boundary values is unique up to an additive constant.

   (d) The total flow of a solution to the problem through the boundary is zero.

3. (a) The fundamental solution of the Laplace equation is a solution to the Laplace equation in the whole space.

   (b) The fundamental solution of the Laplace equation is a bounded function.

   (c) The total flux of the fundamental solution of the Laplace equation through the boundary of every sphere centered at the origin is equal to one.

   (d) The fundamental solution of the Laplace equation is a positive function when $n \geq 3$.

4. Consider the Poisson equation in the whole space.

(a) The convolution of the fundamental solution with the source term is a solution.

(b) The solution is unique.

(c) The solution is harmonic outside the support of the source term.

(d) The solution of the problem in the whole space can be used to construct a solution of the problem in subdomains.

5. Consider the following claims in $\mathbb{R}^n$ with $n \geq 2$.

   (a) $|\partial B(x,r)| = r^{n-1}|\partial B(0,1)|$.

   (b) $|B(x,r)| = r^n|B(0,1)|$.

   (c) $|\partial B(0,1)| = n|B(0,1)|$.

   (d) $|B(0,1)| = \frac{\pi^{\frac{n}{2}}}{\Gamma\left(\frac{n}{2}+1\right)}$.