

MS-C1350 Partial differential equations, fall 2020

Pre-lecture assignment for Tue 10 Nov 2020

Please answer YES or NO, unless otherwise stated.

1. The fundamental solution of the heat equation
 - (a) is a solution to the heat equation in $\mathbb{R}^{n+1}$.
 - (b) is a solution to the heat equation in $\mathbb{R}_+^{n+1}$.
 - (c) is a bounded function in $\mathbb{R}^{n+1}$.
 - (d) converges to zero as $|x| \rightarrow \infty$ or $t \rightarrow \infty$.
2. The fundamental solution of the heat equation
 - (a) models the diffusion of a unit heat distribution concentrated at the origin.
 - (b) induces a family of good kernels that can be used approximation of the identity.
 - (c) has the zero initial values at $t = 0$.
 - (d) is a smooth function in $\mathbb{R}_+^{n+1}$.
3.
 - (a) The initial value problem for the heat equation with the initial values defined in the whole space $\mathbb{R}^n$ is called the Cauchy problem.
 - (b) The convolution of an initial value function with the fundamental solution is a solution to the Cauchy problem for the heat equation.
 - (c) If the initial value is a nonnegative continuous function and strictly positive at some point, then the solution of the Cauchy problem is strictly positive everywhere in the future.
 - (d) If the initial value is an L^1 -function, then the solution of the Cauchy problem converges pointwise to zero as time goes to infinity.
4. The solution of the Cauchy problem for the heat equation
 - (a) depends only on the initial values near the origin.
 - (b) depends only on the initial values far away from the origin.
 - (c) depends on the initial values in the whole space.
 - (d) does not depend on the initial values.
5.
 - (a) Duhamel's principle applies to the Cauchy problem for the nonhomogeneous heat equation.
 - (b) Duhamel's principle applies only for the zero initial values.
 - (c) Duhamel's principle is a process of expressing a solution of a nonhomogeneous problem as an integral of the solutions to the homogeneous problem.
 - (d) The source term is interpreted as an initial condition in Duhamel's principle.