Aalto University School of Science
Department of Mathematics and Systems Analysis — Korte

MS-C1350 Partial differential equations, fall 2020

Pre-lecture assignment for Tue 03 Nov 2020

Please answer YES or NO, unless otherwise stated.

1. (a) The value of any function $u \in C^2(\overline{\Omega})$ can be computed at any point $x \in \Omega$, if u and $\frac{\partial u}{\partial \nu}$ are known on $\partial\Omega$ and Δu is known in Ω.

   (b) The value of any function $u \in C^2(\overline{\Omega})$ can be computed at any point $x \in \Omega$ using Green's function of Ω, if u is known on $\partial\Omega$ and Δu is known in Ω.

   (c) If u is harmonic in the set Ω, then $u(x)$ can be computed at any point $x \in \Omega$, if $\frac{\partial u}{\partial \nu}$ is known on $\partial\Omega$.

   (d) If u is harmonic in the set Ω,then $u(x)$ can be computed at any point $x \in \Omega$ using Green's function of Ω, if u is known on $\partial\Omega$.

2. (a) Green's function of a set Ω depends on Ω.

   (b) The definition of Green's function of a set is based on a solution of a Dirichlet problem.

   (c) For a fixed $x \in \Omega$, Green's function $G(x, y)$ is a harmonic function with respect to the variable $y \in \Omega$.

   (d) For a fixed $x \in \Omega$, Green's function $G(x, y)$ is a harmonic function of $y \in \Omega \setminus \{x\}$ and $y \mapsto G(x, y)$ has zero boundary values on $\partial\Omega$.

3. (a) Green's function of the upper half-space can be written using the fundamental solution of the Laplace equation.

   (b) Green's function of the upper half-space satisfies $G(x, y) = G(y, x)$, $x, y \in \mathbb{R}^{n+1}_+$, $x \neq y$.

   (c) Green's function of the upper half-space is the same function as the Poisson kernel for the upper half-space.

   (d) The derivative with respect to the exterior unit normal of Green's function of the upper half-space is up to a sign the same function as the Poisson kernel of the upper half-space.

4. (a) The mean value formula for harmonic functions asserts that a value of a harmonic function in Ω at a point $x \in \Omega$ is equal to the average of its values over any ball $B(x, r) \subset \Omega$.

   (b) The mean value formula for harmonic functions implies that if a harmonic function in Ω is zero at some point $x \in \Omega$, then it is zero everywhere in every ball $B(x, r) \subset \Omega$.

   (c) The mean value formula for harmonic functions implies that if a nonnegative harmonic function in Ω is zero at some point $x \in \Omega$, then it is zero everywhere in every ball $B(x, r) \subset \Omega$.

(d) The mean value formula holds also for solutions to other PDE in addition to the Laplace equation.

5. (a) The weak maximum principle implies that a harmonic function attains its maximum value at an interior point.

   (b) The weak maximum principle implies that a harmonic function cannot attain its maximum value at an interior point.

   (c) The weak maximum principle implies that a harmonic function cannot attain a strict maximum value at an interior point.

   (d) The weak maximum principle implies that a harmonic function that is continuous up to the boundary attains its maximum on the boundary.